SIX-DIMENSIONAL BUNCH MERGING FOR MUON COLLIDER COOLING*

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Abstract

Muons for a Muon Collider are diffusely produced from pion decay. They are first phase rotated into a train of bunches. The trains are ionization cooled in all six dimensions until they can be merged into single bunches, one of each sign. They are then further cooled in six dimensions before acceleration and injection into the collider. This merging matches more efficiently into the second phase of cooling if the merging is also in six dimensions. A scheme to do this is proposed. Groups of 3, of the initial 12, bunches are merged longitudinally into 4 longer bunches, using rf with multiple harmonics. These 4 are then kicked into 4 separate (trombone) channels of different lengths to bring them to closely packed transverse locations at the same time. Here they are captured into a single bunch with now increased transverse emittance.

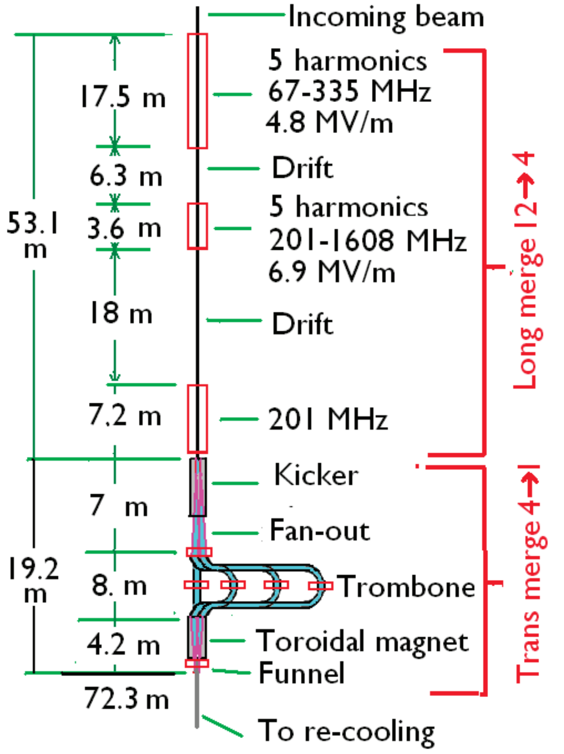

Figure 1: Schematic of bunch merging method.

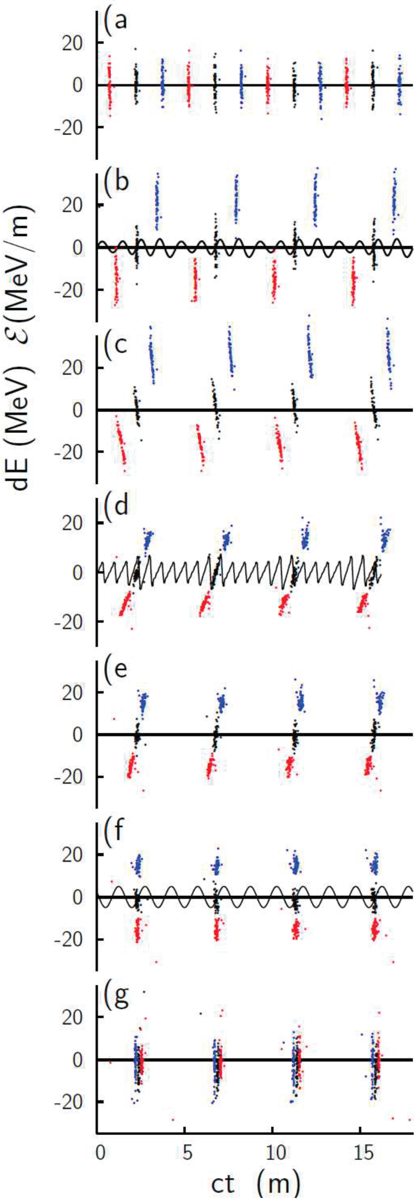

Figure 2: Phase space distribution at locations along the longitudinal merge: a) z=0: start; b) z=17.1 m: end of rf with shifted phase; c) z=23.4 m: end of drift; d) z=28.8 m: start of saw tooth rf; e) z=36.9 m: start of drift; f) z=45.9 m: start of final phase rotation; g) z=53.1 m: end.

INTRODUCTION

Muons, generated by pion decay, have very large emittances. A muon collider requires low emittances, which can be achieved[1] using transverse ionization cooling, combined with emittance exchange using dispersion and shaped absorbers. For efficient capture, muons are first phase-rotated[2] by rf into a train of many bunches. But for high luminosity, we need just one bunch of each sign, so after some initial cooling, these bunches should be merged. They can then be re-cooled to recover from the resulting increase of their emittances.

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Early studies merged only in the longitudinal dimension (2D). A recent design[4], using a helical channel is efficient and compact, but, as with any 2D scheme, the resulting bunches have large longitudinal, but still small transverse emittances, and do not match well into further 6D cooling.

A 6D merge, in contrast, can match very well into a 6D cooling system essentially identical to that used before the merge (see Fig.6 and discussion in the final section).

6D MERGE

Figure 1 shows a schematic of the 6D merge. The system captures 12 bunches. The initial longitudinal merge combines groups of three of these into 4 combined bunches. This is followed by a transverse merge that combines the 4 into one.

Longitudinal Merge

The longitudinal merge is based on the inverse of the 'phase rotation' scheme[2] that breaks a single initial distribution into 12 bunches. rf is used to both hold individual bunches together and, by shifts of phase, to accelerate those to be merged with an earlier bunch or decelerate bunches to be merged with a later bunch. In the phase rotation case, the needed phase rf timing can be achieved by adjusting the frequencies as a function of position. This is not possible here because, to combine groups of 3, the rf is restricted to harmonics of the spacing of those groups (67 MHz). Instead, the needed phase shifts can be generated by sums of harmonics of 67 MHz. This also allows the accelerations and decelerations to be asymmetrical, as required to counter the nonlinearity of particle's velocities vs. energy. Table 1a lists the frequencies and maximum gradients used.

Table 1: Maximum rf gradients for frequencies in a) the initial rf, and b) in the later saw tooth rf in the longitudinal merge.

a) First rf		b) Second rf	
Freq. MHz	Max grad MV/m	Freq. MHz	Max grad MV/m
67	4.3	201	8.4
134	8.7	402	12.8
201	13.0	603	14.2
268	14.0	1005	16.1
335	14.8	1407	17.5

The simulation of this part of the system was done in only one dimension, which is good if the transverse focusing is weak enough to make transport velocities essentially independent of amplitude. Harmonic rf is simulated as though all harmonics were superimposed on a single cavity, although in reality, and assumed for Table 1, separate cavities would be used.

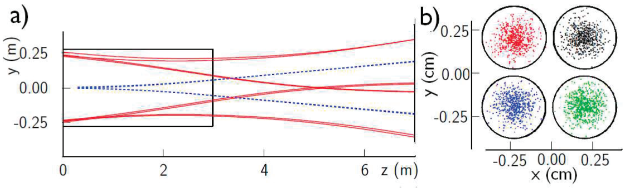

Figure 3: a) 4-sigma outlines of the four bunches in the kicker and fanout. b) transverse positions of simulated tracks at the end of the fanout.

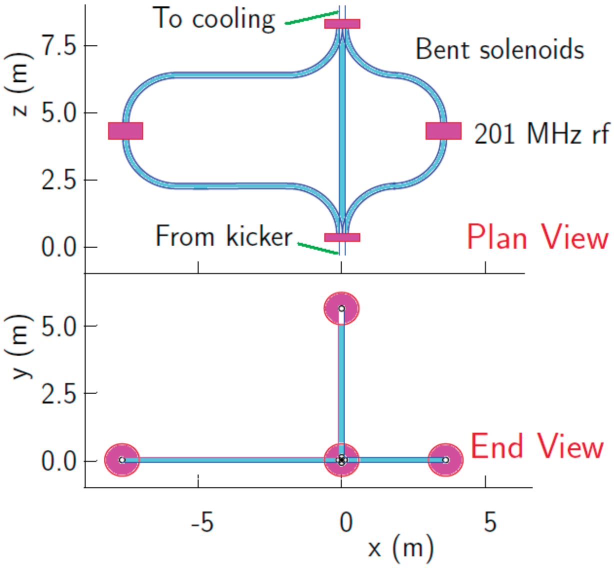

Figure 4: a) plan, and b) end view of solenoid channels of the trombone.

Fig. 2 gives, at different positions along the channel, simulated muon energies vs the time of a 130 MeV reference muon. The colors for the muon energies are to differentiate the sources of muons as they are being combined. When there is rf present, the sums of the rf fields are plotted in black.

Fig. 2a shows the 12 initial bunches spaced at 201 MHz. Fig. 2b shows the bunches at 17.1 m where their energies have been separated into three ranges: low (red), normal (black), and high (blue). Fig. 2c, at 28.8 m, shows them after a drift in which the higher energy muons (blue) have moved to earlier times, and the lower energies have fallen to later times. Fig. 2d, at 28.8 m, shows them after a short section of harmonic generated saw tooth rf whose parameters are given in Table 1b. This rotates the individual bunches so they fall, approximately, on a single curved line. Fig. 2e, at 36.9 m, shows the muons in the second drift section as they are approaching the same times as their neighbors, shown in Fig. 2f, at 45.9 m. At this position, to reduce the energy spreads of the combined bunches, simple 201 MHz is introduced that, by z=53.1 m has phase rotated them as shown in Fig. 2g. We now have the required 4 merged bunches, each of a different group of 3 originals, spaced at 67 MHz.

Table 2: Trombone parameters.

Common Parameters

Solenoid fields	1.59	T
Curvatures	0.5678	m^{-1}
Dipole fields	0.4	T
Arc lengths	2.766	m
rf frequencies	201	MHz
Initial & final rf lengths	0.3	m
Initial & final rf gradient	8.6	MV/m

Individual parameters

Channel	Len.	rf grad.	$\epsilon_\perp$	$\epsilon_\parallel$	Transm
	m	MV/m	mm	mm	%
0	0		1.20	8.92	100
1	8.25	0	1.20	8.90	100
2	12.27	5.8	1.23	8.85	99
3	16.20	12.2	1.25	9.05	98
4	20.30	17.5	1.24	9.21	98
ave.	14.25		1.23	9.00	98.75

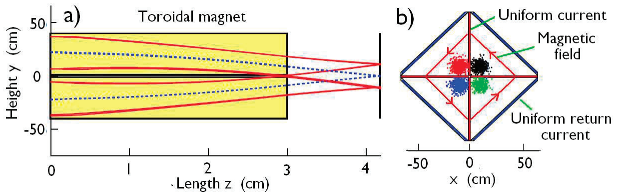

Figure 5: Funnel: a) vertical 4-sigma beam limits vs. length; b) transverse track positions at the end of the toroidal magnet.

Transverse merge

To merge in transverse phase space, the 4 bunches are first separated using a kicker whose transverse field rotates at a frequency of 201/12 = 16.75 MHz. The kicker is 3 m long and 30 cm radius, with a field of 120 Gauss. Following the kicker there is a 4 m fanout drift where the beams separate. The simulated 4-sigma vertical outlines of the four beams are shown in Fig. 3a. The separated beams are then matched into 4 separate solenoid channels (trombones)[3] whose lengths and other parameters are given in Table 2. The plan and elevations of these channels is shown in Fig. 4. Table 2 also gives the transmissions and emittance dilutions obtained from 3D simulations using ICOOL[5]

The outputs from the four trombones are now transversely merged using a toroidal septum magnet whose fields have 4 segments which, ignoring the ends, are pure dipoles. A full 3D simulation of this was performed, but with end effects ignored. Fig. 5a shows the 4-sigma bounds of the four bunches as they are combined into one beam with larger transverse emittance This would be followed by a section matching the combined beam into further 6D cooling.

Table 3: Performance

Initial transverse emittance $\epsilon_\perp$	1.3	mm
Initial longitudinal emittance $\epsilon_\parallel$	1.7	mm
Final transverse emittance $\epsilon_\perp$	3.5	mm
Final longitudinal emittance $\epsilon_\parallel$	9.0	mm
Transm. from particle loss	92	%
Transm. from decay	94.5	%
Overall transmission	87	%

Performance

The initial and final emittances are given in Table 3. The transmission including decay is 87%. As noted, these results are from simulations that are not fully realistic: only 1D simulation with superimposed harmonic rf frequencies in the longitudinal merge, and idealized simulation of the kicker and final funnel ends. More work is needed.

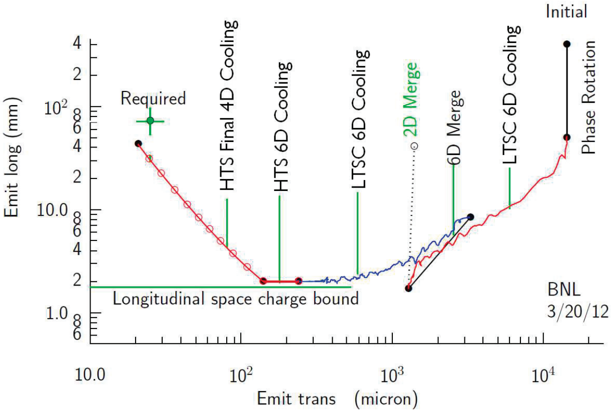

Figure 6: Longitudinal emittances vs. transverse emittances for stages of muon manipulation from capture to start of acceleration. The dotted line is for a 2D merge[4].

APPLICATION

Fig. 6 shows the transverse emittances vs. the longitudinal emittances from the initial muon capture, through initial 6D cooling, the merge discussed here, more 6D cooling after the merge, and final transverse cooling in high field solenoids. It is seen that the output of this 6D merge matches well into a 6D cooling channel essentially identical to that used before the merge.

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