

INVESTIGATIONS OF SCALING LAWS OF DYNAMIC APERTURE WITH TIME FOR NUMERICAL SIMULATIONS INCLUDING WEAK-STRONG BEAM-BEAM EFFECTS

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Abstract

A scaling law describing the time-dependence of the dynamic aperture, i.e., the region of phase space where stable motion occurs, was proposed in previous papers, about ten years ago. It was showed that dynamic aperture has a logarithmic dependence on time, which would be suggested by some fundamental theorems of the theory of dynamical systems. So far, such a law was applied to single-particle effects, only, i.e., the only source of non-linear effects was the magnetic imperfections. In this paper an attempt of extending the scaling law to the case of weak-strong beam-beam effects is made. The results of numerical simulations performed including both non-linear magnetic imperfections and weak-strong beam-beam effects are presented and discussed in detail.

INTRODUCTION

The evolution of the dynamic aperture (DA) with time is a topic of clear interest in the study of dynamical systems, in general, and of accelerator physics, in particular. A scaling law for the DA could be used to extrapolate results obtained by means of numerical simulations performed over a limited number of turns to much longer times. In the past, this topic was considered and some results in this direction obtained [1, 2]. Through the analysis of the optimal way to compute the DA from numerical simulation data [3] the so-called survival plots allowed finding a rather simple scaling law for DA.

Assuming a polar grid in normalised phase space

$$x = r \cos \theta \quad y = r \sin \theta \quad \text{with} \quad 0 < \theta < \pi/2, \quad (1)$$

if $r(\theta; N)$ stands for the last stable amplitude up to N turns in the direction θ , then the dynamic aperture reads:

$$D(N) = \frac{2}{\pi} \int_0^{\pi/2} r(\theta; N) d\theta \equiv r(\theta; t) > . \quad (2)$$

According to the results reported in Refs. [1, 2]

$$D(N) = D_\infty + \frac{b}{[\log N]^\kappa}, \quad (3)$$

where D_∞ represents the asymptotic value of the amplitude of the stability domain, while b and κ are additional parameters. These three quantities can be obtained by fitting the results of numerical simulations.

The interesting point is that such a parametrisation is

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compatible with the hypothesis that the phase space can be partitioned into two regions. A central core, with $r < D_\infty$, where KAM [4] surfaces confine the motion, thus inducing a stable behaviour apart for a set of small measure where Arnold diffusion can take place. An outer part, with $r > D_\infty$, where chaotic motion occurs and the escape rate to infinity is given by a Nekhoroshev-like estimate [5, 6] such as

$$N(r) = N_0 \exp\left(\frac{r^*}{r}\right)^{1/\kappa} \quad (4)$$

where $N(r)$ is the number of turns that are estimated to be stable for particles with initial amplitude smaller than r .

Interestingly enough, two regimes were identified [2]: i) in 4D systems the three quantities D_∞, b, κ are all positive [1]. This corresponds to having a stable region in phase space for arbitrarily long times. ii) In 4D systems with tune modulation or off-momentum dynamics it is possible to have no stable region even for a finite number of turns [2]. This corresponds to having the following cases:

$$\begin{cases} D_\infty > 0 & \kappa < 0 & b < 0 \\ D_\infty < 0 & \kappa > 0 & b < 0 \end{cases} \quad (5)$$

Therefore, rather solid arguments exist to make the scaling law (3) not completely phenomenological. However, the standard Hamiltonian model assumed in Refs. [5, 6] is based on polynomial non-linearities, which is certainly the correct assumption for describing the transverse motion of a charged particle in a lattice including magnetic non-linearities. On the other hand, the beam-beam force has an intrinsically different form and a special treatment is needed to obtain results of the type [5, 6] and, to our best knowledge, such a result is not yet available in the literature. Nevertheless, it is also clear that the extension of the scaling law (3) to the case in which also beam-beam effects are included seems reasonable. One of the characteristics of the Nekhoroshev-like estimates is to provide a description of the dynamics in terms of pseudo-diffusive behaviour. This is certainly compatible with beam-beam (see, e.g., Refs. [7, 8, 9, 10]). In the end, one could also use a purely phenomenological approach and simply check whether the scaling law (3) is capable of describing the evolution of DA when beam-beam effects are included.

It is worth recalling that Eq. 3 has been recently applied to propose a scaling law for the intensity evolution in a hadron machine [11] and also in the interpretation of the luminosity evolution in a hadron collider [12].

SIMULATION RESULTS

The simulations are based on the weak-strong, 4D beam-beam model implemented in the SixTrack code [13]. The lattice used is the one describing the nominal LHC [14] and two different configurations have been considered: i) at injection energy the beams are separated by a crossing angle and a parallel separation in the orthogonal plane; ii) at top energy the beams are colliding in all four experimental points with the nominal crossing angle. The measured magnetic field errors are included in the simulations [15]. In addition to the head-on collisions, 20, 21 (for IP2/8 or IP1/5, respectively) long range collisions are included at each side of the collision point. Finally, the bunch intensity N_b has been varied to observe the changes in the behaviour of the DA.

In terms of parameters for the numerical simulations, 59 phase space angles have been considered to provide a detailed mapping of the phase space. Furthermore, each direction has been probed by distributing 30 pairs of initial conditions every 2σ amplitude range. The momentum offset of the initial conditions was set to 0.75×10^{-3} and to 0.27×10^{-3} for injection and top energy, respectively, while the maximum number of turns was $N_{max} = 10^6$.

In Fig. 1 the DA has been plotted in the normalised phase space for the case of injection. The red markers represent the initial conditions that are stable up to N_{max} , while blue

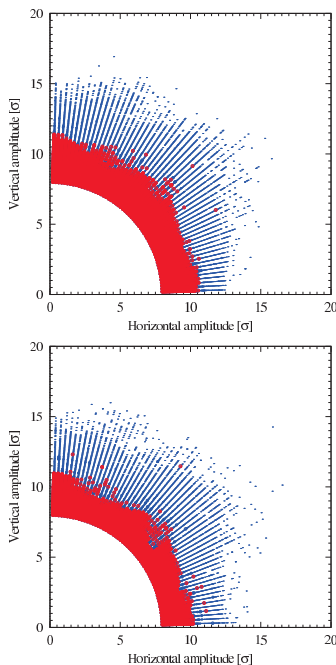


Figure 1: DA for injection energy and $N_b = 1.15 \times 10^{11}$ p (upper) and 1.7×10^{11} p (lower). Beams are separated, but the beam-beam effect is active. The red and blue markers stand for stable or unstable initial conditions, respectively.

markers represent unstable conditions. Furthermore, the size of the blue markers is proportional to the number of stable turns. Two values of N_b have been used, namely

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1.15×10^{11} (upper) and 1.7×10^{11} (lower), representing the so-called nominal and ultimate intensities, respectively. The stable area around the origin was not simulated in order to reduce the CPU-time required. The effect of N_b is visible. The key result is shown in Fig. 2, where the DA vs. time from numerical simulations is compared with the result of the fitted function (3). The agreement is remarkable for both intensities, with a clear reduction of DA in dependence on N_b . The slight difference between the fit and the

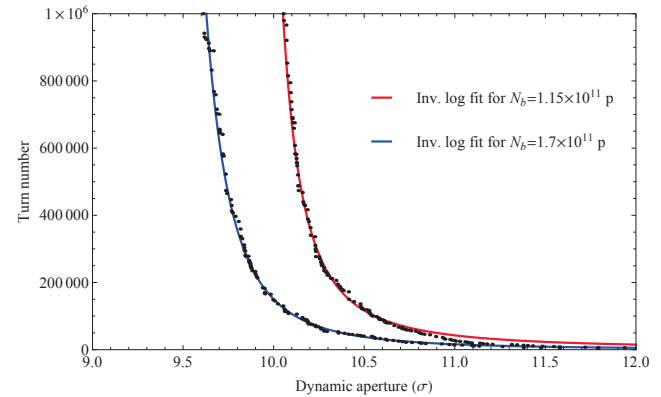


Figure 2: DA vs. time at injection energy (situation plotted in Fig. 1) including the proposed fit. The agreement is very good apart for large-amplitude initial conditions.

data for large amplitudes should not be too surprising as the law (3) has to be considered as a good asymptotic model, i.e., for large number of turns.

At top energy the beams are no more separated and real collisions are taking place. The DA is plotted in Fig. 3 for three typical bunch intensities, with a weak intensity of 0.1×10^{11} added to the nominal and ultimate ones. The shrinking of the stable area is visible. Also for this configuration the key result is plotted in Fig. 4. The agreement is once more remarkable. In this case, the fit reproduces both the large as well the low amplitude behaviour of the DA vs. time.

Finally, in Table 1 the key fit parameters are reported for the five cases presented in this paper. It is worth mentioning that κ is not really computed from a fit, rather a scan over κ is performed and the value retained is the one that minimises the fit residues. Two key features are found by inspecting Table 1: all the fit parameters are positive for the injection energy case, while this is no more the case at top energy. This means that while at injection energy a well-defined asymptotic value of DA, namely D_∞ , exists, at top energy the pseudo-diffusive behaviour is such that $D(\bar{N}) = 0$ for a finite \bar{N} .

CONCLUSIONS

A scaling law for the evolution of the DA has been proposed and successfully applied to the case of numerical simulations of the nominal LHC machine including magnetic non-linearities and weak-strong beam-beam effects. Such a scaling, derived from Nekhoroshev theorem for

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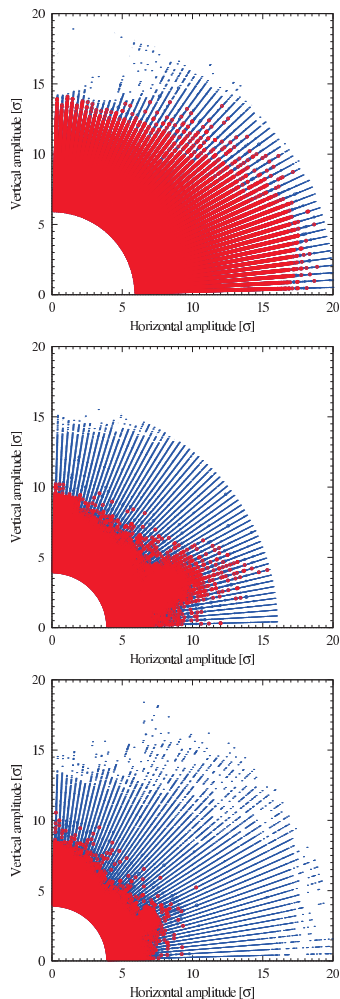


Figure 3: DA for top energy and colliding beams for $N_b = 0.1 \times 10^{11}$ p (upper), 1.15×10^{11} p (middle) and 1.7×10^{11} p (lower). The red and blue markers stand for stable and unstable initial conditions, respectively.

Table 1: Fit parameters for the proposed scaling law (3). The error associated with κ is 0.05. The first two rows correspond to injection energy, while the remaining three to top energy. N_b is expressed in units of 10^{11} p.

N_b	D_∞	b	κ
1.15	9.789 ± 0.004	10231 ± 80	5.90
1.70	9.098 ± 0.006	335 ± 2	3.60
0.10	19.1 ± 0.1	-0.119 ± 0.003	-2.20
1.15	-10.3 ± 0.2	61.1 ± 0.6	0.70
1.70	0.23 ± 0.05	47.7 ± 0.3	1.15

polynomial non-linearities, is currently a simple extension of what was found earlier for the case of beam dynamics under the influence of magnetic multipoles. Theoretical work is required to extend the results of Ref. [6]. On another front, it will be interesting to analyse the behaviour of the fit parameters as a function of the main physical parameters, e.g., N_b , and crossing angle.

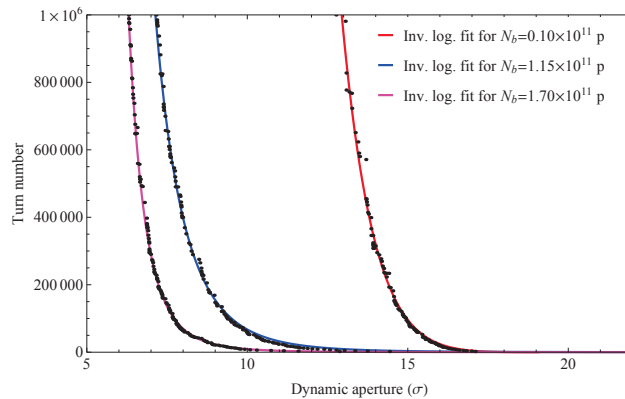


Figure 4: DA vs. time at collision energy (situation plotted in Fig. 3) including the proposed fit. The agreement is very good, and the effect due to N_b is clearly visible.

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