PROGRESS OF EMITTANCE COUPLING CORRECTION AT THE SPring-8 STORAGE RING

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Abstract

The vertical beam spread, or the emittance coupling, is one of the most important parameters for the high brilliance light source storage ring. By the precise alignment of the magnets and the proper COD correction, at the commissioning phase of the SPring-8 storage ring we succeeded in achieving the small coupling a few tenth % without correction. However, the coupling had grown large with the years, so recently we have corrected it and recovered the initial performance. The scheme of the coupling correction at the SPring-8 storage ring is the global one, which is based on the perturbation theory with single resonance approximation. In the beginning of the correction the coupling was corrected by means of minimizing the vertical beam size. Then the performance of the coupling correction has been further improved by changing the scheme to minimizing the betatron coupling mode in the vertical oscillation of the horizontally kicked beam. This result implies that the higher order coupling contributes to the emittance coupling, which could be corrected by the higher skew multi-pole magnet. The present status of the coupling correction at the SPring-8 storage ring will be reported.

PRELIMINARIES

Perturbation Theory with Single Resonance Approximation for Linearly Coupled System

First we review the theoretical background of the coupling correction, i.e. the perturbation theory of linear coupling resonance with the single resonance approximation [1, 2, 3, 4, 5]. The Hamiltonian of the linearly coupled system is given by

$$H = H_0 + H_1, \tag{1}$$

where H_0 is the unperturbed Hamiltonian for the betatron motion

$$H_0 = \frac{1}{2}\left[p_x^2 + p_y^2 + G_x(s)\,x^2 + G_y(s)\,y^2\right], \tag{2}$$

and H_1 is the perturbing Hamiltonian giving the coupling between the transverse oscillations

$$H_1 = K(s)\,xy. \tag{3}$$

Here $G_{x,y}$'s are the coefficients of the restoring potentials and K is that of the coupling one.

The source of the perturbing Hamiltonian is random error of optics functions so that the coupling effect distorts the beam behavior very little except for the case that the errors resonates to the beam motion. On the other hand, in the case near resonance the small distortion can act coherently on the beam and give the significant effect. Then, near the difference resonance the perturbing Hamiltonian H_1 can be approximated as (i.e. single resonance approximation)

$$H_1 = \frac{\pi}{L}\left(C a_x \bar{a}_y e^{2\pi i\Delta s/L} + \bar{C}\bar{a}_x a_y e^{-2\pi i\Delta s/L}\right), \tag{4}$$

where C is the strength of the coupling driving term (CDT) for differential resonance

$$C = \frac{1}{2\pi}\oint ds\sqrt{\beta_x(s)\,\beta_y(s)}K(s)\,e^{i\left[\phi_x(s)-\phi_y(s)-\frac{2\pi\Delta s}{L}\right]} \tag{5}$$

with $\Delta = \nu_x - \nu_y - q$ the distance from resonance with an integer q and L the circumference as well as the betatron functions $\beta_{x,y}(s)$, the phases $\phi_{x,y}(s) = \int_0^s d\tilde{s}/\beta_{x,y}(\tilde{s})$, and the tunes $\nu_{x,y}$. Note that the optics parameters are those of the unperturbed system. Here $a_{x,y}$'s are the amplitude functions of the betatron motion, which give the solution of the equation of motion:

$$z = a_z(s)\,w_z(s) + c.c., \tag{6}$$

$$p_z = a_z(s)\,w_z'(s) + c.c., \tag{7}$$

where $c.c.$ denotes complex conjugate of the preceding term and w_z ($z = x$, or y) the solution of the unperturbed betatron motion

$$w_{x,y}(s) = \sqrt{\frac{\beta_{x,y}(s)}{2}}e^{i\phi_{x,y}(s)}. \tag{8}$$

The eigen mode tunes of the coupled betatron motion are given by

$$\nu_{1,2} = \frac{1}{2}\left(\Delta \pm \sqrt{\Delta^2 + |C|^2}\right). \tag{9}$$

With based on the perturbation theory and ignoring the vertical dispersion, we can derive the approximated formula for the equilibrium beam profile parameters, i.e. the sizes $\sigma_{x,y}$ and the tilt angle θ [1, 2, 3]:

$$\sigma_x^2 = \frac{\Delta^2 + \frac{1}{2}|C|^2}{\Delta^2 + |C|^2}\beta_x\epsilon_{x0}, \tag{10}$$

$$\sigma_y^2 = \frac{\frac{1}{2}|C|^2}{\Delta^2 + |C|^2}\beta_y\epsilon_{x0}, \tag{11}$$

$$\tan 2\theta = \frac{\Re(C)\,\Delta\sqrt{\beta_x\beta_y}}{\Delta^2\beta_x + \frac{1}{2}|C|^2(\beta_x - \beta_y)}, \tag{12}$$

where ϵ_{x0} is the natural emittance.

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Tune Survey

At the SPring-8 storage ring we frequently performed the tune survey to check the beam performance. The nominal working point of the storage ring is $(40.14, 18.35)$, so the nearest neighbor difference resonance is $\nu_x - \nu_y = 22$. Increasing the horizontal tune with keeping the vertical one constant, we perform the tune survey so as to cross the resonance. Figure 1 shows the measurement results, i.e. the difference of the tunes, the beam sizes, and the tilt angle. At the SPring-8 storage ring the parameters of the beam profile are measured by the two-dimensional visible light interferometer [6] and the x-ray beam profile monitor [7].

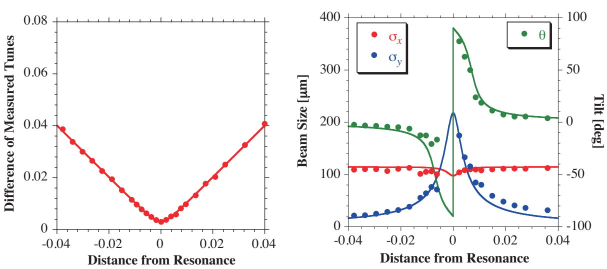

Figure 1: Tune survey at the SPring-8 storage ring.

Equation (9) implies the minimum tune separation gives the strength of the CDT, so in this measurement we find $|C| = 0.0030$. The solid lines in right plot of Fig. 1 indicate the estimated beam parameters expected by the formula (10)-(12) with the above CDT. From the result we can be convinced that the perturbation theory fairly well describes the beam parameters of the coupled system.

COUPLING CORRECTION

Indirect Scheme through the Vertical Beam Size

Since the perturbation theory with a single resonance approximation well stands up in the coupled betatron motion at the SPring-8 storage ring, we can control the coupling by means of the properly arranged skew quadrupole magnets. There are 54 skew quadrupole magnets in the SPring-8 storage ring, which we use to correct the betatron coupling and the vertical dispersion. The latter correction is performed after the betatron coupling correction under the constraint of not exciting the linear coupling resonances near the operation point any more [8].

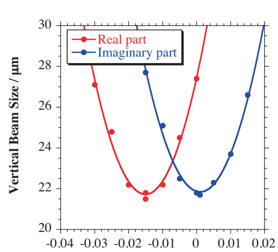

Figure 2: Coupling correction through vertical beam size.

The betatron coupling correction is carried out in accordance with the response of the measured vertical beam size. Using Eq. (5), we can independently control the real and imaginary parts of the CDT by means of the appropriate setting of the skew quadrupole magnets. Figure 2 shows the tuning process of the betatron coupling, which reduces the vertical beam size from 27.4μm without correction to 21.7μm. The strengths of real part and imaginary part of the CDT induced by the corrector skew quadrupole magnets are -0.015 and 0.001, respectively.

The tune survey given in the previous subsection is done under the condition of correcting the betatron coupling by the present scheme. As a result, by means of the present correction scheme, we reduce the strength of the CDT from 0.015 to 0.003, which is not so small in spite of after the correction.

Direct Scheme by Using Turn-by-turn Beam Position Monitor

By using the turn-by-turn beam position monitor (BPM) for the purpose of measuring the dynamic aperture and the resonance excitation, we observe the beam oscillation after kicked by the injection bump magnets so as to give initial amplitude. The left and right plots of Fig. 3 indicate the Fourier transforms of the horizontal and the vertical oscillations, respectively. This measurement were conducted with changing the amplitude, i.e. the bump height.

In the horizontal oscillation, in addition to the horizontal tune component, the second harmonics induced by the sextupole magnetic field is observed. On the other hand, in the vertical oscillation, the horizontal tune component induced by the linear coupling, the higher order couplings induced by the normal and skew sextupole magnetic fields, and the faint one by the skew octupole magnetic field are observed as well as the vertical tune component.

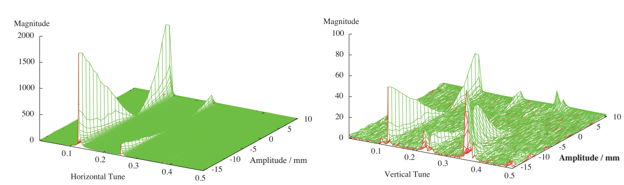

Figure 3: Fourier transform of horizontally kicked beam oscillation.

The measurements were done under the condition of correcting the linear coupling by the previous indirect scheme. The horizontal tune mode in the vertical oscillation is very proportional to the horizontal betatron oscillation, which implies that the linear coupling remains after the correction. This fact agrees with the remaining CDT mentioned in the preceding subsection. Hence we change the correction scheme from that based on the response of the vertical beam size to the one based on the response of the horizontal tune component in the vertical oscillation induced by the horizontal kick, i.e. the linear coupling mode.

Figure 4 shows the process of correcting the linear coupling by the latter, or direct scheme. The plots in the upper line represent the response to the real part of the CRD, and those in the lower line to the imaginary part. The plots in the left (right) column indicate the horizontal (vertical) oscillation. The amplitude of the horizontal betatron oscillation E_x and the amplitude E_y of the coupling mode are related to the strength of the CRD just like the beam sizes, respectively [1, 4, 5]:

$$E_x^2 = \frac{\Delta^2 + \frac{1}{2}|C|^2}{\Delta^2 + |C|^2} E_0^2, \quad E_y^2 = \frac{\frac{1}{2}|C|^2}{\Delta^2 + |C|^2} E_0^2, \qquad (13)$$

with the amplitude of the unperturbed betatron oscillation E_0. Although the horizontal oscillation scarcely changes, the horizontal tune component in the vertical oscillation varies with corresponding to the strength of the CDR. Thus the optimized strength of the CDR makes the linear coupling mode vanishing.

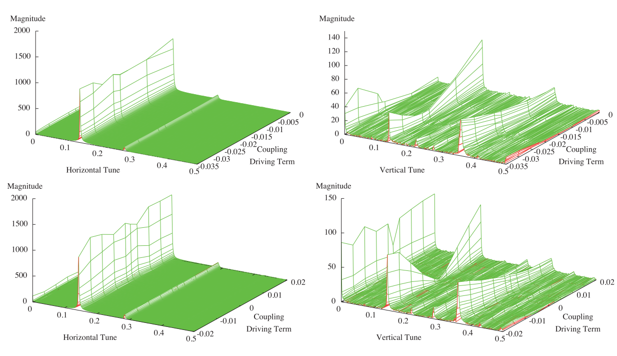

Figure 4: Fourier transform of beam oscillation in coupling correction.

For the purpose of comparing the effect of the correction schemes we perform the tune survey, whose result is shown in Fig. 5. The left plot in Fig. 5 indicates the difference between the measured betatron (eigen mode) tunes, and the right the vertical beam size in the neighborhood of the linear coupling resonance. The measured strengths of the CRD are 0.0006, 0.0030 and 0.0178 for the direct and the indirect schemes and the case without the correction, respectively. By changing the correction scheme from the indirect to the direct one, we improve the effect of the coupling correction by five times.

STATUS OF COUPLING CORRECTION

After the linear betatron coupling correction, we carry out the vertical dispersion correction [8] by using the same skew quadrupole magnets with the constraint of not exciting the linear coupling resonance. The residual vertical dispersion before the vertical dispersion correction is about 4 or 5 mm r.m.s., which in usual is reduced to from 1.2 mm r.m.s. to 1.8 mm r.m.s..

Figure 6 shows the beam profiles at the source point of the bending magnet in the SPring-8 storage ring before

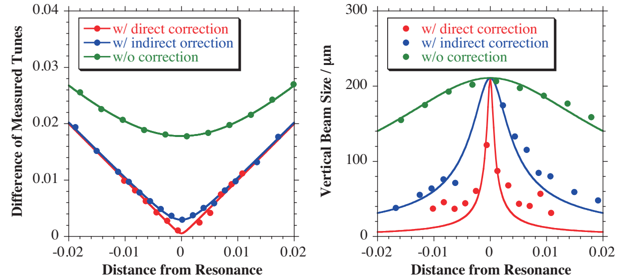

Figure 5: The tune differences and the vertical beam sizes near the linear coupling resonance for various coupling correction schemes.

(left) and after (right) the correction measured by the x-ray profile monitor [7]. The beam profile parameters are listed in Table 1. The estimated emittance coupling ratio is about 1 % and 0.3 % for the cases without and with the correction, respectively. The linear betatron coupling is almost completely corrected to disappear by means of the direct scheme, so we expect the remaining vertical beam spread might come from the higher order coupling resonances, which in practice are observed as shown in Fig. 4. To correct the higher order coupling, the plan of introducing the skew multi-pole magnets is in progress.

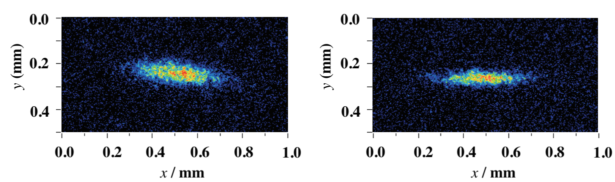

Figure 6: The beam profile before (left) and after (right) the correction.

Table 1: Beam profile parameters at the source point of the bending magnet in the SPring-8 storage ring.

	w/o correction	w/ correction
Horizontal beam size	115.3 μm	113.3 μm
Vertical beam size	27.5 μm	18.4 μm
Tilt angle	- 6.4 degree	$\sim$ 0 degree

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