NUMERICAL STUDY OF BEAM TRAPPING IN STABLE ISLANDS FOR SIMPLE 2D MODELS OF BETATRONIC MOTION

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Abstract

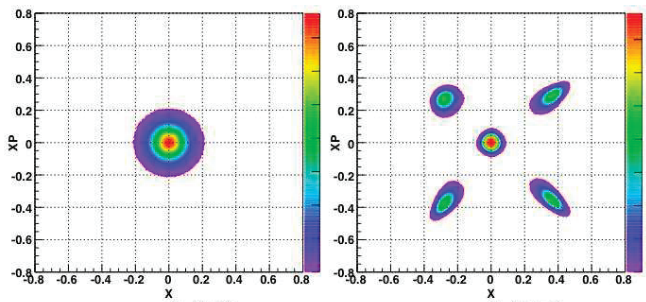
An essential ingredient for the proposed Multi-Turn Extraction (MTE) at the CERN PS is the beam trapping in stable islands. The control of the trapping process is essential for the quality of the final beam in terms of intensity sharing and emittance. In this paper, the splitting process is studied quantitatively by means of numerical simulations performed on 2D model representing the horizontal non-linear betatronic motion. The results are reviewed and discussed in details.

INTRODUCTION

The multi-turn extraction (MTE) is based on the trapping of a fraction of particle beam in stable islands created in the horizontal phase space by means of sextupoles and octupoles [1, 2]. The beam is split by crossing a resonance using a variation of the tune. Different resonances have been successfully tested at the CERN PS [3] and the actual implementation uses the fourth order resonance. The resulting beam is made of two different structures: the core and four beamlets corresponding to the particles trapped inside the stable islands. By inducing a closed orbit bump around the extraction septum it is then possible to extract the beam in five turns. A crucial point of this method is to obtain an equal intensity and emittance sharing among the core and the four islands. Such a sharing depends on the strengths of the non-linearities, the tune variation, and the initial beam distribution. The effect of these parameters has been studied by means of a simple numerical model. The motion in the accelerator is represented by a linear transformation except for a sextupole and an octupole, located at the same place, and represented by a single, non-linear kick [2]. In this paper we discuss the results of numerical simulations based on a 2D generalised Hénon mapping given by

$$\begin{pmatrix} \hat{X} \\ \hat{X}' \end{pmatrix}_{n+1} = R(\omega_n) \begin{pmatrix} \hat{X} \\ \hat{X}' + \hat{X}^2 + \kappa \hat{X}^3 \end{pmatrix}_n \quad \kappa = \frac{2}{3}\frac{K_3}{K_2^2}\frac{1}{\beta_x} \tag{1}$$

where $\omega_n = 2\pi\nu(n)$ is time-varying and κ represents the ratio between the strengths of the non-linear elements, weighted by the value of the beta function at that location. These coordinates are dimensionless and are related to the usual Courant-Snyder coordinates by the scaling factor $\frac{1}{2}K_2\beta_x^{3/2}$. The trapping of a particle inside the island occurs when it crosses the separatrix, which is moving due to the tune variation. At that point the particle has a certain probability to be trapped in the island. That probability increases if the adiabaticity of the crossing is increased, i.e., if the motion of the separatrix is slow compared to the motion of the particle in the phase space. Fig. 1 represents the evolution of the beam distribution during the tune variation.



$\nu = 0.252$ $\nu = 0.245$

Figure 1: Beam distribution at the beginning ($n = 0$) and end ($n = 20000$) of the trapping process. In all simulations 10^6 particles have been used.

TRAPPING MODEL

The trapping fraction T, defined as the percentage of particles trapped in one island, is crucial to assess the performance of the splitting process as the goal is to reach $T = 20\%$, which corresponds to beamlets and core equally populated. Tracking simulations using the model (1) were performed to obtain T as a function of the total number of turns N over which the tune variation is performed, the strength κ, and the initial emittance ϵ_i. The resonance is crossed from above with a tune varying linearly from 0.252 to 0.245.

Fig. 2 displays T for an initial Gaussian beam distribution of 10^6 initial conditions as a function of ϵ_i for a given set of N and κ. The trapping increases with ϵ_i in a similar way for the range of variation of N and κ that was used. A model for T is proposed with the following form

$$T(\epsilon, N, \kappa) = A(\kappa, N)\left[1 - e^{-B(\kappa,N)\epsilon_i + C(\kappa,N)}\right] \tag{2}$$

where the results show that the following simple expressions can be assumed for the functions A, B and C:

$$\begin{aligned} A(\kappa, N) &= (c_1\kappa + c_2)(d_1\sqrt{N} + d_2) \\ B(\kappa, N) &= (c_3\kappa + c_4)(d_3 N + d_4) \\ C(\kappa, N) &= (c_5\kappa + c_6)(d_5 N + d_6). \end{aligned} \tag{3}$$

Simulations performed for a large range of parameters set[1] allowed to obtain the numerical values of the coefficients listed in Table 1 together with the error associated with the fit. The increase of the trapping with a slower tune vari-

[1] Beam distributions with $2 \times 10^{-4} \le \epsilon_i \le 80 \times 10^{-4}$, $-1.9 \le \kappa \le -1.1$ and $5 \times 10^3 \le N \le 25 \times 10^3$.

ISBN 978-3-95450-115-1

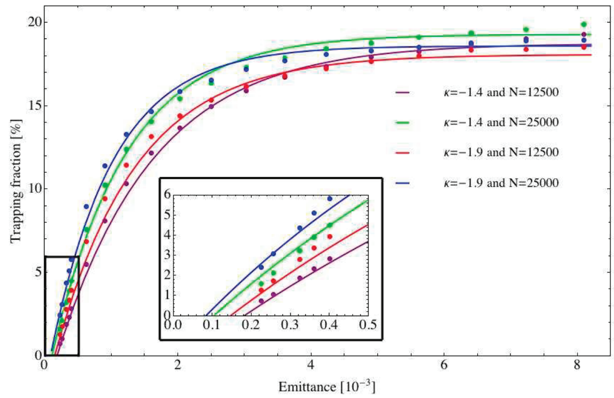


Figure 2: Trapping fraction as a function of ϵ_i for different sets of parameters N and κ. The markers represent the simulation results, while the lines the fitted curves.

Table 1: Coefficients of the fit defined in Eqs. (2) and (3)

c_1	0.288	±	0.009	c_2	3.588	±	0.007
d_1	0.44	±	0.01	d_2	5.400	±	0.007
c_3	1.742	±	0.056	c_4	−1.322	±	0.074
d_3	−0.054	±	0.002	d_4	−0.125	±	0.001
c_5	−0.111	±	0.007	c_6	−0.383	±	0.011
d_5	0.341	±	0.006	d_6	−1.217	±	0.004

ation is easily understood as an improvement of the adiabaticity of the process as the separatrix moves slower. We observe also a reduced effect for large emittances: indeed the trapping of large amplitude particles already occurs in conditions where the adiabaticity is restored as the motion of the islands is more sudden when they are close to the origin. That behaviour does not depend much on the value of κ, justifying the factorisation in the coefficients of Eq. (3).

We also conclude from Fig. 2 and from our model that the effect of a small $|\kappa|$ is to improve the trapping for large emittances while it decreases for small emittances. Using again the argument of the adiabaticity of the separatrix crossing, it is observed that for small $|\kappa|$ the stable fixed points move away from the origin faster as the tune varies close to the resonant value. On the other hand, small $|\kappa|$ allows the islands to move further away from the origin thus improving the trapping in the tails of the Gaussian distribution, which improves the trapping for large emittances.

The zoom of Fig. 2 close to the origin also reveals an important feature of the trapping process: below a certain amplitude the trapping fraction is zero. That indicates the presence of a region of phase space around the origin where no trapping can occur due to the non adiabatic motion of the separatrices. Simulations with uniform distributions (therefore with sharp edges) were performed to characterise better that region. That allowed to find the minimal trapping amplitude R_{min} as a function of κ and N. Fig. 3 shows the obtained results. The minimal amplitude decreases following an exponential with the total number

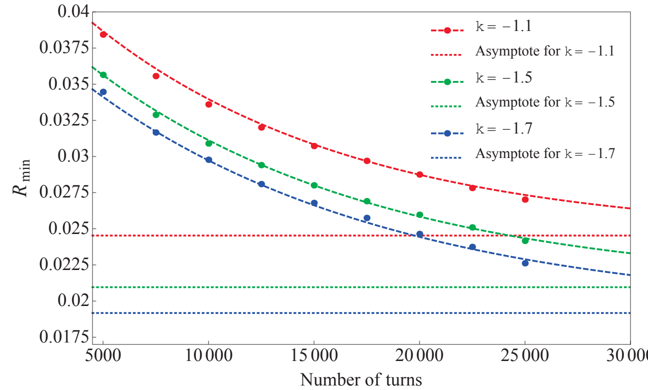


Figure 3: R_{min} as a function of N, comparison of different values of κ.

of turns. A fitting model of the form $\tilde{A}e^{-\tilde{B}(\kappa)N} + \tilde{C}(\kappa)$ can be proposed for the whole range of values of κ used in the simulations. $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ are positive quantities and it turns out that $\tilde{B}$ and $\tilde{C}$ are linear in κ. Fig. 3 also displays the value of the asymptote $\tilde{C}(\kappa)$. The value of $\tilde{C}(\kappa)$, i.e., the region of phase space were no trapping can occur due to the lack of adiabaticity even for a very large N, is a decreasing function of $|\kappa|$. Indeed, in these cases the islands stay closer to the origin, the motion of the fixed point is thus slower for a given variation of the tune. It is worth stressing that the integral of the beam distribution over R_{min} provides the fraction of particles that will be never trapped in the beamlets and hence prevents reaching $T = 20\%$.

The value of T has also been evaluated using a time dependent variation of κ. The results show that the effect of κ on the trapping is mainly given by its value at the beginning of the trapping, i.e., when the islands are growing close to the origin. In addition, using a non-linear tune variation where the slope of the tune curve is zero at the resonance crossing allowed to confirm our argument of adiabaticity, as it is clearly observed that the trapping fraction is increased in such a case. Such a phenomenon was already reported in Ref. [4]

EMITTANCE EVOLUTION

After the splitting, the emittances of the core ϵ_{core} and of the beamlets $\epsilon_{beamlets}$ are different and are reduced compared to ϵ_i. The value of the resulting ϵ_{core} depends on two counteracting effects: the reduction due to the trapping of an important part of the initial intensity in the islands and the growth due to particles crossing the separatrix without being trapped. The $\epsilon_{beamlets}$ depends on the topology of the phase space, the density of the beam located in the islands can be varied if the islands are squeezed and dilated by the non-linearities.

The values of $\epsilon_{beamlets}$ and ϵ_{core} have been computed from the simulations and the ratios to ϵ_i have been computed. Fits like $\hat{A}(N,\kappa)e^{-\hat{B}(N,\kappa)\epsilon_i} + \hat{C}(N,\kappa)$ for the core emittance ratio and of the form $\hat{A}(N,\kappa)\epsilon_i^{-\hat{B}(N,\kappa)} +$

$\hat{C}(N,\kappa)$ for the island emittance ratio have shown good agreement with the results at least for ϵ_i not too small. The functional form for $\epsilon_{beamlets}$ features $\hat{B} > 0$. In the case of small ϵ_i the trapping will be very small, due to the presence of the no-trapping zone of size R_{min}, and large emittance growth is observed. Fig. 4 (upper) shows one case. Results for different sets of κ and N shows that the core emittance ratio reaches less than 20% for large enough ϵ_i. For large values of $|\kappa|$ the island emittance ratio also reaches a value close to 20% but for small $|\kappa|$ the resulting emittance ratio can be close to 50% even for large initial emittances. The

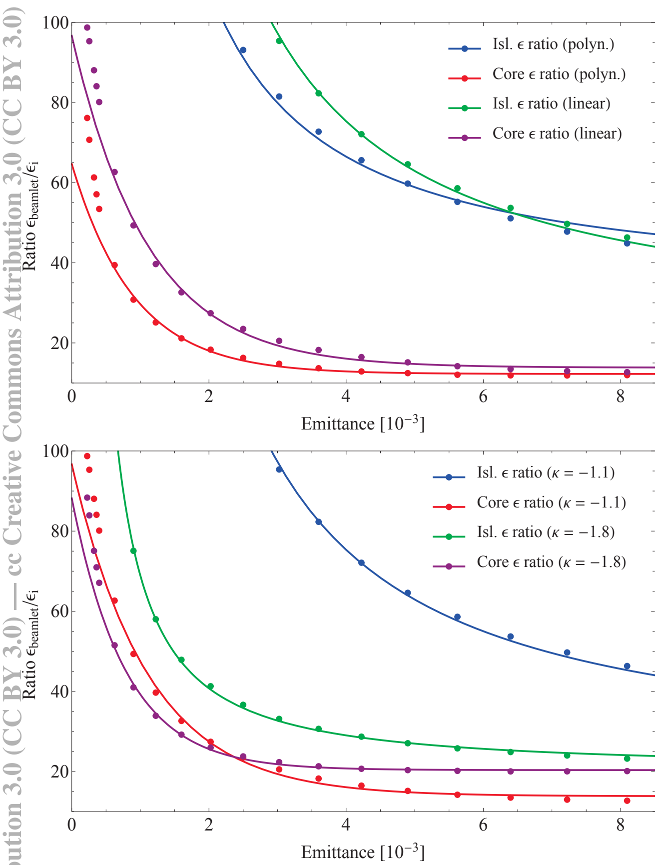


Figure 4: Emittance ratios for $N = 20000$ and different κ (upper). Emittance ratios for $\kappa = -1.1$ and $N = 20000$ and non-linear variation of the tune (lower).

impact of a non-linear tune variation on the emittance ratios has also been analysed. Fig 4 (lower) shows the results where the slope of the tune curve is zero at the resonance crossing. The case of the core seems to feature a smaller final emittance independently on ϵ_i. On the other hand, the beamlets feature a smaller $\epsilon_{beamlets}$ only when ϵ_i is not too large.

Simulations where a time variation of κ is applied led to interesting conclusions. The results show that it allows to improve both the trapping fraction (which is found to be better for smaller $|\kappa|$) as well as the emittance ratios. The time variation of κ was linear from -1.1 to -1.9. Results for one case are shown in Fig. 5. The trapping fraction reaches a value close to the static case $\kappa = -1.1$ while the emittance ratios are lowered to values corresponding to cases in-between $\kappa = -1.1$ and -1.9.

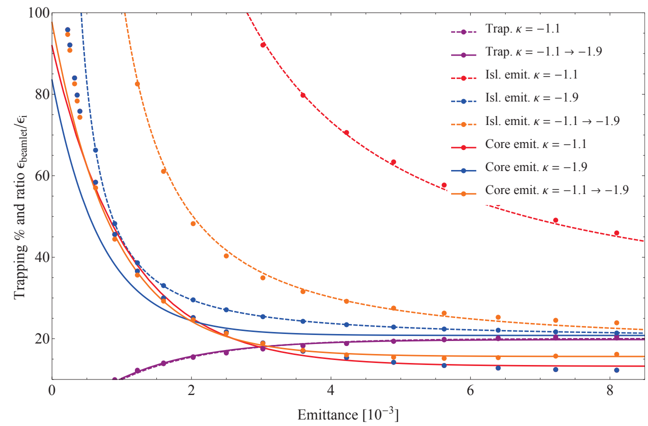


Figure 5: Trapping, core emittance ratio and island emittance ratio for different values and variations of κ.

CONCLUSIONS

Parametric studies of the trapping fraction and the emittance are presented, based on detailed and massive numerical simulations using a 2D model of the MTE dynamics. A numerical fitted model is proposed for the trapping as a function of the free parameters of the model, namely initial beam emittance, non-linear strength κ and number of turns N over which the resonance crossing is performed. Furthermore, simple models were derived also for the size of the small area around the origin in which trapping can never occur, as well as for the final emittance of the core and beamlets. More refined models in which the tune variation is non-linear and also the parameter κ is time-dependent showed further option to optimise the overall trapping process. The next step will be the link between the fitted models and analytical properties of the system (1) and the analysis of the impact of the vertical motion.

ACKNOWLEDGEMENTS

One of the authors (CH) would like to thank D. Manglunki and A. Dubus for their encouragements and support during the preparation of this work.

Several interesting discussions with A. Bazzani are warmly acknowledged.

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