EXTRACTION OF LIE MAP FROM REALISTIC 3D MAGNETIC FIELD DATA*

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Abstract

We present a method to extract the Lie map from the realistic magnetic field data of arbitrary magnet. The multi-particle tracking results are fitted into Taylor maps, which can be factorized into a Lie map by using Dragt-Finn's method [1]. This method is validated by comparing against the COSY-infinity code [2] for a soft-edge quadrupole model. Examples of extracting the symplectic maps for the SPEAR and the NSLS-II dipoles are shown. Good agreements have been achieved in comparing the Lie map tracking results against the direct field integration.

INTRODUCTION

In designing and modelling accelerators, some simplified magnet models have been used. But it is desirable to have the accurate and realistic transfer maps help us to better understand beam dynamics behaviour and to improve machine performance. Actually, the realistic field data of magnets can be easily obtained by various existing three-dimensional magnetic field codes, like OPERA. But the computation of high-order transfer maps based on this data is not easy. Many efforts have been put on solving such problems. One usually starts from fitting the magnet field into a closed analytical formula, and then calculates its high derivatives with respect to the reference orbit. But the fitting process is not easy either, and sometimes time-consuming. In this paper, we present a method to extract the transfer map by using direct particle tracking through the realistic magnet field instead of fitting the field. Some examples will be given to illustrate the application of this method.

METHOD OF EXTRACTION LIE TRANSFER MAP

Consider a particle state represented by a vector with 3 pairs canonical coordinates in phase space

$$X = \left[x, p_x, y, p_y, -ct, \delta\right]^T.$$

The transfer map of a magnet brings the coordinates of the particles from its entrance to the exit. The map must satisfy the symplecticity condition because it is a Hamiltonian system. And Lie map is a special map which can maintain the symplecticity at any order, which is widely used in accelerator physics community now.

Our method of extracting the Lie map from a realistic field map can be summarized in the following 5 steps:

Step 1: Obtaining 3D field map in the fine grid from calculation or measurement. The field map can be in different coordinates. We are using the Cartesian system. The only requirement for the field map is that the grid's dimensions should be fine enough for implementing 3D data interpolation in particle tracking.

Step 2: Calculating the reference orbit by tracking nominal particle. For some magnets, the reference orbit is obvious, i.e. it is a straight line for a quadrupole with fringe field. But in some cases it is not so straightforward, i.e. a dipole with fringe field or transverse gradient. Therefore the realistic reference orbit has to be determined by direct tracking.

Step 3: Tracking multiple particle trajectories. After obtaining the reference orbit, we need to track multi-particles with different initial conditions with respect to the reference orbit. Because high-order transfer map is usually used to determine and optimize dynamic aperture, particles can be populated evenly in an area which is large enough to cover the dynamic aperture in phase space. The accuracy of trajectories is important, so one has to use a fine longitudinal step-size. The tracking can be implemented in the Cartesian coordinate, and particle coordinates at the entrance and the exit need to be converted into the canonical coordinates $(x, p_x = \frac{x'}{\sqrt{1+x'^2+y'^2}}, \cdots)$ with respect to the reference orbit. The tracking can be implemented by the non-symplectic integrators, e.g. Runge-Kutta integrator. This is because the transfer map will be symplecified in the step 5.

Step 4: Fitting tracking results into Taylor maps. From multi-particle tracking result, the particle final coordinates at the exit can be fitted into 6 Taylor maps of initial coordinates at the entrance:

$$X_{1,i} = \sum a_{ij} X_{0,j} + \sum b_{ijk} X_{0,j} X_{0,k} + \cdots$$

Here subscript 0 and 1 refer to the coordinates at the entrance and exit respectively, and *i, j* and *k* are coordinate index. The order of fitted Taylor maps depends on the convergence of particle tracking, the number of particles, and the order of Lie map to be extracted, etc.

Step 5: Factorizing the fitted transfer map into a Lie map. Dragt-Finn factorization method [1] is used to extract symplectic Lie map from the Taylor maps. After the factorization, the Taylor maps become a symplectic map automatically:

$$M = e^{:f_2:} e^{:f_3:} e^{:f_4:} \cdots$$

where f_n is a n^{th}-order homogeneous polynomial.

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APPLICATION 1: SOFT FRINGE QUADRUPOLE

The first example shows the extracted Lie map for a quadrupole with a soft edge. The soft fringe field is described by the Enge function with six parameters. Actually this quadrupole's Lie map can be calculated analytical by the COSY-Infinity code [2]. The purpose to show this example is to validate our method. For simplicity, we consider a quadrupole with only one side fringe field as show in Figure 1. The linear transport matrix from COSY and our numerical mode are perfectly agreeable. The first order nonlinear coefficients of f_3 are zeros due to symmetry. There are 19 non-zero second order geometry terms in f_4. The comparison of our numerical result against the COSY is illustrated in Figure 2. One can find they have a good agreement.

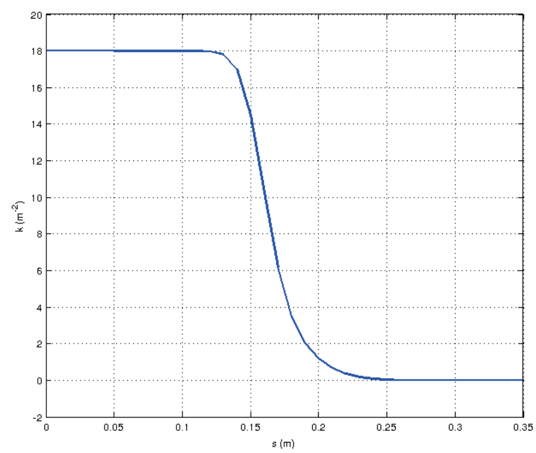

Figure 1: Longitudinal profile of a quadrupole with one side soft fringe. Here we choose quadrupole parameters as $K = 18m^{-2}$ and $L_{effective} = 0.155m$.

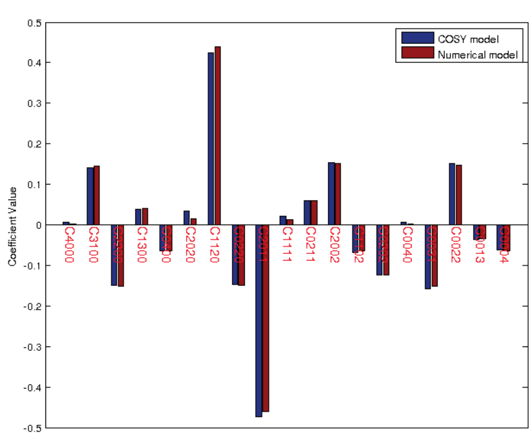

Figure 2: Comparison of the 2[nd] geometry terms in f_4 between the COSY code and the numeric model.

APPLICATION 2: SPEAR AND NSLS-II DIPOLES

In this section we apply this method to extract SPEAR and NSLS-II dipoles. The SPAER's dipole has a Cartesian gradient and NSLS-II's has end-noses. So far only few codes can track particles through such magnet using non-symplectic method. For example, the ELEGANT code provides a magnet element NISEPT to implement direct tracking by using non-symplectic R-K integrator. MATLAB based Accelerator Toolbox (AT) [3] also implement similar function.

First we study SPAER's dipole with Cartesian transverse gradient. The field profile of vertical magnetic component in the mid-plane is given in Figure 3. The reference orbit is determined with initial condition (x, x', y, y', δ) = (0, 0, 0, 0, 0) in the local coordinate, which needs to be transferred into Cartesian coordinate for trajectory tracking. A Taylor maps are fitted from multi-particle tracking results. So far, up to second order (f_4) nonlinear Lie map coefficients have been extracted from the Taylor maps, which are shown in the format of Table 1.

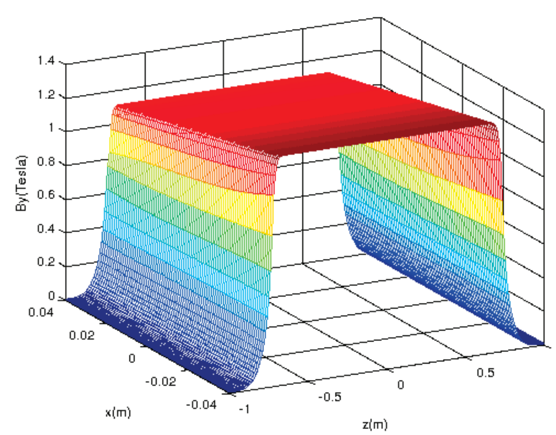

Figure 3: SPEAR dipole profile with transverse Cartesian gradient.

Table 1: Part of Lie coefficients for the SPEAR's dipole with gradient

index	coefficient	
010002	1.5390289210188e-02	
010200	-2.7575417781664e-01	1st
011100	3.7714791615780e-01	
…	…	
010003	-8.1219745887434e-02	
010201	4.8139109256354e-01	2nd
011101	-3.8051718734419e-01	
…	…	

We have applied the same method to study the NSLS-II 35mm gap dipole with two noses attached to the end-faces yoke body (Figure 4). Since the existence of end-noses, the longitudinal field profile (Figure 5) is irregular

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and difficult in modeling. The extraction of its linear model has been presented in [4]. With this new method, we can obtain its higher order terms.

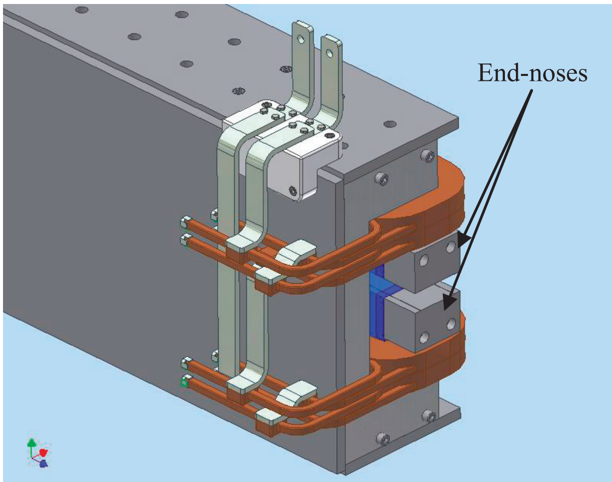

Figure 4: NSLS-II dipole with end-nose.

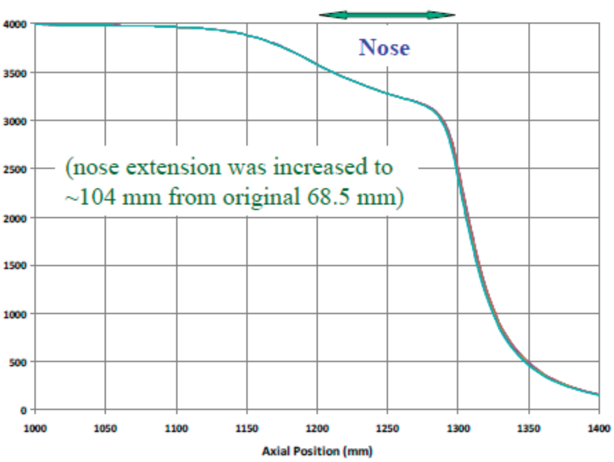

Figure 5: NSLS-II dipole's longitudinal profile. Notice the irregularity due to the end-nose.

Our method of extracting Lie map can be used widely in single particle beam dynamics studies, for example, to investigate the interaction between adjacent magnets if their fringe fields are overlapped and coupled; to model insertion devices for the storage ring based light sources; and even to explore some unusual magnets, like the snake magnet to help preserve beam polarization in the high energy colliders.

INCORPORATED INTO ACCELERATOR TOOLKIT

The extracted Lie map can be used for dynamic aperture optimization and particle tracking. Its linear part can be used for linear lattice design or match, and higher order term can be used to optimize the higher order driving terms, and the tune-dependence-on-amplitude coefficients through the standard normal form technique.

As for tracking, we may separate the terms of f_3 and f_4 and evaluate them individually. This procedure is symplectic since the monomial maps have exact solutions [5]. Separating f_3 terms yields some additional f_4 terms. But this can be done before tracking. We compared the Lie map tracking results with the non-symplectic numeric integration (Figure 6). If we use the 2nd order Lie map, the trajectory accuracy can achieve to the order of $10^{-7}m\ (or\ rad)$, which may be further improved by using more higher order terms. The benefits of using Lie map in tracking are that symplecticity can be kept, and tracking speed is much faster than the numeric integration. Implanting a Lie map module into the MATLAB based accelerator toolbox - AT [3] has been implemented, and some preliminary results have been obtained in modelling the SPEAR ring at SLAC [6].

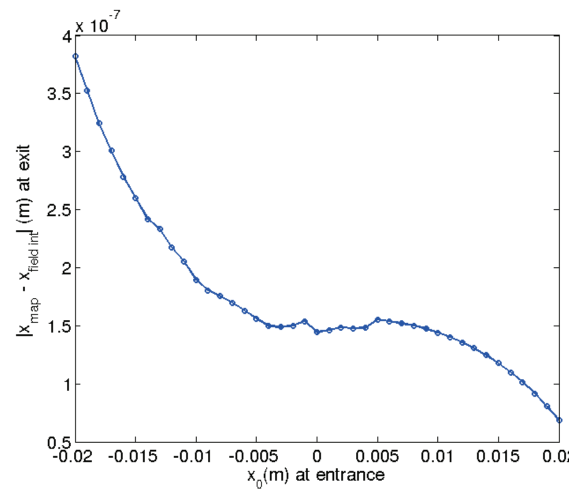

Figure 6: Comparison between the Lie map tracking and with the non-symplectic R-K integration. The trajectory differences at the magnet exit are at the order of $10^{-7}m$ with the same initial conditions.

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