EFFECTS OF MULTIPOLES ON DYNAMIC APERTURE OF THE ILSF STORAGE RING*

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Abstract

The main limitation of dynamic aperture is due to the chromatic sextupoles. However, small multipole errors in magnetic elements can reduce the dynamic aperture by generating high order resonances at the aperture boundary. The ILSF dynamic aperture in the presence of systematic magnetic multipoles was studied and simulated by BETA [1] tracking code and the results are presented in this paper.

INTRODUCTION

The general expansion of magnetic field is given by [2]

$$B_y + iB_x = \sum_{n=1}^{\infty}(b_n + ia_n)(x+iy)^{n-1} \quad , \tag{1}$$

where b_n and a_n are the normal and the skew components respectively. According to this equation, every multipole produces the magnetic field with the amplitude at the given radius r_0 as

$$\Delta A_n(r_0) = a_n r_0^{n-1}, \Delta B_n(r_0) = b_n r_0^{n-1}. \tag{2}$$

The field of each component provides an additional kick to the beam as,

$$\Delta x' = \frac{\Delta B.L}{B\rho}, \tag{3}$$

where $B\rho$ is the magnetic rigidity and L is the magnet length. The kick increases the particle betatron amplitude by [3]

$$\Delta x = \beta_x \Delta x'. \tag{4}$$

The magnetic field expansions of combined dipole, quadrupole and sextupole magnet including systematic field error have a form which are given in Eq. (5) – Eq. (7) respectively

$$B(x) = B_0 + gx + b_3x^2 + b_5x^4 + ... \tag{5}$$

$$B(x) = b_2x + b_6x^5 + b_{10}x^9 + b_{14}x^{13} ... \tag{6}$$

$$B(x) = b_3x^2 + b_9x^8 + b_{15}x^{14} + b_{21}x^{20} ... \tag{7}$$

where

$$b_n = \frac{1}{(n-1)!}\left(\frac{\partial^{n-1}B_y}{\partial x^{n-1}}\right).$$

The relative multipole errors for a combined dipole, quadrupole and sextupole magnets are

$$\left(\frac{\Delta B}{B_{main}}\right)_n = \frac{b_n x^{n-1}}{B_0 + gx} \text{ (n=3,5,7,...)}$$

$$\left(\frac{\Delta B}{B_{main}}\right)_n = \frac{b_n x^{n-2}}{b_2} \text{ (n=6,10,14,...)}$$

$$\left(\frac{\Delta B}{B_{main}}\right)_n = \frac{b_n x^{n-3}}{b_3} \text{ (n=9,15,21,...)}$$

CRITERIA FOR HIGHER MULTIPOLES IN ILSF MAGNETS

The boundary of stable area is defined by nonlinear parameters specially sextupole strength. If the displacement of betatron amplitude exceeds this boundary, the stable motion of the particle will lost and the dynamic aperture will decrease. For *n* magnets with statistically independent field errors, one can roughly estimate that [3]

$$\Delta x \approx \frac{\sqrt{n}\Delta B.\bar{L}\bar{\beta}_x}{B\rho}, \tag{8}$$

where ΔB is the rms field error, $\bar{L}$ is the average of magnet length, $\bar{\beta}_x$ is the average of horizontal beta function. If we are concerned about the dynamic aperture reduction by $\Delta x=1$ *mm*, in the case of ILSF (*n=264*), $\bar{L} = 0.37\,m$ and $\bar{\beta}_x = 6.1\,m$ and for magnetic rigidity of 10 T.m, the rms field error should be 2.7E-4 T.

In spite that this calculation is very rough the order of magnitude seems to be true, so for investigation of reasonable decreasing of dynamic aperture, magnitude field error should be higher than 1E-4 T. By considering the phase space of bare ILSF lattice [4] the maximum stable amplitude is $x=25$ *mm* and maximum slope is $x'=2$ *mrad*. The kick according to the higher multipoles in the magnets should not be larger as 5% of the maximum slope x'[5]

$$\delta x' \leq 0.05 * 2\,mrad \leq 100\,\mu rad\,.$$

Using Eq. (8)

$$\Delta B \approx \frac{\delta x'.B\rho}{l},$$

the change of ΔB should be smaller than 2.7 mT in ILSF lattice.

For the quadrupole with the highest gradient of 23 T/m and good field region of ±18 mm,

$$\frac{\Delta B}{B_2} = \frac{2.7}{414} = 6.52 * 10^{-3}\cdot$$

For the sextupole with the maximum sextupole component of 700 T/m^2 and the kick of 1.1 mrad in good field region of ±12 mm

$$\frac{\Delta B}{B_3} = \frac{2.7}{50.4} = 5.35 * 10^{-2}\cdot$$

*The work is supported by ILSF.

Thus in order to have some margin (factor 2), the maximum relative field error in quadrupole and sextupole magnets must be less than 3E-3 and 2.5E-2 respectively.

$$\frac{\Delta B}{B_2} \le 3*10^{-3}$$

$$\frac{\Delta B}{B_3} \le 2.5*10^{-2}.$$

MULTIPOLE ERRORS OF DIPOLE MAGNETS

The systematic multipole contents of the magnetic flux for two families of ILSF storage ring bending magnets [4] with good field region of ±10 mm have been estimated in median plane by Fourier analysis in 2-D POISON code. The simulated systematic allowed multipole errors for the two families of bending (BE1 and BE2) [4] are listed in the Table 1 and Table 2. The kicks for each multipole component are listed in these tables. The absolute normalized multipole errors for both types of bending magnets at good field region of ±10 mm are shown in Fig. 1.

Table 1: Systematic multipole component field at r_0=10 mm for the bending magnet BE1

n	Pole	b_n [T/m^{n-1}]	ΔB/B	Kick (rad)
3	6-Pole	-1.73E+00	-1.26E-04	-2.40E-05
5	10-Pole	-2.39E+03	-1.73E-05	-3.31E-06
6	12-Pole	-6.97E+04	-5.05E-06	-9.65E-07
7	14-Pole	-2.28E+06	-1.65E-06	-3.15E-07
9	18-Pole	-4.61E+08	-3.34E-08	-6.38E-09
10	20-Pole	3.35E+11	2.43E-07	4.63E-08
11	22-Pole	-3.58E+13	-2.60E-07	-4.96E-08
13	26-Pole	4.94E+16	3.58E-08	6.83E-09
14	28-Pole	-3.30E+18	-2.39E-08	-4.57E-09

Table 2: Systematic multipole component field at r_0=10 mm for the bending magnet BE2

n	Pole	b_n [T/m^{n-1}]	ΔB/B	Kick (rad)
3	6-Pole	5.79E-01	4.25E-05	8.01E-06
5	10-Pole	4.99E+03	3.66E-05	6.91E-06
6	12-Pole	1.49E+05	1.09E-05	2.07E-06
7	14-Pole	2.25E+06	1.65E-06	3.12E-07
9	18-Pole	-3.39E+10	-2.49E-06	-4.70E-07
10	20-Pole	1.21E+12	8.90E-07	1.68E-07
11	22-Pole	1.76E+13	1.29E-07	2.43E-08
13	26-Pole	-1.32E+17	-9.69E-08	-1.83E-08
14	28-Pole	2.48E+18	1.82E-08	3.44E-09

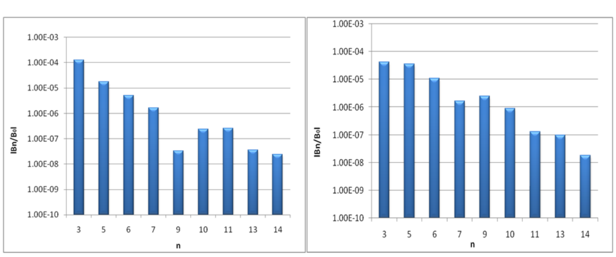

Figure 1: Absolute normalized multipoles errors for bending magnet BE1 (left) and BE2 (right).

The maximum value of kick produced with multipole errors in BE1 and BE2 is 0.024 mrad. It is worthwhile to mention that the kick generated by the strongest sextupole (700 T/m^2) with the length of 220 mm at the same radius (10 mm) is about 0.77 mrad which is roughly 32 times larger than bending component kick. This means that the higher systematic multipoles of bending should not have any influence on the dynamic aperture. Fig. 2 gives a comparison between dynamic aperture of ILSF ring for the cases of without multipole errors and with multipole errors. As depicted, reduction of dynamic aperture is ignorable.

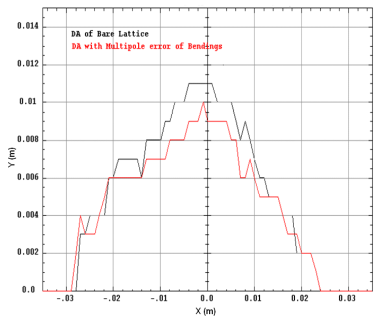

Figure 2: Dynamic aperture of ILSF ring for two cases of without multipole errors and with multipole errors in dipoles.

MULTIPOLE ERRORS OF QUADRUPOLE MAGNETS

Since only infinitely wide hyperbolic poles create a pure quadrupole field, we expect the appearance of higher multipole field errors characteristic for a finite pole width. The systematic multipole contents of the magnetic flux in the good field region of ±18 mm have been estimated by Fourier analysis of the flux distribution in the median plane of quadrupole magnet. The simulated systematic allowed multipole errors and their kicks for the ILSF quadrupole magnets are listed in the Table 3 and plotted in the Fig. 3 (left).

Table 3: Multipole contents of ILSF focusing quadrupole magnets at r_0=18 mm

n	Poles	b_n [T/m^{n-1}]	ΔB/B	Kick (rad)
6	12-pole	1.49E+04	6.77E-05	1.487E-06
10	20-pole	4.96E+09	2.37E-06	5.209E-08
14	28-pole	-1.01E+16	-5.05E-07	-1.109E-08
18	36-pole	-1.37E+22	-7.22E-08	-1.585E-09
22	44-pole	-2.30E+27	-1.27E-09	-2.796E-11
26	52-pole	-5.41E+33	-3.14E-10	-6.902E-12

The maximum value of kick produced with multipole errors in the strongest quadrupole 1.48 µrad which is too small in comparison with the kick of the strongest sextupole (1.1 mrad). This means that the systematic higher multipoles of quadrupole should not have any influence on dynamic aperture, Fig. 4.

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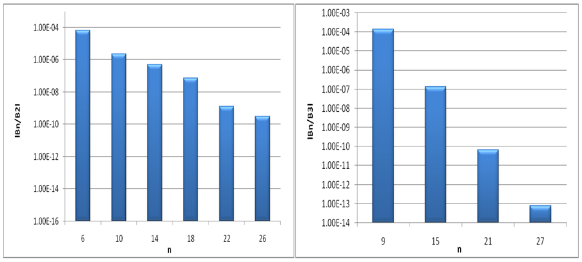

Figure 3: Absolute normalized multipoles errors for quadrupole magnets at the ± 18 mm good field region (left) and sextupole magnets at the ±12 mm good field region (right).

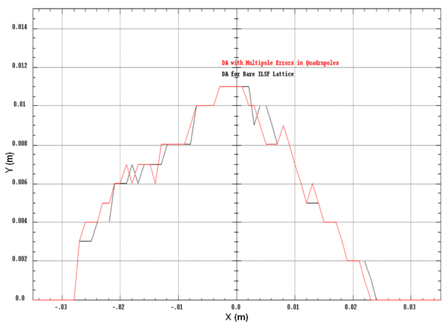

Figure 4: Dynamic aperture of ILSF ring for two cases of without multipole errors and with multipole errors in quadrupoles.

MULTIPOLE ERRORS OF SEXTUPOLE MAGNETS

The relative multipole contents for ILSF sextupole at ±12 mm good field region are listed in Table 4 and plotted in the Fig. 3 (right). It is clear that the relative kick of each component to main kick of sextupole field is too small to affect the dynamic aperture, Fig. 5.

Table 4: Multipole contents of ILSF sextupole magnet

n	Poles	b_n [T/m^{n-1}]	ΔB/B	Kick
9	18-pole	-1.608E+10	-1.372E-04	-1.52E-07
15	30-pole	-5.267E+18	-1.342E-07	-1.49E-10
21	41-pole	8.866E+26	6.7443E-11	7.47E-14
27	54-pole	3.435E+35	7.802E-14	8.65E-17

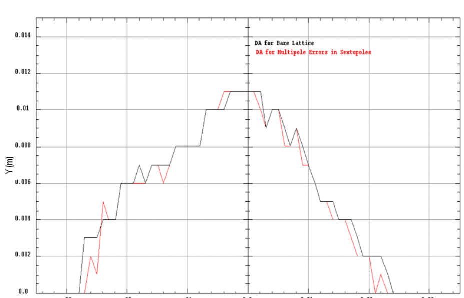

Figure 5: Dynamic aperture of ILSF ring for two cases of without multipole errors and with multipole errors in sextupoles.

MULTIPOLE ERRORS OF ALL MAGNETS

The systematic errors in the magnets reduce the dynamic aperture especially in vertical direction. Fig. 6 shows dynamic aperture by 3000 turns tracking of on and off momentum electrons with multipole errors in all kind of magnets which is comparable with dynamic aperture of bare lattice. To clarify these effects, Fig. 7 gives a comparison of transverse tune shift with amplitude for bare ILSF lattice and ILSF lattice with higher multipole errors.

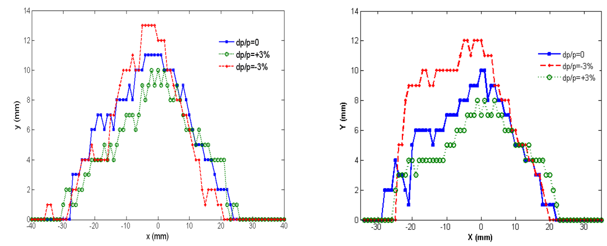

Figure 6: Dynamic aperture with multipole errors in all kind of magnets (left), and without any errors (right).

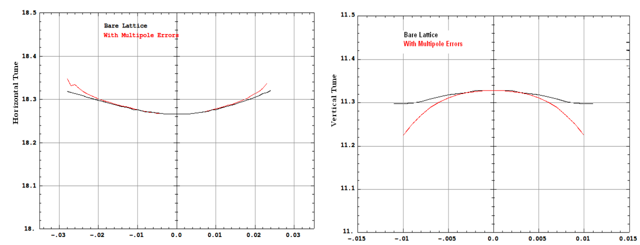

Figure 7: Tune shift with horizontal (left) and vertical (right) amplitude with and without multipole errors.

CONCLUSIONS

Results of the systematic errors in the magnets of ILSF storage ring are presented. The sextupole component in the bending magnets as it seems does not provide significant effect on the dynamic aperture. Multipole field components in the quadrupoles and sextupoles also seem not too dangerous up to the relative field error 3E-3 and 2.5E-2 respectively.

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