MULTIPOLE EFFECTS STUDY FOR PROJECT X FRONT END CAVITIES*

P. Berrutti#, M. Awida, I. Gonin, F. Ostiguy, N. Solyak, V. Yakovlev
Fermilab, Batavia, IL 60510, USA

Abstract

Effects of RF field asymmetry along with multipoles have been studied in Project X front end cavities. One family of half wave resonators operating at 162.5 and two of spoke resonators operating at 325 MHz have been analysed. HWR and spoke resonators unlike elliptical cavities, do not have axial symmetry, hence a quadrupole perturbation to the beam is present. The purpose of this paper is to explain the approach and the calculation method used to understand and overcome the drawbacks due to the RF field asymmetry.

INTRODUCTION

Project-X is a multi-MW proton source under development at Fermilab[1]. The facility is based on 3 GeV 1 mA CW and on a 8 GeV pulsed superconducting linac. The CW linac starts with a 162.5 MHz half wave resonator section having optimal β = 0.11 (2.1MeV – 10.8 MeV). After that there is a single spoke resonator part at 325 MHz, SSR1 having β = 0.21 (10.8MeV - 35 MeV). SSR2 is another single spoke resonator operating at 325 MHz and having optimal β = 0.47, beam energy goes from 35MeV to 165 MeV. The high energy part of the CW linac is made of two families of 650 MHz, five cells elliptical cavities having β = 0.61 and 0.9 (165MeV - 3 GeV). Simulating the whole linac lattice using TraceWin a RF field asymmetry effect has been noticed for HWR and SSR2 cavities. In the low energy section of Project X the focusing is provided by solenoids, these devices provide radial symmetric focusing, which does not correct asymmetry into the x-y beam envelope. SSR2 and HWR designs are being finalized now while SSR1 is already built and tested [3]. In this paper a method to study the quadrupole and higher order multipoles is presented, and it is suitable for any cavity mentioned above. In addition, the dependence of the field asymmetry effect on the particle β has been investigated.

EM FIELD ASYMMETRY

Spoke and half wave resonator geometries have a central electrode that lies on one of the axes perpendicular to the particles motion, breaking axial symmetry of the cavity. Figure 1 shows HWR [2] and SSR2 geometries, in these structures the fundamental mode is quasi-TEM, the electric field is perpendicular to the central conductor while the magnetic field lines surround the spoke, spinning around it, Figure 2.

*Work supported by D.O.E.
#berrutti@fnal.gov

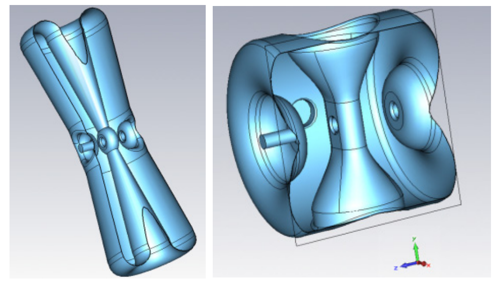

Figure 1: HWR and SSR2 3D geometries.

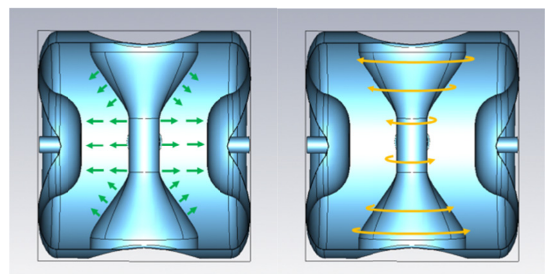

Figure 2: electric and magnetic field SSR2 fundamental mode.

Having asymmetric transverse fields, these cavities introduce a perturbation to beam dynamic since a particle will be subject to non-uniform radial kick. This is an issue since the focusing in HWR, SSR1 and SSR2 cryomodules relies upon solenoids, which provide uniform radial correction. The effect of field asymmetry on beam dynamic has been studied for SSR2 section [4] and further analyses have been conducted including HWR cavity as well. Lorentz's force and Panofsky-Wenzel theorem are the two methods of calculation used to evaluate the transverse momentum gain for a particle traveling through a cavity. Considering the particle velocity constant along z axis, β is constant, one can write:

$$\Delta p_x(r,\alpha)c = \int_{z_i}^{z_f}\left(\frac{E_x(r,\alpha)}{\beta} - \mathbf{Z}_0 i H_y(r,\alpha)\right) e^{i\frac{kz}{\beta}} dz \quad (1)$$

$$\Delta p_y(r,\alpha)c = \int_{z_i}^{z_f}\left(\frac{E_y(r,\alpha)}{\beta} + \mathbf{Z}_0 i H_x(r,\alpha)\right) e^{i\frac{kz}{\beta}} dz \quad (2)$$

using Lorentz, and

$$\Delta p_\perp(r,\alpha)c = \frac{i}{k}\int_{z_i}^{z_f} \nabla_\perp E_z(r,\alpha)\, e^{i\frac{kz}{\beta}} dz \quad (3)$$

using Panofsky-Wenzel theorem.

$\Delta p_x(r,\alpha)c$ and $\Delta p_y(r,\alpha)c$ are functions of the radius and the angle α on the x-y plane, α is taken with respect to the x axis: α=0 corresponds to x axis and α=π/2 refers to y axis.

The radial component variation will be maximum between the kick on x and on y axis, so to have an

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estimation of the asymmetry one can define a parameter called Q:

$$Q = \frac{\Delta p_x(r,0)c - \Delta p_y(r,\pi/2)c}{(\Delta p_x(r,0)c + \Delta p_y(r,\pi/2)c)/2}. \qquad (4)$$

This parameter has been used as an indicator of field asymmetry during the design of SSR2 and HWR cavities. Q is a ratio between the quadrupole amplitude and an averaged kick value equivalent to a monopole, which leads to a uniform radial kick. Q does not carry information regarding higher order multipoles, if one wants to have a complete picture of the radial component $\Delta p_R c(r,\alpha)$, a multipoles expansion has to be done. The behaviour of Q with the RF phase has been studied and since $\Delta p_x(r,0)c$ and $\Delta p_y(r,\pi/2)c$ are synchronous in phase the asymmetry parameter is constant in phase, Figure 3 shows results of calculation made for SSR2 cavity at particle β=0.47.

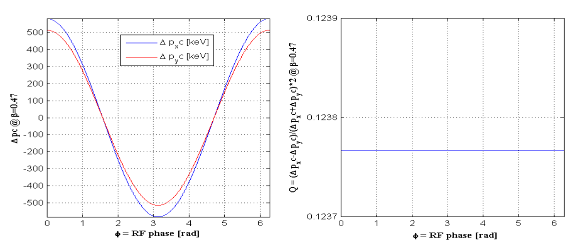

Figure 3: transverse momentum and Q parameter as a function of RF phase, SSR2 cavity at β=0.47.

Since each cavity in Project X front end will be used in a wide range of energy it was necessary to investigate the behaviour of the Q parameter in the whole β range. Figure 4 shows Q vs β/ β_{opt} in the whole range of usage of every cavity in the front end.

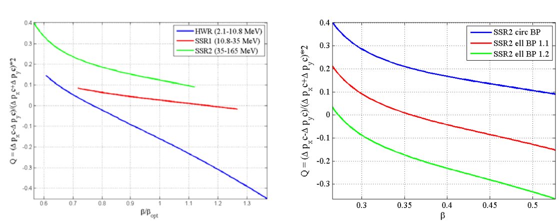

Figure 4: Asymmetry parameter Q vs β: HWR, SSR1 and SSR2 (left), BP shapes comparison for SSR2 (right).

SSR1 field asymmetry is the least significant since Q is smaller than 0.1 in the whole range of energy. HWR geometry used for this calculation is the very first one, the actual version shows a much better Q dependence on β, SSR2 plot refers to the actual cavity geometry, that has circular beam pipe and circular beam aperture in the spoke. The area around the beam pipe is the one that influences the most EM transverse field asymmetry: changing the shape of the beam tube holes, makes possible to change Q characteristic versus β. Using a racetrack or an elliptical shape, with an optimized x/y ratio, allows to set the zero point of the Q vs β curve in the range of energy in which the cavity is used. Figure 4 on the right shows SSR2 Q vs beta for different x/y ratios. This modification will set a zero point, but it will give only local compensation. To partially overcome the symmetry limitation due to SSR2 and HWR geometries, one can think about shaping the central part of the spoke using a radial symmetric shape, for example a ring. This modification leads to symmetry within a certain radius, it helps having a symmetric electric field around the beam area. The ring geometry of HWR and SSR2 are compared with regular spoke geometry in Figure 5.

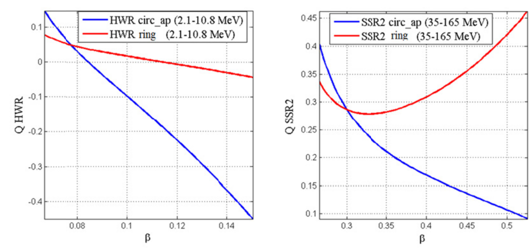

Figure 5: Q vs β for HWR and SSR2, comparison between regular spoke and ring [2].

From Figure.5, it is evident how HWR asymmetry is reduced by the radial symmetric central electrode. The ring shape symmetrizes the transverse electric field but it does not modify the magnetic, this is the reason why SSR2 ring shows more asymmetry than the previous version. To understand this fact, one should look at the electric and magnetic kick separately. The magnetic field is not influenced by the ring shape because it spins around the spoke electrode, one of its components will always be greater than the other near the beam tube aperture, Figure 2. Moreover the magnetic contribution to $\Delta p_R c$ has a different dependence on β than the electric one: the electric transverse kick decrease significantly while the magnetic contribution is of the same order of magnitude in the whole β range.

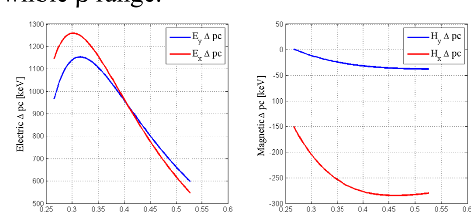

Figure 6: SSR2 electric and magnetic transverse kick as function of β.

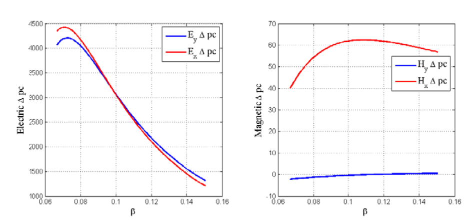

Figure 7: HWR electric and magnetic transverse kick as function of β.

In Figure 6 electric and magnetic transverse kick components are shown, this plot is for SSR2 having optimal β=0.47, in HWR cavity, which is designed for a

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lower optimal β, the magnetic field does not contribute much to the transverse momentum gain and the ring shape for the central electrode is suitable to minimize the field asymmetry, see Figure 7.

MULTIPOLES FIELD EXPANSION

Asymmetry parameter Q gives partial information on the transverse kick, evaluating $\Delta p_R c$ on the whole x-y plane, Figure 8, it is possible to calculate the multipole expansion of the transverse momentum gain.

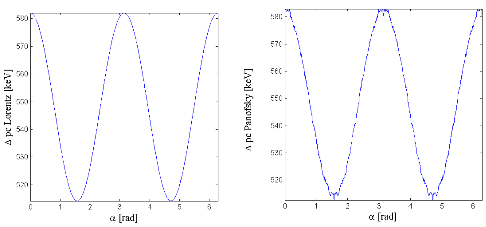

Figure 8: $\Delta p_R c$ as a function of the angle α, SSR2 cavity, Lorentz force and Panofsky-Wenzel theorem evaluations.

Expanding $\Delta p_R c$ one gets:

$$\Delta p_R c(r,\alpha)=A_0(r)+\sum_{n=1}^{\infty} A_n(r)\cos(n\alpha)+B_n(r)\sin(n\alpha) \quad (6)$$

where $A_n(r) \propto nr^{n-1}$ and $B_n(r) \propto nr^{n-1}$ are the Fourier series coefficient of $\Delta p_R c$ while $A_0(r) \propto r$ and it is the mean value; one set out of two has non zero values and the other vanishes. Normal or skew components are due to the symmetry of the problem and to the choice frame of reference to define x-y plane. This expansion can be applied to either Lorentz's force or Panofsky-Wenzel theorem, it gives information on the amplitude of dipole, quadrupole, sextupole, octupole, decapole, dodecapole and so on. The transverse kick has been expanded up to 16-pole in this paper, corresponding to $n = 8$, because higher components are not significant. $\Delta p_R c$ shows a periodic dependence on the angle α, this is due to the spoke and half wave resonator geometries: the central electrode lies on one of the two transverse directions breaking the symmetry of the cavity. Multipoles coefficients, for a particle traveling through SSR2 cavity (circular aperture) at $\beta = 0.47$, are reported in table 1, they are divided by r^{n-1} to show the dependence on r; Fourier kick components calculated at $r = 10$ mm and at $\beta = 0.47$ as a function of the angle α are in Figure 9. Looking at Figure 8 and table 1, it is clear that quadrupole is the main component of the kick expansion. Changing the beam aperture shape, either in the central electrode or in the end walls, helps reducing the 2nd harmonic only for a very small beta range. Higher order multipoles are not significant in SSR2 cavity and, as expected, odd order multipoles are completely negligible. Starting from the 6th harmonic coefficients become very small and they can be considered noise as odd amplitudes.

Table 1: Normalized Multipole Amplitude, SSR2 Circular BP Aperture at $\beta = 0.47$.

A_n/r^{n-1}	r=5 mm	r=10 mm	r=15 mm
1st [keV]	1.199e-14	5.812e-14	9.140e-14
2nd [keV/mm]	3.385	3.391	3.402
3rd [keV/mm^2]	7.998e-16	4.331e-16	3.351e-16
4th [keV/mm^3]	6.962e-4	4.747e-4	4.757e-4
5th [keV/mm^4]	4.935e-17	4.957e-18	1.805e-18
6th [keV/mm^5]	5.060e-06	2.338e-08	1.135e-08
7th [keV/mm^6]	1.133e-18	1.367e-19	4.413e-21
8th [keV/mm^7]	1.2039e-07	2.719e-10	2.162e-10

It is easy to show that $Q = 2 * A_2(r)/A_0(r)$, so the asymmetry parameter is a measure of the quadrupole amplitude over the monopole. As a proof of what is reported in the previous section table 2 compares quadrupole amplitudes of HWR straight and ring electrode geometries:

Table 2: HWR Quadrupole Comparison = 0.11 .

HWR straight spoke	HWR ring spoke
18.8 keV/mm	1.43 keV/mm

CONCLUSIONS

A method for a complete study and understanding of field asymmetry and field multipoles has been presented, for half wave resonator and spoke cavities as well. Quadrupole component is present in these resonators because of the post conductor that breaks axial symmetry of the structure. It has been tried to achieve better field symmetry for HWR and SSR2 cavities, because of the solenoidal focusing in Project X front end. HWR field can be symmetrized using a ring shape for the inner conductor, because it affects the electric field, which gives the major contribution to $\Delta p_R c$ in the whole β range. SSR2 cavity shows a strong magnetic component that cannot be corrected with a ring shaped spoke. Changing the beam pipe transverse cross section locally compensates the field asymmetry, but only for a narrow beta range. The possibility of rotating of π/2 one cavity with respect to the neighbour is being considered at FNAL. Morover, this study showed that multipoles higher than quadrupole are not relevant for HWR, SSR1 and SSR2 cavities.

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