INFLUENCE OF ELECTRON BEAM PARAMETERS ON COHERENT ELECTRON COOLING*

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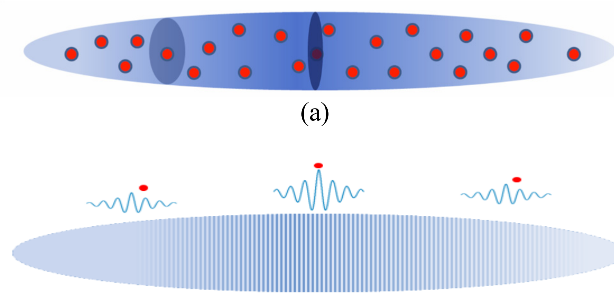
Abstract

Coherent electron cooling (CeC) promises to revolutionize the cooling of high energy hadron beams. The intricate dynamics of the CeC depends both on the local density and energy distribution of the beam. The variations of the local density (beam current) are inevitable in any realistic beam. Hence, in this paper we propose a novel method of beam conditioning. The conditioning provides compensation of effect from such variation by a correlated energy modulation. We use our analytical FEL model for an electron bunch with Gaussian line charge density and cosine-type energy variation along bunch. We analyze the phase variation between the electron density modulation at the exit of the FEL-amplifier and the ions inducing it in the modulator as a function of the peak current and the electron beam energy. Based on this analysis, electron bunch parameters for optimal CeC cooling are found numerically.

INTRODUCTION

In a CeC system, electron beam serves both as a pick-up and a kicker to provide correcting forces for the circulating ions [1]. Consequently, the performance of a CeC system relies on the properties of the electron beam. The CeC system will use bunched electron beam (for example, as in the prototype we are building for the proof of principal experiment in RHIC) and ions interacting with various portion of the electron bunch will experience different cooling (or even anti-cooling) effects due to the variation in local properties of the electron beam.

In the modulator, assuming the local temperature of the electrons does not vary along the bunch, an ion interacting with the center of the electron bunch modulates the beam density more efficiently than an ion interacting with the tail of the bunch. For example, the higher electron density at the bunch center leads to faster plasma oscillation and smaller Debye length. The dependence of FEL amplification process on the local properties of electron bunch is more complicated. Besides the line charge density variation, an electron bunch accelerated in rf cavities also has cosine-type energy variation along the bunch. Amplitude and phase of wave-packet originated from an ion and amplified by FEL depends on the electron density and energy overlapping with the wave packet. Since by design the optimal gain and phase occur at the center of the electron bunch, towards the tails, not only the amplification gain will decrease but the phase of wave packet will also change. Since the phase of the correcting force is directly connected to the phase of the density modulation, it will slip away from the optimal phase and even can lead to



(a)

(b)

Figure 1: Illustration of CeC process dependence on local electron density. (a) Dependence of Debye shielding on local electron density. Red dots represent ions and blue ellipse represents electron bunch with darker color representing higher electron density. An ion sitting at the center of the electron bunch creates a denser electron cloud due to higher background electron density and hence shorter Debye length; (b) Dependence of FEL amplification on local electron bunch current. Density modulation wave-packets (blue curves on top of the ellipse) originated from ions (red dots) located at different longitudinal locations along the bunch have different amplitude and phases due to the variation of both local electron density and energy.

anti-cooling. These effects due to the variations of local electron parameters can lead to reduction of the average cooling rate and hence need to be investigated.

In this work, we apply a FEL model recently developed for a uniform electron beam locally to study the influence of the parameter variation along the electron bunch. This requires assuming that the variations of the electron beam properties at the FEL coherence length (slippage) are small and hence can be neglected. For a specific illustration we use parameters of the prototype coherent electron cooling system built for the proof of principle experiment at RHIC. We calculate the amplitude and phase variation resulting from local density and energy variation. Based on the calculation, the rms bunch length and total charge of the electron bunch are adjusted such that the phase variation of the correcting force along the bunch is minimized. In this process the maximum amplification amplitude at the bunch center remains un-affected.

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THE MODEL

The analytical FEL model applied in this study assumes an infinite electron beam with uniform spatial density and κ-2 energy distribution [2, 3]. The amplification process in a FEL only involves electrons within a coherence length. Hence, the model is applicable to an electron bunch with non-uniform line charge density and varying energy along bunch as long as the relative variations at the FEL coherence length remain small. This assumption is correct for short wavelength FEL and relatively long electron bunches we plan using for CEC. In this section we briefly describe the analytical model and equations for calculating the amplified electron density perturbation.

After dropping the fast oscillation term, the slowly varying amplitude of the radiation field in an FEL is described by the following parabolic integro-differential equation[4]:

$$\left(\nabla_\perp^2 + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\widetilde{E}(\vec{r}_\perp, z)$$
$$= ij_0(\vec{r}_\perp)\int_0^z dz'\left[\frac{e\omega\theta_s^2}{2c^2\varepsilon_0}\widetilde{E}(\vec{r}_\perp, z') + \frac{e}{\varepsilon_0\omega}\left(\nabla_\perp^2 + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\widetilde{E}(\vec{r}_\perp, z')\right]$$
$$\times\int_{-\infty}^{\infty} e^{i\left(C+\frac{\omega}{c\gamma_z^2 \mathrm{E}_0}P\right)(z'-z)}\frac{\partial}{\partial P}F(P)dP + \frac{i\theta_s\omega}{c\varepsilon_0}e\int_{-\infty}^{\infty}\widetilde{f}_1(\vec{r}_\perp, P, 0)e^{-i\left(C+\frac{\omega}{c\gamma_z^2 \mathrm{E}_0}P\right)z}dP \tag{1}$$

where $\widetilde{E}(\vec{r}_\perp, z)$ is the complex amplitude of the radiation field, ω is the radiation frequency, C is the detuning, E_0 is the nominal electron energy, P is the electron energy deviation, θ_s is the electron deflection angle, $F(P)$ is the energy distribution function, $j_0(\vec{r}_\perp)$ is the transverse spatial distribution of the unperturbed electron beam and $\widetilde{f}_1(\vec{r}_\perp, P, 0)$ is the initial phase space density perturbation. Assuming that $j_0(\vec{r}_\perp) = j_0$ is uniform in space, the energy distribution of the background electrons is κ-2, i.e.

$$F(P) = \frac{2}{\pi q}\left(1 + \frac{P^2}{q^2}\right)^{-2}, \tag{2}$$

and the initial perturbation takes the form

$$f_1(z_0, t, x, y, P) = \frac{1}{2\pi\sigma_\perp^2\sqrt{2\pi}\sigma_t}e^{-\frac{r^2}{2\sigma_\perp^2}}e^{-\frac{(z_0-\beta ct)^2}{2\sigma_z^2}}\delta(P) \tag{3}$$

at the entrance of the FEL $z = z_0$, the electron density wave-packets at the exit of the FEL can be calculated from the following expression[3]:

$$\widetilde{j}_1(\vec{x}, t) = -\frac{ec\Gamma^3\gamma_z^4}{2\pi^2\rho}e^{-i\hat{k}_w\xi}\sum_{(i,j,k,l)}\int_{-\infty}^{\infty}d\hat{C}_{3d}e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}}I_x(\hat{r}, \hat{C}_{3d})$$
$$\times\frac{\lambda_i\left(B_{jkl} + \frac{\hat{q}^3}{\lambda_i + i\hat{C}_{3d}}\right)e^{\lambda_i\hat{z}} + \frac{i\hat{q}^3\hat{C}_{3d}}{\lambda_i + i\hat{C}_{3d}}e^{-i\hat{C}_{3d}\hat{z}}}{(\lambda_i - \lambda_j)(\lambda_i - \lambda_k)(\lambda_i - \lambda_l)}, \tag{4}$$

where $\Gamma \equiv \left[\frac{\pi j_0\theta_s^2\omega}{c\gamma_z^2\gamma I_A}\right]^{1/3}$, $\hat{\Lambda}_p \equiv \frac{1}{\Gamma^2}\left[\frac{4\pi j_0}{\gamma_z^2\gamma I_A}\right]^{1/2}$, $I_A \equiv m_e c^3/e$, $\hat{C}_{3d} \equiv \hat{C} - \hat{k}_\perp^2$, $\hat{C} \equiv C/\Gamma$, $\hat{z} \equiv \Gamma z$, $\rho = \gamma_z^2\Gamma c/\omega$, $\hat{q} \equiv q/\rho$, $\hat{r} \equiv r\sqrt{2}\gamma_z\Gamma/\sqrt{\rho}$, $\hat{k}_\perp \equiv k_\perp\sqrt{\rho}/(\sqrt{2}\gamma_z\Gamma)$, $\xi = -[\hat{z} + 2\gamma_z^2(\hat{z} - c\hat{t})]$, λ_i are roots of the polynomial equation,

$$\lambda^4 + 3(i\hat{C}_{3d} + \hat{q})\lambda^3 + \left[\hat{\Lambda}_p^2 + 3(i\hat{C}_{3d} + \hat{q})^2\right]\lambda^2$$
$$+ \left[(i\hat{C}_{3d} + 3\hat{q})\hat{\Lambda}_p^2 + (i\hat{C}_{3d} + \hat{q})^3 - i\right]\lambda = i(i\hat{C}_{3d} + 3\hat{q}),$$

$$B_{jkl} \equiv \lambda_j\lambda_k + \lambda_j\lambda_l + \lambda_k\lambda_l + i\hat{C}_{3d}(\lambda_j + \lambda_k + \lambda_l) - \hat{C}_{3d}^2,$$

$$I_x(\hat{r}, \hat{C}_{3d}) \equiv \int_0^\infty e^{-\left(\frac{\hat{\sigma}_\perp^2}{2} - i\xi\right)x}e^{\frac{-\hat{\sigma}_z^2}{2}(x + \hat{C}_{3d} - \hat{k} - \hat{k}_w)^2}J_0(\hat{r}_\perp\sqrt{x})dx,$$

and the summation is over the cyclic permutation of the four indices. The double integral in eq. (4) is difficult to be carried out analytically and numerical integration has to be applied in order to proceed further.

APPLICATION TO BUNCHED ELECTRON BEAM

If the variation of electron line charge density and energy along bunch are small over a FEL coherence length, eq. (4) can be applied to obtain the amplified electron density wave-packet with the local electron parameters being used for the calculation. Consequently, all variables involving $j_0(\tau)$ and $\gamma(\tau)$ become function of the longitudinal location τ, with $\tau = 0$ being the bunch center. Although analytical solution for initial perturbation due to Debye screening in infinite electron plasma has been found [5], for simplicity, we will use eq. (3) to approximate the initial perturbation with the local transverse and longitudinal Debye length being taken as $\sigma_\perp(\tau)$ and $\sigma_z(\tau)$ respectively.

The contents of this section are organized as follows. We first investigate the influence of cosine energy variation by taking a uniform line charge density in the calculation. In the second subsection, the effects of density variation are studied without energy variation. Then we take both density and energy variation into account and calculate the amplitude and phase of the wave-packets as a function of the longitudinal location, τ. Based on the calculation, the bunch charge and bunch length are adjusted such that optimal cooling condition is achieved.

Influence of Energy Variation

Due to the wave form of the rf voltage, after acceleration, the energy of the electron bunch varies as

$$\gamma(\tau) = \gamma_0 \cos(2\pi f_{rf}\tau) \quad , \tag{5}$$

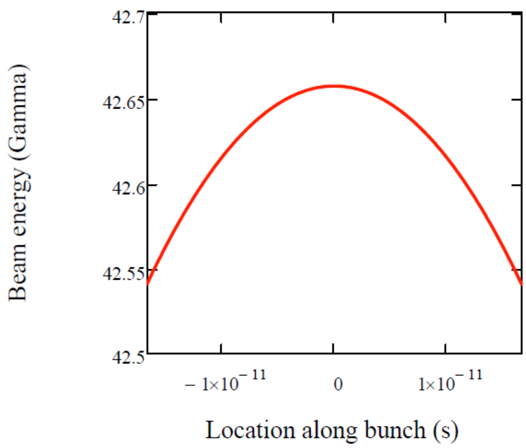

Figure 2: Energy variations along the electron bunch in prototype CeC system with 704MHz accelerating rf cavities.

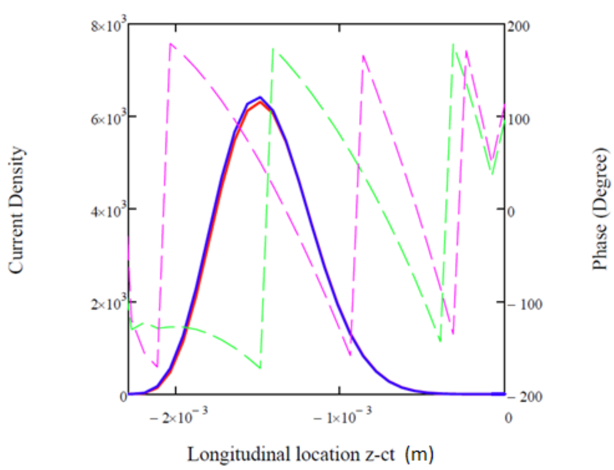

Figure 3: Amplitude and phase of wave-packets at the exit of a FEL amplifier with e-beam energy dependence shown in Fig. 2. . The abscissa is $l_{FEL} - c \cdot (t - t_0)$ with l_{FEL} being the length of the FEL amplifier, t being the observation time and t_0 being the time when the perturbation arrives at the entrance of the FEL. The first ordinate is the current density in units of $\frac{e \cdot c \cdot \Gamma^3 \gamma_z^5}{2\pi^2 \rho}$. The second ordinate is the phases of wave-packets in degrees. The red solid curve and green dash curve are the amplitude of a wave-packet originated from a perturbation located at the center of the electron bunch. The blue solid curve and purple dash curve are amplitude and phase of a wave-packet induced by a perturbation 5 mm away from the center.

where $\gamma(\tau)$ is the instantaneous average energy at location τ and f_{rf} is the frequency of the acceleration rf system. Figure 2 shows the electron bunch energy variation after acceleration with the 704 MHz rf system of BNL prototype CeC system. Taking a uniform current density of $8\times10^7\, A\cdot m^{-2}$, the amplitudes and phases of the electron current density wave-packets as calculated from eq. (4) are shown in fig. 3. Figure 4 shows the amplitude and phase of the wave-packets at the observation time $= t_0 + (l_{FEL} + 1.46mm)/c$. As shown in fig. 3 and fig. 4, the dependence of the amplitude of wave-packet due on the beam energy is weak but the phase dependence is strong. This results from the resonant wavelength dependence on the beam energy.

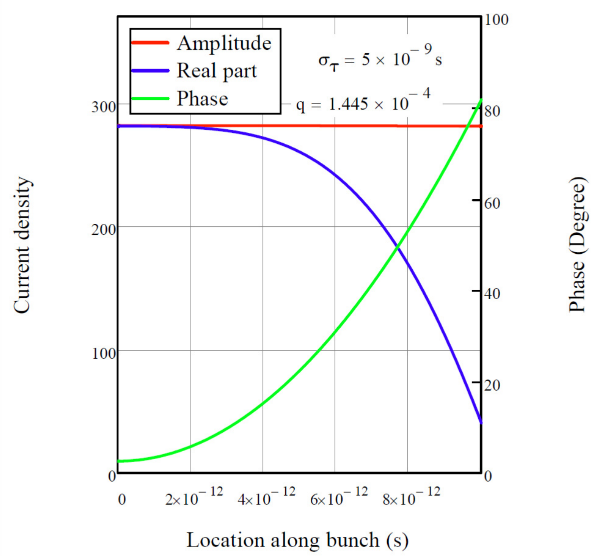

Figure 4: amplitude and phase variation of the wave-packets due to energy variation as a function of longitudinal location along the bunch. The observation time is chosen at $t = t_0 + (l_{FEL} + 1.46mm)/c$. The abscissa is location along bunch in units of second and the ordinate is current density in unit of $A \cdot m^{-2}$.

Since the peak of the wave- packets is usually a few tens of resonant wavelengths away from the initial perturbation, the phase difference accumulates for a few tens of periods leading to a substantial phase variation. On the other hand, the amplitude depends on the energy through the 1-D gain parameter, Γ . As Γ inversely proportional to energy, the relative difference in amplitude due to energy variation can be estimated by

$$\frac{\Delta e^{\Gamma z}}{e^{\Gamma z}} = z\Gamma \frac{\Delta E}{E} . \tag{6}$$

For the parameters that we considered, eq. (6) is around 4%.

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Influence of Density Variation

To investigate the influence of density variation, we consider a Gaussian bunch with constant energy along bunch. Taking 5ps of rms bunch length, $8\times10^{7}\,A\cdot m^{-2}$ of peak current density as shown in fig. 5, and using the local Debye length as the initial width of the density modulation, the wave-packets calculated from eq. (4) is shown in fig. 6. As shown in fig. 6 and fig. 7, local density variation significantly changes both the amplitude and phase of the amplified wave-packet. Since both the Debye length and 1-D gain length depend on local current density and their effects to the wave-packet amplitude are in the same direction. As shown in fig. 7, the amplification process ceases towards the tail of the electron bunch when initial seeding is off from the bunch center by one sigma or more.

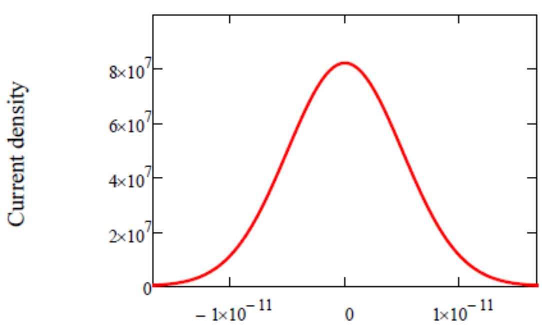

Figure 5: electron current density along bunch. The abscissa the location along bunch in unit of second and the ordinate is the current density in unit of $A\cdot m^{-2}$.

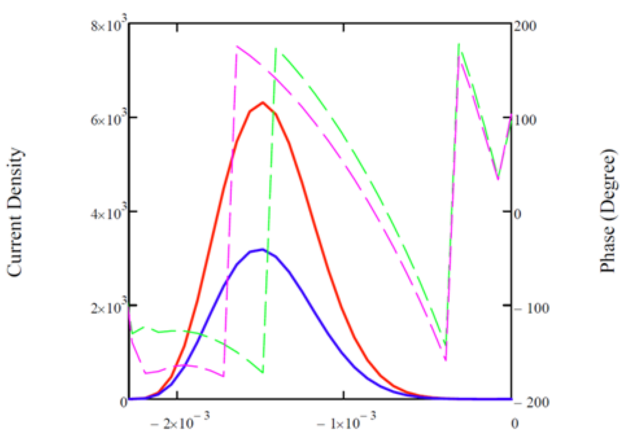

Figure 6: amplitude and phase of wave-packets at the exit of a FEL amplifier with density varying electron beam. The abscissa and ordinate are the same as fig. 3. The red solid curve and green dash curve are the amplitude of a wave-packet originated from a perturbation sitting at the center of the electron bunch. The blue solid curve and purple dash curve are amplitude and phase of a wave-packet induced by a perturbation 1 mm away from the center.

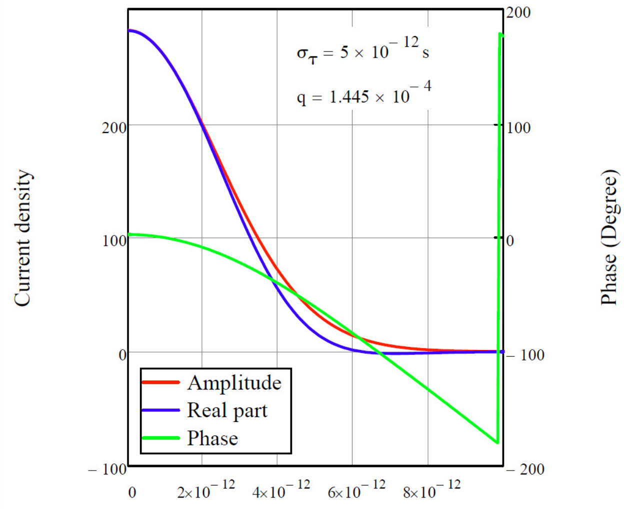

Figure 7: Amplitude and phase variation of the wave-packets due to local density variation as a function of longitudinal location along the bunch. The observation time is chosen at $t = t_0 + (l_{FEL} + 1.46mm)/c$. The abscissa and the ordinate is the same as in fig. 4.

Since the phase velocity of the wave-packet depends on the local current density, at the exit of the FEL amplifier, the phase advance of the wave-packets depends on the location along the bunch. More importantly, the phase variation of the density variation has the opposite sign compared with that originated from the cosine-like energy variation (see fig. 4 and fig. 7). The fact that the energy variation and the density variation work against each other provides us with the possibility of beam conditioning, i.e. minimizing the phase variation of the wave packets along the electron bunch to achieve optimal cooling.

Optimum Electron Bunch Parameters

Let's now consider an electron bunch with both cosine energy variation and Gaussian current density distribution. The resulting effect on the density modulation wave-packets is shown in fig. 8. As expected, the phase variation is significantly reduced compared with the beam without conditioning.

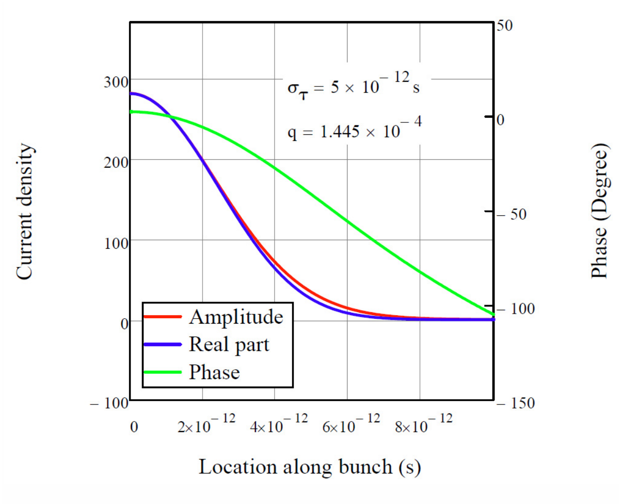

Figure 8: Amplitude and phase variation of the wave-packets for the conditioned e-beam The observation time and axis are the same as fig. 4.

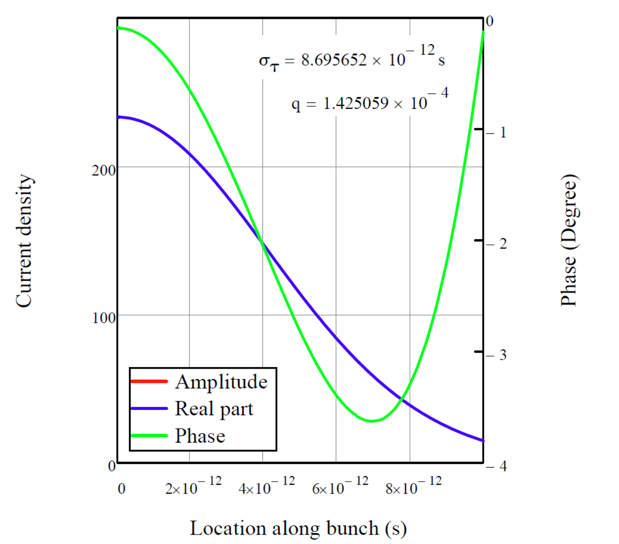

Figure 9: Amplitude and phase variation of the wave-packets after optimizing the rms bunch length and bunch charge. The observation time and the axis are the same as fig. 4.

In order to further reduce the phase variation, we optimized the bunch length to increase the amplitude of the energy modulation sufficiently such that it cancels the influence from local density variation. This can be achieved by increasing the bunch charge without increasing peak current. Fig. 9 shows the amplitude and phase variation of the wave-packet for a conditioned e-beam with the rms bunch length increased from 5 ps to 8.7 ps (and bunch charge being increased from 1 nC to 1.67 nC).

SUMMARY

As we mentioned in previous sections, our model is applicable only when the electron parameters do not vary significantly at the scale of FEL coherence length. In the prototype CeC system, the rms bunch length is 1.5mm and the FEL wavelength is 13 μm, which are only a factor of 100 apart. Hence, the applicability of our model may be marginal. Nevertheless, the analysis provides qualitative understandings of the influence of local electron parameters variations. Furthermore, Genesis simulations have shown similar behavior as disclosed by our analytical analysis [6].

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