ELECTROMAGNETIC FIELD OF CHARGED PARTICLE BUNCH MOVING IN WIRE METAMATERIAL*

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Abstract

We consider the field of bunch flying through a "wire metamaterial" Analytical and computational investigations are carried out. In the case of motion perpendicularly to the wires it is shown that the radiation concentrates in a small vicinity of the determined lines behind the bunch and the Pointing vector is directed along the wires. This phenomenon can be useful for charged bunch examination. Some calculations show that the measurements of electrical field intensity and energy flow density allow determining the length of the bunch and its velocity. The case of bunch moving along the wires is also examined. It is shown that the radiation can be generated only for the wires possessing non-conducting coating. The radiation is directed at a sharp angle to the wires.

STRUCTURE DESCRIPTION

Nowadays, a lot of detectors of charged particles are based on Cherenkov radiation (CR) [1]. But all traditional methods of CR generation produce waves that essentially decrease with distance from the charge. As we will show further, this defect can be partially overcome in case of using some special structure called wire metamaterial (Fig. 1). It is a periodic structure made of long conductive wires with radius r_0; they can have dielectric coating with radius r_d and permittivity ε. The spacing d is assumed to be small in compare with distance of an average field variation. Therefore the structure can be considered like a medium.

Such structure without coating was investigated in many papers [2, 3, 4]. The case of coated wires was analyzed in works [5, 6]. The effective permittivity tensor of the structure has the following form (x-axis is parallel to the wires) [6]:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\| & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \varepsilon_\| = 1 - \frac{\omega_p^2}{\omega^2 - \chi^2 k_x^2 c^2 + 2i\,\omega_d\,\omega},$$

(1)

where $\omega_p^2 = 2\pi c^2 d^{-2} \left[\ln(d/r_0) - C\right]^{-1}$ $\chi^2 = 1 + \left(\varepsilon^{-1} - 1\right)\ln(r_d/r_0)\left[\ln(d/r_0) - C\right]^{-1}$, ω_d is a small constant responsible for energy losses ($\omega_d \ll \omega_p$), and C is some constant (in the literature, different estimations of C can be found; however, all of them yield values on the order of 1). One can see that the "meta-medium" under consideration possesses both frequency and spatial dispersion.

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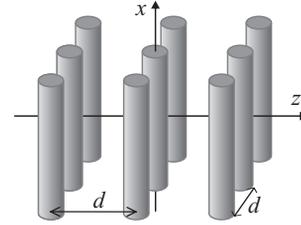
Figure 1: Scheme of the structure.

PARALLEL MOVEMENT

At first, parallel movement of the charge with velocity $\vec{V} = V\vec{e}_x = c\beta\vec{e}_x$ is considered. One can show that, in this case, radiation can be generated only under condition $\beta > \chi$. Thus, the wires should possess non-conducting coating (otherwise $\chi = 1$ and mentioned condition cannot be met). It can be shown that the wave part of the electromagnetic field has only components E_ρ, E_x and H_φ in cylindrical coordinate system ($y = \rho\cos\varphi$, $z = \rho\sin\varphi$).

In order to describe the behavior of radiation we use the full energy emitted by the charge through the unity square for the whole time of motion: $\vec{W} = \int_{-\infty}^{+\infty} \vec{S}dt$, where $\vec{S} = c/(4\pi)\left[\vec{E} \times \vec{H}\right]$ is a Poynting vector. Using some transformations we can obtain the spectral decomposition: $\vec{W} = \int_0^{+\infty} \vec{w}(\omega)d\omega$. The dependence of spectral density of radiation $w = \sqrt{w_\rho^2 + w_x^2}$ on the angle $\theta_g = \arctan(w_\rho/w_x)$ is shown on the figure 2. As one can see, the angle of the maximum radiation is quite sharp, and it is sharper than higher the velocity of the charge. This angle is less than typical angles of radiation in ordinary media with refractive index > 1.
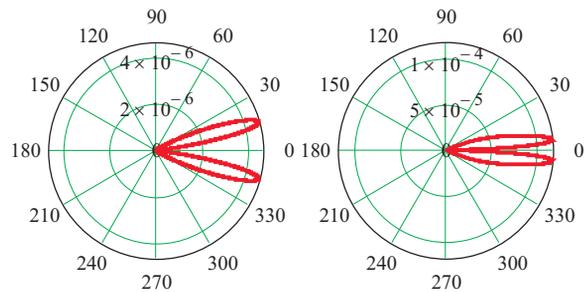
Figure 2: Polar diagram $w(\theta_g)$ (in unity J · s/m²) for the case of parallel movement at velocity $\beta = 0.95$ (left) and $\beta = 0.99$ (right). $r_d = 0.07$mm, $r_0 = 0.05$mm, $d = 1$mm, $\varepsilon = 5$, $|q| = 1$nC.

Note that the case of parallel motion is considered as well in works [7, 8] where the structure is assumed to be put into a cylindrical waveguide. The waveguide modes generated in this situation can be used for determination of energy of particles.

PERPENDICULAR MOVEMENT

Now we consider the case of charge moving perpendicularly to the wires with a constant velocity $\vec{V} = V\vec{e}_z = c\beta\vec{e}_z$ then the current density is $\vec{j} = \vec{e}_z c\beta q\delta(x, y, \zeta)$, where $\zeta = z - c\beta t$. In this case radiation exists even for wires without coating. Therefore, for simplicity, we consider the structure with $\chi = 1$. It is important as well that radiation is generated at arbitrary velocity of the charge.

We suppose that the observation point is far enough from the line of charge motion so that "quasi-static" part of the field is negligible. It can be shown that propagating waves have the following form [9, 10]:

$$E_y^W = -2\beta q \frac{y(\zeta + \beta|x|)}{\xi^4} \left[1 - \frac{\omega_p^2}{2c^2}\xi^2 K_2\left(\frac{\omega_p \xi}{c}\right)\right],$$
(2)

$$E_z^W = -\beta q \left[\frac{y^2 - (\zeta + \beta|x|)^2}{\xi^4} - \frac{\omega_p}{\xi c} K_1\left(\frac{\omega_p \xi}{c}\right) - \frac{(\zeta + \beta|x|)^2 \omega_p^2}{\xi^2 c^2} K_2\left(\frac{\omega_p \xi}{c}\right)\right],$$
(3)

where $\xi^2 = y^2 + (\zeta + \beta|x|)^2$. The magnetic field is simply expressed through electric one: $H_z^W = E_y^W \operatorname{sgn} x$, $H_y^W = -E_z^W \operatorname{sgn} x$.
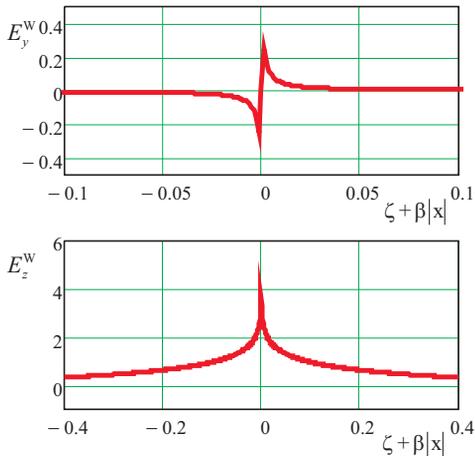
Figure 3: Electric wave field components of a point charge as a function of $\zeta + \beta|x|$ for $y = 0.001c/\omega_p$. Distances are given in units of c/ω_p, and field intensity in units of $q\omega_p^2/c^2$.

One can show that components E_y^W and H_z^W tend to zero on the lines $\zeta = -\beta|x|$, $y = 0$, and compo-

nents H_y^W and E_z^W each have a logarithmic singularity on these lines. When the viewpoint is shifted along these lines, the wave field does not vary. The Poynting vector $\vec{S} = c(4\pi)^{-1}\left[\vec{E} \times \vec{H}\right]$ consists only of x-component and proportional to the full electric wave field squared. The energy flow through some square $\int_\Sigma \vec{S}d\vec{\Sigma}$ is limited and does not depend on the distance from the charge trajectory. Wave field components decrease proportionally to $R^{-2} = \left(x^2 + y^2 + \zeta^2\right)^{-1}$ far from the lines $\zeta = -\beta|x|$, $y = 0$, under the condition $\omega_p \xi/c \gg 1$.

Thus, a charge moving perpendicularly to the wires emits CR concentrated within some small vicinity of the lines $\zeta = -\beta|x|$ in the plane $y = 0$ behind the charge. These waves propagate along the wires and exist for an arbitrary velocity of the charge. This phenomena can be explained by properties of plane waves in the medium under consideration [9]. Figure 3 shows the behavior of the electric field components as a function of $\zeta + \beta|x|$ (note that the dependencies on y are similar).

FINITE CHARGED PARTICLE BUNCH

Further we analyze the field of cylindrical bunch moving perpendicularly to the wires (along z-axis). Charge density is $\rho = q/(2\sigma\pi\eta^2) = const$ for $|\zeta| \leq \sigma$ and $\sqrt{x^2 + y^2} \leq \eta$, and $\rho = 0$ outside the cylinder. The field of the bunch can be written in the form

$$\vec{E}(\vec{r}, t) = \iiint\limits_{-\infty}^{+\infty} \rho(\vec{r'}) \vec{E}_\delta(\vec{r} - \vec{r'}, t) d^3\vec{r'},$$
(4)

where $\vec{E}_\delta(\vec{r}, t)$ is the field of the point charge, described above, with $q = 1$.

Figure 4 represents computational results for bunches with different parameters. Here are presented the electrical components and the Poynting vector magnitude for the wave (Cherenkov) field on some plane $x = const$. This field coincides with the full field if the observation plane is far from z-axis. As we can see, the location of maximums and minimums of different components depend on parameters of the bunch. They partly "repeat" the bunch shape. It is especially notable for Poynting vector value.

Thus, the measurement of energy flow and field components can be used to determine the length, width and shape of the bunch. Note as well that the inclination of the lines of the radiation concentration can be applied to estimate the bunch velocity.

In conclusion we say some words on validity of results described above. The model of medium is valid if the distance of the averaged field variation is much greater than the structure period. One can show that it is equivalent to limitation $\omega < \omega_p$. Using the same energy characteristic as for charge moving parallel to the wires, we can estimate the part of the radiation spectrum which fits into this limitation (Fig. 5). As one can see, the most part of radiation even from the point charge satisfies this restriction. In the
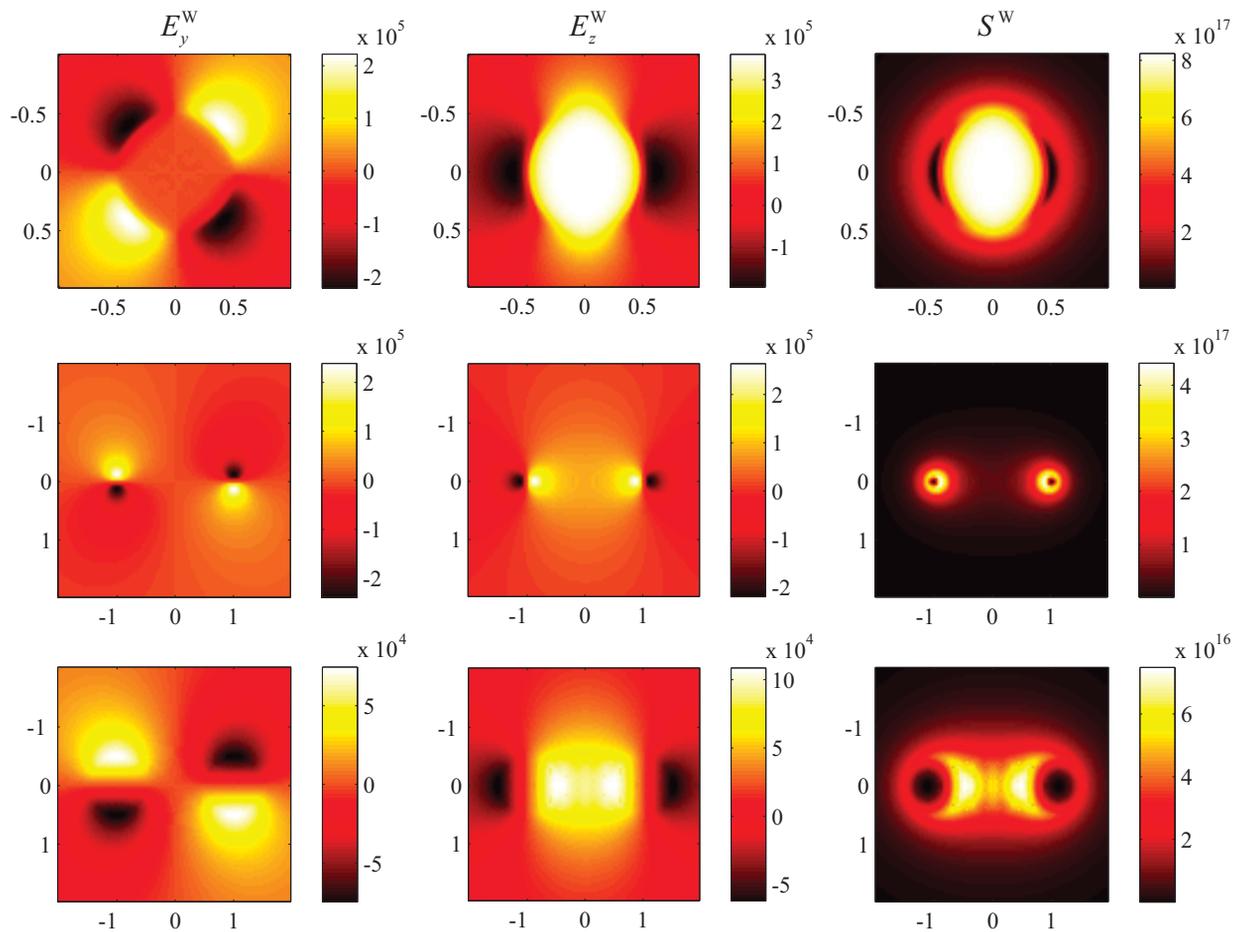
Figure 4: "Snapshots" of E_y^W (left column), E_z^W (middle column), and the absolute value of the Poynting vector $\vec{S}^W$ (right column) for different cylindrical bunches on the plane $x = \text{const}$. The first row: $\sigma = 0$, $\eta = 5\text{mm}$; the second row: $\sigma = 10\text{mm}$, $\eta = 1\text{mm}$; the third row: $\sigma = 10\text{mm}$, $\eta = 5\text{mm}$. $\zeta + \beta|x|$ is on the horizontal axis and y is on the vertical axis for each image. Energy flow is in W/m^2, electric field is in V/m. $\beta = 1$, $q = -1\text{nC}$, $r_0 = 0.05\text{mm}$, $d = 1\text{mm}$.

case of the finite bunch the role of high-frequency part of spectrum is less, and the medium model works better.
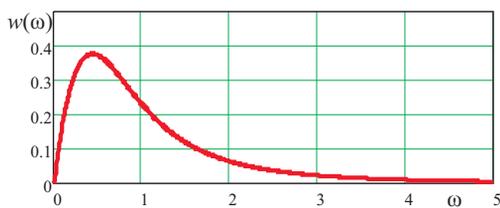
Figure 5: The spectral radiation density for a point charge moving perpendicularly in wire metamaterial. ω is given in units of ω_p and spectral density $w(\omega)$ is in normalized units.

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