STUDIES OF THE TWIN HELIX PARAMETRIC-RESONANCE IONIZATION COOLING CHANNEL WITH COSY INFINITY*

J.A. Maloney[#], B. Erdelyi (Northern Illinois University, DeKalb, IL, USA), S.A. Bogacz, Y. S. Derbenev, V.S. Morozov (JLAB, Newport News, VA, USA), A. Afanasev (George Washington University, Washington, D.C., USA) K.B. Beard, R. P. Johnson (Muons, Inc., Batavia, IL, USA)

Abstract

A primary technical challenge to the design of a high luminosity muon collider is an effective beam cooling system. An epicyclic twin-helix channel utilizing parametric-resonance ionization cooling (EPIC) has been proposed for the final 6D cooling stage. A proposed design of this twin-helix channel is presented that utilizes correlated optics between the horizontal and vertical betatron periods to simultaneously focus transverse motion of the beam in both planes. Parametric resonance is induced in both planes via a system of helical quadrupole harmonics. Ionization cooling is achieved via periodically placed wedges of absorbing material, with bi-periodic rf cavities restoring longitudinal momentum necessary to maintain stable orbit of the beam. COSY INFINITY is utilized to simulate the theory at first order. The motion of particles around a hyperbolic fixed point is tracked. Comparison is made between the EPIC cooling channel and standard ionization cooling effects. Cooling effects are measured, after including stochastic effects, for both a single particle and a distribution of particles.

INTRODUCTION

A proposed next-generation muon collider will require major technical advances to achieve the rapid beam cooling requirements [1]. A twin-helix cooling channel design has been proposed for the final 6-D cooling stage [2]. This channel utilized pairs of helical harmonic magnetic fields with matching field strengths and phase shifts, but equal and opposite helicities. Continuous multipole fields are also superimposed on the channel. This channel maintains a condition of correlated optics where the horizontal and vertical betatron tunes are integer multiples of each other and of the dispersion function [3]. Using the correlated optics condition, wedge absorbers are placed at locations of small, but nonzero, dispersion. RF cavities are also used to maintain the momentum of the reference particle [4]. Using additional pairs of helical harmonic magnets, a parametric resonance is induced inside the channel's absorbers to achieve parametric resonance ionization cooling (PIC) [5]. PIC offers the potential to increase cooling by a factor of 10 over standard ionization cooling [6]. The twin helix channel is simulated using COSY Infinity, a DA-based code that allows for calculation of non-linear effects to arbitrary order [7]. This paper details a linear simulation of this channel, with and without stochastic

effects, and studies cooling efficiency with and without the effects of PIC. The linear simulation provides a baseline for ideal cooling in the channel if nonlinear aberrations in the channel have been fully corrected.

SIMULATION PARAMETERS

Table 1 details the parameters used in this linear simulation. The basic cell consists of a continuous straight quadrupole field superimposed upon a pair of helical harmonic dipole fields to establish the correlated optics condition. Wedge absorbers, made of beryllium with a central thickness of 2 cm and a gradient of 30%, are placed every 4 meters in the channel at a location of small but non-zero dispersion. Idealized RF cavities are placed 3 cm after the center of each wedge. COSY INFINITY calculates the transfer map for a 4-meter long cell (from the center of a wedge absorber through the center of the next wedge absorber). Figure 1 illustrates the geometry of this cell.

Figure 1. Schematic of single twin helix cell.

The total transfer map for the twin helix channel is obtained by composing the maps for each of the cells that make up the channel upon one another. The orbit of the reference particle (a 250 MeV/c muon) is periodic from the beginning to end of each cell.

Table 1: Four Meter Cell Parameters

H. Dipole Field	1.63 T
H. Dipole wavelength	1 meter
Continuous Quadrupole Field	.72 T/m
H. Quadrupole Field (Horizontal Lenses)	.02 T/m
H. Quadrupole wavelength	2 meters
H. Quadrupole Field (Vertical Lenses)	.04 T/m
H. Quadrupole wavelength	1 meter
RF Voltage	-12.5 MeV
RF Frequency	201.25 MHz
RF Phase	30 Degrees

# SIMULATION OF THE PARAMETRIC RESONANCE CONDITION IN THE TWIN HELIX CHANNEL

To induce the resonance condition for PIC in the twin helix channel, two independent pairs of helical harmonic quadrupole fields (parametric lenses) are used; one pair induces resonance in the horizontal plane, the other in the vertical plane. The resonances induced by these fields create a hyperbolic fixed point; i.e., motion of particles relative to the reference orbit at the center of the wedge absorber becomes hyperbolic rather than elliptical. Figures 2a-b show this condition in the basic cell (without wedge absorber or RF) when a test particle that is offset both horizontally and vertically in both position and angle relative to the reference orbit by 2 cm and 130 mrads respectively. With the parametric lenses, the position offset is quickly minimized at the expense of a rapid blowup in the angle offset.
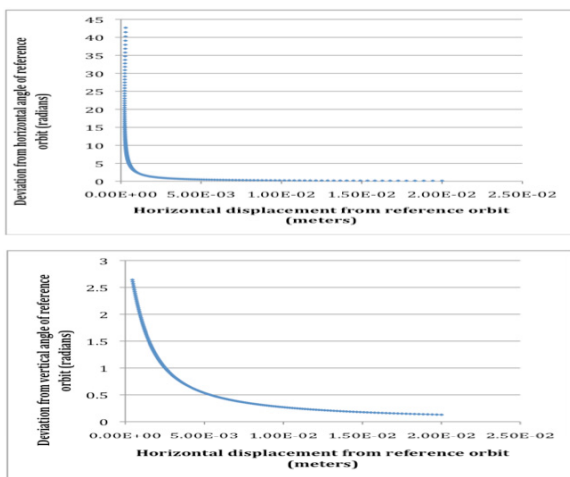
Figures 2a-b. Single particle launched with a horizontal and vertical offsets of 2 cm and .130 radians from the reference orbit and tracked for 200 cells.

# SIMULATION OF IONIZATION COOLING IN THE TWIN HELIX CHANNEL

The next stage in the simulations adds wedge absorbers and RF every 4 meters. Figures 3a-b show simulations with and without the parametric lenses to demonstrate the effects of ionization cooling with and without the PIC condition. The simulations demonstrate the expected results. With only ionization cooling, initial cooling effects are primarily to angle, and only later to the positional offset. With PIC the cooling effects are primarily to position offset, i.e., spot size of the beam. The increase in angle offset is minimized thru the ionization cooling effects of the wedge absorber.
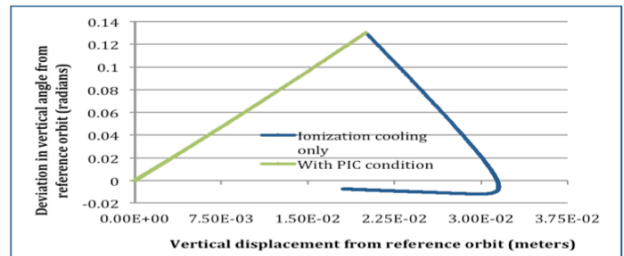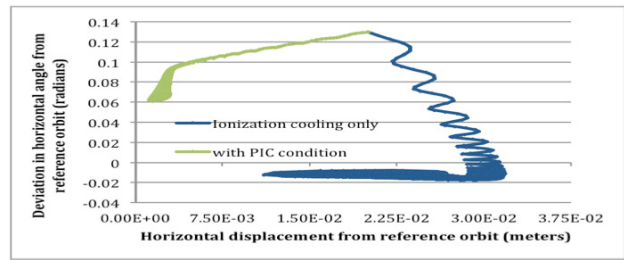
Figures 3a-b. Single particle tracked thru 1000 cells with PIC condition and with ionization cooling only.

SIMULATION OF STOCHASTIC EFFECTS IN THE TWIN HELIX CHANNEL

Stochastic effects of multiple scattering and energy straggling within the wedge absorber were then added to the simulations. Figures 4a-b show the results of combining ionization cooling with stochastic effects on a single particle initially offset both horizontally and vertically in both position and angle relative to the reference orbit by 2 cm and 130 mrads respectively.
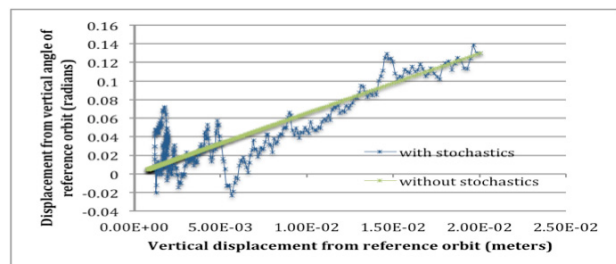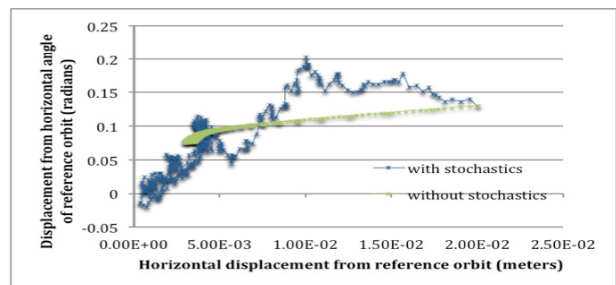
Figures 4a-b. Single particle tracked showing cooling thru 350 cells with and without stochastic effects.

As expected, the test particle is cooled until equilibrium is reached when cooling has been balanced with the effects of multiple scattering and energy straggling [8].

A distribution of test particles was also used to test cooling effects in the full simulation of the linear channel. The initial distribution uses a sigma of 2 cm in positions, 130 mrad in angles, and 1% spread in energy from the reference particle. The distribution is also spread over a bunch length of ± 3 cms relative to the reference particle. Figure 5 shows the 2D emittance change in the system calculated from the distribution. The horizontal and vertical 2D emittances are both reduced until equilibrium is reached. Longitudinal emittance is determined from deviation in path length and energy from the reference particle. Once the transverse emittance has reached equilibrium, the increases in longitudinal emittance contribute to heating in the beam distribution. Total 6D emittance for the distribution is plotted in Figure 6 with and without parametric lenses to induce the PIC condition. Figure 7 shows the cooling factor for the channel with and without the PIC condition.
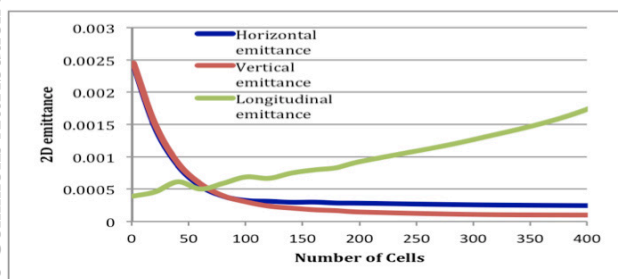

Figure 5: Emittance reduction for a distribution of 1000 particles tracked thru the twin-helix channel with the PIC condition and stochastic effects.
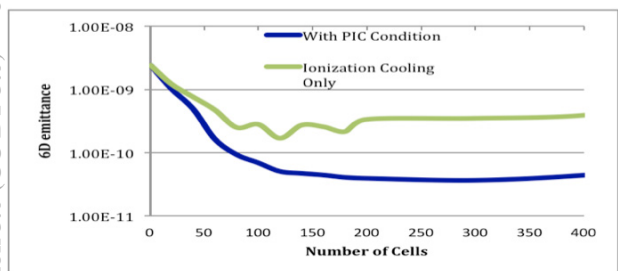

Figure 6: Comparison of 6D emittance reduction with and without the PIC condition.
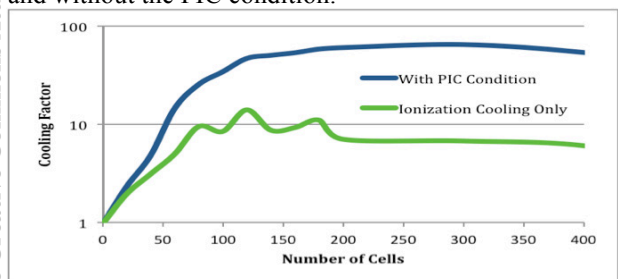

Figure 7: Comparison of cooling factor (ratio of initial to final 6D emittance) with and without the PIC condition.

The determinant of the transfer map for the cell, a 6x6 matrix in the linear case, can also be used to show transverse and 6D cooling in the system [9]. The determinant for the transfer matrix for this test channel is 0.945372. The determinant of the transverse 4x4 quadrant of the transfer matrix is 0.986054.

CONCLUSIONS AND FUTURE WORK

Current simulations in COSY INFINITY have demonstrated that the linear model with stochastic effects of the twin helix channel achieves the resonance condition for PIC, as well as 6D cooling. Future simulations will determine the optimal parameters for this linear model, including cell length, magnet strengths, helicity and phase shifts for the helical harmonic magnets. Wedge gradients and thickness, as well as RF placement and parameters, will also be optimized. Next, studies will determine the largest nonlinear aberrations affecting this optimized twin-helix channel and the dependence these aberrations have on higher order helical harmonic and continuous multipole fields of differing strength, helicity and phase. The cooling efficiency of a system with corrected higher order effects can then be measured and compared with competing 6D cooling methods.

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