# STRAIGHT SCALING FFAG EXPERIMENT* 

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## Abstract

Straight scaling FFAG experiment has been done at Ky oto University Research Reactor Institute (KURRI). Details and results are presented here.

## INTRODUCTION

Scaling FFAG accelerators can be designed not only in a ring shape, but also with no overall bend. It is indeed possible to build straight scaling FFAGs, with a guide field following in the mid-plane [1, 2]

$$
\begin{equation*}
B_{z}=B_{0} e^{m\left(x-x_{0}\right)} \mathcal{F}(s), \tag{1}
\end{equation*}
$$

with $x$ and $s$ the horizontal and longitudinal coordinates, respectively, $B_{0}=B_{z}\left(x_{0}\right), \mathcal{F}$ an arbitrary function of $s$ and $m$ the normalized field gradient

$$
\begin{equation*}
m=\frac{1}{B} \frac{d B}{d x} \tag{2}
\end{equation*}
$$

To verify and study this new field law, an experiment has been done at Kyoto University Research Reactor Institute. It measured the horizontal phase advance for two different energies in a straight scaling FFAG prototype cell. In order to do it, the Courant-Snyder parameters as well as position and angle of the beam are measured at the exit of the Straight Cell when it is launched off its reference trajectory (corresponding term to a closed orbit for circular machines). The linear transfer matrix of the line is

$$
\binom{x_{1}}{x_{1}^{\prime}}=\left(\begin{array}{cc}
\sqrt{\frac{\beta_{1}}{\beta_{0}}}\left(\cos \psi+\alpha_{0} \sin \psi\right) & a_{12} \\
\frac{\left(\alpha_{0}-\alpha_{1}\right) \cos \psi-\left(1+\alpha_{0} \alpha_{1}\right) \sin \psi}{\sqrt{\beta_{1} \beta_{0}}} & a_{22}
\end{array}\right) \cdot\binom{x_{0}}{0}
$$

with $x_{1}$ and $x_{1}^{\prime}$ the distance and the angle to the reference trajectory at the exit of the cell, respectively. $x_{0}$ is the distance to the reference trajectory at the entrance of the cell. No angle with the reference trajectory is added at the entrance of the cell. $\alpha_{0}$ and $\beta_{0}$ are the Courant-Snyder parameters at the entrance of the cell, while $\alpha_{1}$ and $\beta_{1}$ are the Courant-Snyder parameters at the exit of the cell. The linear phase advance $\psi$ is then calculated by:

$$
\begin{equation*}
\tan \psi=\frac{\left(\alpha_{0}-\alpha_{1}\right) x_{1}-\beta_{1} x_{1}^{\prime}}{\left(1+\alpha_{0} \alpha_{1}\right) x_{1}+\alpha_{0} \beta_{1} x_{1}^{\prime}} \tag{3}
\end{equation*}
$$

[^0]
## LAYOUT OF THE EXPERIMENT

For the Straight Experiment, the injector of the 150 MeV FFAG complex is used, a Linac delivering H- particles of 7 MeV and 11 MeV in the KURRI FFAG complex. A schematic view of the setup is presented in Fig. 1.

Two collimators are installed at the entrance of the Straight Cell to set the emittance and the Courant-Snyder parameters. The size of the collimators and the distance between them indeed determine the Courant-Snyder parameters and the emittance. The emittance $\epsilon$ is a function of the collimator size $a$ and the distance between the two collimators $b$ by

$$
\begin{equation*}
\epsilon_{100 \%}=\frac{a}{2} \cdot \frac{a}{b}=\frac{a^{2}}{2 b} \tag{4}
\end{equation*}
$$

At the center of the collimator system, we have

$$
\begin{align*}
& \alpha=0 \\
& \beta=\frac{\left(\frac{a}{2}\right)^{2}}{\epsilon_{100 \%}} . \tag{5}
\end{align*}
$$

The size of the collimators has however to be small compared with the incident beam size. The collimator size is then 2 mm and the distance between the two collimators is set to 1530 mm . The $100 \%$ emittance is defined as 9 times the rms emittance. We then have at the entrance of the system

$$
\begin{align*}
& \epsilon_{100 \%}=1.3 \pi \mathrm{~mm} . \mathrm{mrad} \\
& \epsilon_{r m s}=\frac{\epsilon_{100 \%}}{9}=0.14 \pi \mathrm{~mm} . \mathrm{mrad}  \tag{6}\\
& \beta=0.77 \mathrm{~m} \\
& \alpha=0
\end{align*}
$$

A Courant-Snyder measurement system is added after the Straight Cell to determine the exit parameters of the beam. This system is similar to the emittance measurement "slit-grid method" [3], but the grid is replaced by a fluorescent screen. A slit is used to measure the horizontal angle of the beam, and the slope of the line $x^{\prime}$ vs. $x$. When the slit is removed, the size of the beam on the screen fitted with a gaussian gives access to the beta value, and the center of the gaussian is taken as the position of the beam.

Since each energy has a different reference trajectory, the cell is able to move horizontally to match the different trajectories without changing the magnetic field. Bellows before and after the cell allows the vacuum chamber to move along with the magnets without breaking the vacuum.

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Figure 1: Schematic view of the Straight Experiment.

## STRAIGHT CELL DESIGN AND FIELD MEASUREMENT

The prototype for the straight scaling FFAG cell is a FDF triplet. The parameters of the cell are summarized in Table 1. The pole shape has been designed with TOSCA code [4]. A "C" type magnet has been chosen to have an easier access to the pole for field measurement.

Table 1: Parameters of the Straight Cell.

| Type | FDF |
| :--- | :---: |
| $m$-value | $11 \mathrm{~m}^{-1}$ |
| Total length | 4.68 m |
| Length of F magnet | 15 cm |
| Length of D magnet | 30 cm |
| Max. B Field | 0.35 T |
| Horizontal phase advance | 87.7 deg. |
| Vertical phase advance | 106.2 deg. |

Field measurement has been conducted with a 3dimensional magnetic probe in order to verify the manufacturing. The resulting measured field has a good agreement with TOSCA field (see Fig. 2), with a difference less than $1 \%$. It validates the manufacturing.


D


Figure 2: Difference of the vertical magnetic field in the mid-plane in Gauss between the TOSCA field map and measured field map of the magnets F1 (top left), F3 (top right) and D (bottom).

## TRACKING IN FIELD MAP

Tracking step by step using Runge Kutta integration has been done in TOSCA and measured field maps. The local horizontal phase advances in the Straight Cell are plotted in Fig. 3. the variation in the local $m$-value (see Fig. 4) is so small that the variation in the horizontal phase advance is negligible. The horizontal betafunctions in the system are shown in Fig. 5.


Figure 3: Horizontal local phase advances versus kinetic energy in the Straight FFAG line with TOSCA map (plain) and with measured map (dotted).

## EXPERIMENT

The experiment consists of 6 series of data, taken at a different magnet position. The first position magnet corresponds to the reference trajectory position (for 11 MeV and 7 MeV ), from which the Courant Snyder parameters are extracted. Then the position +10 mm and the position -10 mm (for 11 MeV and 7 MeV ) gives the angle and position of the beam off the reference trajectory. The second order is canceled by adding the 2 results. To have more statistical data, Courant Snyder parameters are extracted from all magnet positions. Data consist of pictures of the fluorescent screen with different slit positions. The experimental results are presented in Table 2. For 11 MeV the resulting 05 Beam Dynamics and Electromagnetic Fields


Figure 4: Local $m$-value versus horizontal abscissa in the Straight FFAG line with TOSCA map (plain) and with measured map (dotted).


Figure 5: Horizontal betafunctions in the Straight FFAG line with TOSCA map (plain) and with measured map (dotted).
phase advance $\psi_{\text {exp }}(11 \mathrm{MeV})$ is then

$$
\begin{equation*}
\psi_{\exp }(11 \mathrm{MeV})=87.5 \mathrm{deg} \tag{7}
\end{equation*}
$$

and for 7 MeV the phase advance $\psi_{\exp }(7 \mathrm{MeV})$ is

$$
\begin{equation*}
\psi_{\exp }(7 \mathrm{MeV})=86.1 \mathrm{deg} \tag{8}
\end{equation*}
$$

The final results are summarized in Table 3. A difference of $1.6 \%$ between the two energies is then found, which is in the acceptable range. There is no difference of phase advance with tracking in the 11 MeV case, and it is $1.7 \%$ for the 7 MeV case. This good agreement clarifies the straight scaling law.

Table 2: Results of the position $\left(x_{1}\right)$ and angle $\left(x_{1}^{\prime}\right)$, Courant Snyder parameters ( $\beta_{1}$ and $\alpha_{1}$ ) in the Straight Scaling FFAG experiment.

|  | $x_{1}(\mathrm{~mm})$ | $x_{1}^{\prime}(\mathrm{mrad})$ | $\beta_{1}(\mathrm{~m})$ | $\alpha_{1}$ |
| :--- | :---: | :---: | :---: | :---: |
| 11 MeV | 2.0 | -2.4 | 17.7 | -1.5 |
| 7 MeV | 1.8 | -2.1 | 11.7 | -1.0 |

The errors, presented in Table 3, are dominated by the beta function error, due to the fluctuation of the beam size. 05 Beam Dynamics and Electromagnetic Fields


Figure 6: Picture of the straight scaling FFAG magnets prototype.

Table 3: Calculated experimental phase advance ( $\psi_{\text {exp }}$ ) in the Straight Scaling FFAG experiment and tracking phase advance $\psi_{t r a c k}$ in TOSCA field map.

|  | $\psi_{\exp }(\mathrm{deg})$ | $\psi_{\text {track }}(\mathrm{deg})$ |
| :--- | :---: | :---: |
| 11 MeV | $\mathbf{8 7 . 5} \pm 3.3$ | 87.5 |
| 7 MeV | $\mathbf{8 6 . 1} \pm 9.6$ | 87.6 |

## SUMMARY

To verify the straight scaling field law, an experiment has been conducted at KURRI. Design of the prototype of the cell has been done. The magnets have been manufactured, and field measurement has been done. Tracking has been computed in TOSCA field map and in measured field map, with similar results. Horizontal phase advance has been measured in the prototype cell, clarifying the straight scaling law.

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