# A MODEL OF THE AGS BASED ON STEPWISE RAY-TRACING THROUGH THE MEASURED FIELD MAPS OF THE MAIN MAGNETS*

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*Abstract*

Two-dimensional mid-plane magnetic field maps of two of the main AGS magnets were produced, from Hall probe measurements, for a series of different current settings. The analysis of these data yielded the excitation functions [1] and the harmonic coefficients [2] of the main magnets which have been used so far in all the models of the AGS. The constant increase of the computation power makes it possible today to directly use a stepwise ray-tracing through these measured field maps with a reasonable computation time. We describe in detail how these field maps have allowed the generation of models of the 6 different types of AGS main magnets, and how they are being handled with the Zgoubi ray-tracing code [3]. We give and discuss a number of results obtained regarding both beam and spin dynamics in the AGS, and we provide comparisons with other numerical and analytical modelling methods.

## INTRODUCTION

The Alternating Gradient Synchrotron is used in the BNL accelerator complex as the injector of the Relativistic Heavy Ions Collider. For the polarized proton operations the AGS accelerates the protons from 2.2 to 23.8 $GeV/c$. Since the AGS uses combined function main magnets, in charge of most of the focusing strength, their modelisation is critical.

Currently all the AGS models use the same excitation functions and harmonic contents for the AGS main magnets. A step by step measurement of the magnetic field was made [2] for two magnets over a number of current settings to generate the excitation functions and harmonic contents by integration along the reference orbit in the generated field maps.

This paper develops some work briefly introduced in earlier conference publications [4].

Firstly the AGS main magnets will be described as well as their measured field maps and the method used to extract the harmonic contents using the Zgoubi tracking code. Secondly some results of the AGS model using the measured field maps with the Zgoubi tracking code will be presented.

## THE AGS MAIN MAGNETS

The AGS main magnets are combined function C-shaped magnets with two different lengths. Depending on the side of the wide gap the magnet is called open (A and B-type) or closed (C-type) as illustrated in Fig. 1. The iron of A-type and C-type is 90 inches long and it is 75 inches long for the B-type .

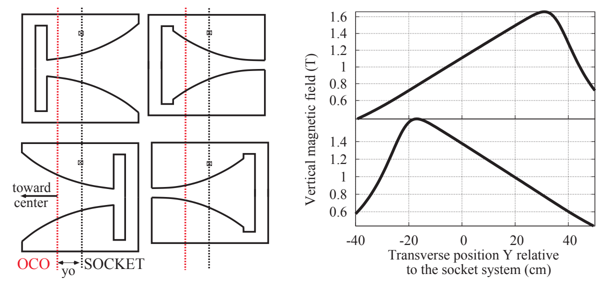


Figure 1: Left side : sketches of the relation between ”OCO” and ”SOCKET” system for A-type (or B-type) magnet (D and F) at right and C-type magnet (F and D) at left for a positive particle going toward the plan of the sheet. Right side : field profile at the center of an A-type magnet at 5150 A, for F and D.

### The Main Magnet Measured Field Maps

Point-to-point field maps were made for a A-type and a C-type magnet, from the fringe field region to the middle of the magnet, using hall probe. The maps are produced in the median plane of the magnet for the vertical field component and over 13 different currents from 15 to 5850 $A$.

The original mesh size of these maps was $\Delta x = 0.1 inch$ in the transverse direction and $\Delta s = 0.25\ inch$ or $\Delta s = 1\ inch$ in the longitudinal one, depending on the gradient. However Zgoubi requires a uniform mesh so the maps were converted to a constant longitudinal mesh of $\Delta s = 0.25\, inch$ [4].

The field maps of the B-type magnets are derived by shortening the central region of the A-type maps. The two functions of each magnet type (focusing and defocusing) are obtained by flipping the map with respect to the longitudinal axis.

### Extraction of the Harmonic Coefficients

The quadrupole and sextupole strengths of the main magnets were extracted from the measured field maps to build an interpolation polynomial for the current dependence [2].

The integral of the vertical field over one integration step can be established from the ray-tracing considering the particle trajectory in the field map :

$$\frac{1}{B_0\rho_0}\frac{dB_y}{dx}.l_{step} \tag{1}$$

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Summing (Eq. 1) along a trajectory with length $l_{traj}$ produces the integrated quadrupole strength $k_1^{map}.l_{traj}$. Moreover as the models of the AGS use the magnetic length of these magnets, $k_1^{map}$ needs to be scaled to the magnetic length of the magnet in order to compare our results to those computed in [2] which are given by the polynomial function. The scaled quadrupole strength is :

$$k_1^{mag} = k_1^{map} \frac{l_{traj}}{l_{mag}} \tag{2}$$

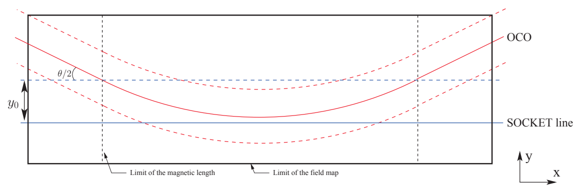


Figure 2: Sketch of the ideal trajectory in the field map of a main magnet (plain line) and two shifted trajectories (dashed lines).

A way of extracting both the quadrupole and sextupole strengths from the measured field maps is to use $k_1^{mag}$ determined along the ideal trajectory [1] in the field map and along few other trajectories, transversally shifted from the ideal one (see Fig. 2). This gives the value of $k_1^{mag}$ as a function of the orbit shift $y_{shift}$ from the ideal trajectory, which is then fitted by a second order polynomial :

$$k_1^{mag}(y_{shift}) = a_0 + a_1 y_{shift} + a_2 y_{shift} \tag{3}$$

The values of the quadrupole and sextupole strengths are directly deduced from Eq. 3 :

$$k_1^{mag} = k_1^{mag}(y)|_{y=0} = a_0 \tag{4}$$

and

$$k_2^{mag} = \left.\frac{dk_1^{mag}(y)}{dy}\right|_{y=0} = a_1 \tag{5}$$

A comparison between the quadrupole and the sextupole strength determined either using Zgoubi or Ref. [2], shows very close results (Fig. 3). This study is handled for currents ranging from 108 to 5150 $A$ which is equivalent to a range of protons momentum between 0.7 and 29 $GeV/c$.

## AGS MODEL USING THE FIELD MAPS

### *Field Maps Positioning*

The measured field maps use a coordinate system called the "Socket System" whereas Zgoubi (and MADX as well) uses the OCO in defining their reference frame. The different coordinate systems and their relationship are detailed in references [6] [7]. Moreover the AGS model using the measured field maps employs the same conventions as the classical MADX model : a counter-clockwise circulation of the particles and the positive transverse direction toward the center of the ring.

[1] The ideal trajectory is defined by tracking a particle (with an adjusted momentum) that coincides with the Optimum Closed Orbit [5] at the entrance and exit of the field map.

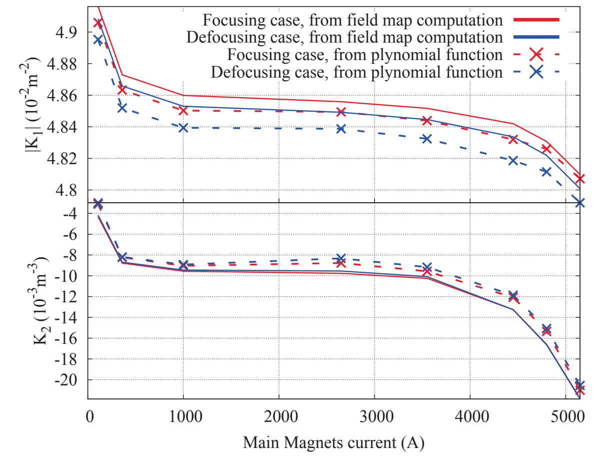


Figure 3: Quadrupole (top plot) ans sextupole (bottom plot) strengths as a function of the magnet current for a C-type magnet.

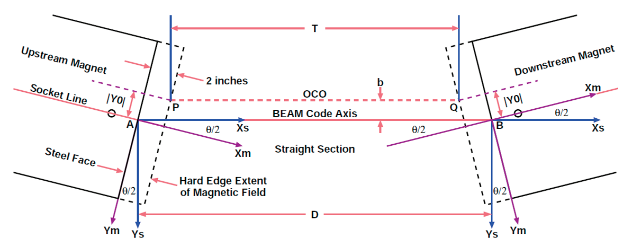


Figure 4: Positioning and relation between the various referentials across a straight section between two main magnets [7].

### *Comparison of Optical Parameters at 20.5 $GeV/c$*

The Table 1 outlines the comparison of some optical parameters in the case of the bare machine running at 20.5 $GeV/c$, equivalent to a current in the main magnets of 3550 $A$.

Since MADX is using an hard-edge model, it could be the source of the difference between Zgoubi using the field maps and the MADX model using the Zgoubi-computed harmonic coefficients. But it has been proved that Zgoubi and MADX gives very close results when they use the same harmonic coefficients [4]. These differences between the field maps, harmonic coefficients and measurements are under investigation.

### *Long Term Tracking*

The Figure 5 displays a tracking computed by the Zgoubi code in the AGS lattice using the measured field maps at 3550 $A$. Four particles were launched in each plane. One notices that the single particle emittance is not conserved during such a long tracking. This may have various origins



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Table 1: Relevant parameters for different models of the AGS at 20.5 $GeV/c$.

| Model | $Q_x$ | $Q_y$ | $Q'_x$ | $Q'_y$ |
|---|---|---|---|---|
| Zgoubi using the measured field maps at 3550 $A$ | 8.69 | 8.76 | $-18.2$ | 1.02 |
| MADX with Zgoubi-computed harmonic coefficients (see Fig. 3) | 8.71 | 8.74 | $-20.5$ | 1.31 |
| MADX with harmonic coefficients from the polynomial function | 8.70 | 8.75 | $-21.4$ | 2.12 |
| Measurement | 8.74 | 8.75 | $-17$ | 0 |

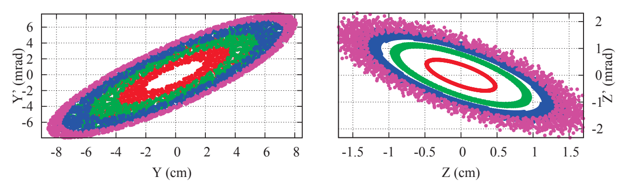


Figure 5: Phase spaces for few trackings in the bare AGS using the field maps at $3550A$ for $20,000$ turns with only horizontal non zero initial coordinates (left plot) and with only vertical non zero initial coordinates (right plot).

as field map smoothness, coupling, or numerical precision, investigations are in progress. This model can efficiently be used to extract the optical parameters of the lattice or to determine the strength of a spin resonance which requires roughly 1000 turns tracking.

### *Crossing of a Strong Intrinsic Spin Resonance*

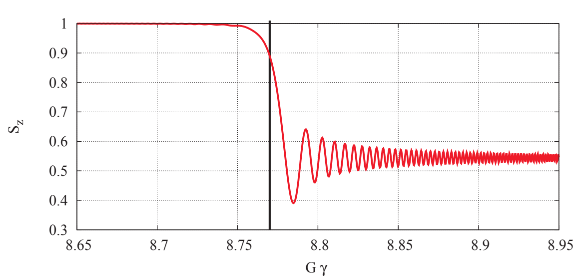


Figure 6: Vertical component of the spin across the $G\gamma = 0 + Q_z$ strong intrinsic resonance.

As an example Fig. 6 presents the evolution of the vertical component of the spin across the 0+ spin resonance, in the case of the bare AGS. The strength of the resonance can be determined by the amount of depolarisation induced on the particle following the Froissart-Stora formula [10] :

$$\frac{P_f}{P_i} = 2e^{-\frac{\pi}{2}\frac{|\epsilon|^2}{\alpha}} - 1 \qquad (6)$$

With $\epsilon$ the resonance strength and $\alpha$ the crossing speed.

Table 2 summarizes the computed resonance strength for some resonance, in the AGS using only their main magnets. The resonance strength computed using the "thin lens model" [8] from the lattices functions delivered by MADX is shown for reference. This procedure can be systematized over the all energy range of the AGS and also for the imperfection resonances [8] [9].

Table 2: Comparison of the computed strength for the strongest intrinsic spin resonance

| Resonance | Thin lens | Zgoubi | relative difference (%) |
|---|---|---|---|
| $0 + Q_z$ | 125 | 114 | 8.8 |
| $12 + Q_z$ | 39 | 28 | 28.2 |
| $36 - Q_z$ | 320 | 281 | 12.2 |
| $36 + Q_z$ | 1950 | 1760 | 9.74 |

## CONCLUSION

The tracking through the measured field maps allows to take into account realistic fields including longitudinal fringe fields and non-linear harmonics. The discrepancies between measurements and simulations by either the Zgoubi or MADX models have to be explained.

In order to track over the AGS cycle using field maps, we plan to build a set of computed TOSCA field maps for a wide range of current including field-dependent permeability, that would also allow taking into account the time variation of the field.

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