MEASUREMENTS OF MAGNETIC PERMEABILITY OF SOFT STEEL AT HIGH FREQUENCIES *

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Abstract

The Fermilab Booster does not have a vacuum chamber, which would screen the beam from laminations of its dipoles cores. Therefore the Booster impedance is mainly driven by the impedance of these dipoles. Recently an analytical model of the laminated dipole impedances was developed. However, to match the impedance measurements with calculations one needs an accurate measurement of soft steel magnetic permeability. This paper presents the measurements of high frequency magnetic permeability for soft steel similar to the steel of Booster laminations. The measurements were performed in a frequency range from ~10 MHz to 1 GHz and were based on a study of electro-magnetic wave propagation in 30 cm long transmission line built with help of steel strip. Measurements were performed in a DC magnetic field to observe the effect of steel saturation on the high frequency permeability. Both real and imaginary parts of the permeability were measured. As expected their values were decreasing with frequency increase from 10 MHz to 1 GHz and with saturation of steel DC permeability.

INTRODUCTION

Combined functions dipoles occupy 75 % of the Booster 474 meters circumference and make the major contribution to the Booster impedances at high frequencies. There are two types of dipoles: focusing and defocusing with half-apertures of 2.08 and 2.85 cm, correspondingly. Booster dipoles are driven at frequency of 15 Hz. That determines the thickness of laminations to be 0.025"=0.635 mm. In difference to most synchrotrons the Booster does not have a "normal" vacuum chamber inside dipoles, and instead, the laminations of dipole cores form "vacuum" chamber walls visible by the beam. That greatly amplifies the Booster impedance.

Recently an analytical model of the laminated dipole impedances was developed [1]. Combining its results with the impedance measurements performed with wire [2] we estimated high frequency magnetic permeability of the steel to be about 40. However an accurate decomposition of magnetic permeability dependence on the frequency with other details of dipole geometry was impossible. To address this deficiency we performed dedicated measurements of soft steel magnetic properties. They were based on the measurements of electromagnetic wave reflection and propagation in a 30 cm long micro-strip transmission line built from DI MAX M-22 steel, which properties are close to the steel used in Booster dipoles. Such a method allows one to perform measurements in a wide frequency range. In our measurements the frequency range of interest was 10 MHz to 1 GHz where the impedances are maximized.

In addition to the finite steel resistivity its magnetic permeability μ also affects the resistive losses. Thus, accurate measurements of signal damping in the transmission line allows one to decompose contributions of resistivity and magnetic permeability into the loss and compute the magnetic permeability as a complex function of frequency.

THEORETICAL BACKGROUND

A comparison of one-dimensional (1-D) and two-dimensional models of wave propagation in a flat transmission line showed that 1-D approximation is sufficiently accurate for chosen geometry of electrodes. Therefore only the 1-D model is considered below. It describes obtained experimental data sufficiently well.

The 1-D transmission line can be represented by an effective scheme, which comprises of series of infinitesimally small elements consisting of resistance, R, inductance, L, and capacitance, C, per unit length. The solution of equations obtained in a quasi-static approximation yields the so-called telegrapher's equations. In order to obtain the expressions, which describe the reflection and transmission of electromagnetic waves, one needs to satisfy the boundary conditions. Performing calculations one obtains the S-parameters, which are the amplitude reflection (S_{11}) and transmission coefficients (S_{21}):

$$S_{11} = \frac{i(\kappa^2 - 1)\tan kl}{2\kappa + i(\kappa^2 + 1)\tan kl}$$

$$S_{21} = \frac{2\kappa}{2\kappa \cos kl + i(\kappa^2 + 1)\sin kl} \quad (1)$$

where $\kappa = \rho / Z_0$, $Z_0 = 50\,\Omega$, $\rho = \sqrt{(R + i\omega L)/(i\omega C)}$ is a wave impedance of our transmission line, l – its length, $k^2 = -i\omega C(R + i\omega L)$ - wave number, ω is an angular frequency, i is an imaginary unit.

At each frequency point we have four non-linear equations (real and imaginary parts of S-parameters) and four parameters to determine (real and imaginary parts of the wave impedance and the wave number). By using numerical procedure these parameters can be restored and then we can calculate R (and μ), provided C and L are known. Capacitance and inductance were determined experimentally by using copper micro-strips, in which resistive losses are so small, that can be neglected. In addition, analysis of copper micro-strips S-parameters allowed us to make necessary corrections to 2-port calibration of the network analyzer used in the measurements [3]. The magnetic permeability was

determined from the resistance per unit length with the following expression [4]:

$$R = (1+i)\frac{r}{W}\sqrt{\frac{\omega\mu\mu_0}{2\sigma}}\left(\frac{1}{\pi}+\frac{1}{\pi^2}\ln\frac{4\pi W}{T}\right) , \qquad (2)$$

where for $W/H > 0.5$ the loss ratio is equal to

$$r = 0.94 + 0.132(W/H) - 0.0062(W/H)^2 ,$$

W is the width of a micro-strip, T is its thickness, and σ is its conductivity.

EXPERIMENTAL SETUP AND CALCULATION TECHNIQUE

It is more convenient to rewrite Eq. (1) in terms of $\chi = \exp(-i\omega\tau)$, where $\omega\tau = kl$; and numerically solve the obtained system of equations with respect to χ and ρ, since in this case there is no discontinuity in the arguments of trigonometric functions caused by their periodicity.

Once the wave impedance ρ_c and time delay τ_c of copper micro-strip are determined, the steel resistance R can be calculated from the following formula:

$$\tau_s = \tau_c\sqrt{1+\frac{R}{i\omega L}} , \qquad (3)$$

where τ_s is the time delay in steel micro-strip, $L = \rho_c \cdot \tau_c / l$ - inductance per unit length.

Agilent E5061B network analyzer was used in the measurements. The 85033D calibration kit was used for network analyzer calibration. Obtained data were analyzed with the help of MATLAB.
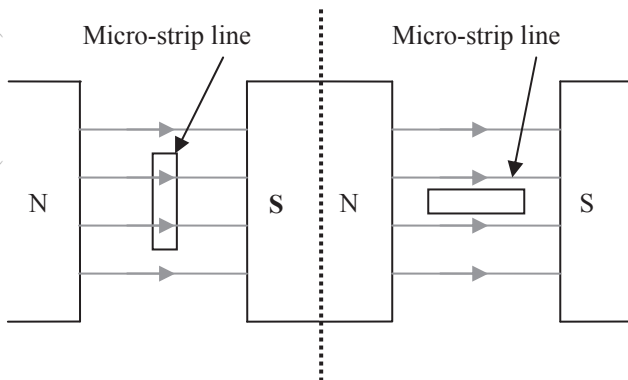

Figure 1: Schematics of the experiment with steel in DC magnet. The normal orientation is represented on the left, and the parallel one on the right.

For initial measurements with copper strip-line, the line was etched on one side of the G10 double sided printed circuit board with 1.4 mm thickness. For the final measurements this strip-line was peeled off and replaced by a copper or steel electrode having the same sizes. The other side of the printed board was used as a ground plane. The length of the lines was 30 cm, the width 12 mm and the thickness 0.5 mm. Both steel and copper lines had the same size and were attached to G10 board in the same way, so the only difference between the copper and steel measurements was in much higher resistive losses of the steel.

Measurements of the steel micro-strip line were carried out inside the DC magnet at different values of the magnetic field (0, 1 T and 2 T) and different orientations (parallel to the strip plane and normal to the strip plane) (see Figure 1).

For all measurements it has been crucial to make a very good, tight contact between a strip conductor and the board dielectric. In the case of normal magnetic field orientation the steel strip was pressed to the board with a fiberglass plate of the same length by movable poles of the magnet. The width of the fiberglass plate was smaller than the width of the micro-strip, so its effect on wave propagation would be small. Both copper and steel lines were pressed to the board in the same way. Therefore the effect of the fiberglass plate was cancelled out.

In the case when the magnetic field was parallel to the strip plane the steel strip was attached to the dielectric by epoxy glue, which influenced S-parameters. However, one can use the resistance at zero magnetic field as the reference and calculate the resistance at higher magnetic fields:

$$\tau_s = \frac{\tau_{0T}}{\sqrt{1+\frac{R_{ref}}{j\omega L}}}\sqrt{1+\frac{R}{j\omega L}} , \qquad (4)$$

where R_{ref} is the reference resistance at 0T, taken from normal magnetic fields measurements, τ_{0T} is the time delay in steel micro-strip at 0T, determined from parallel magnetic field measurements.
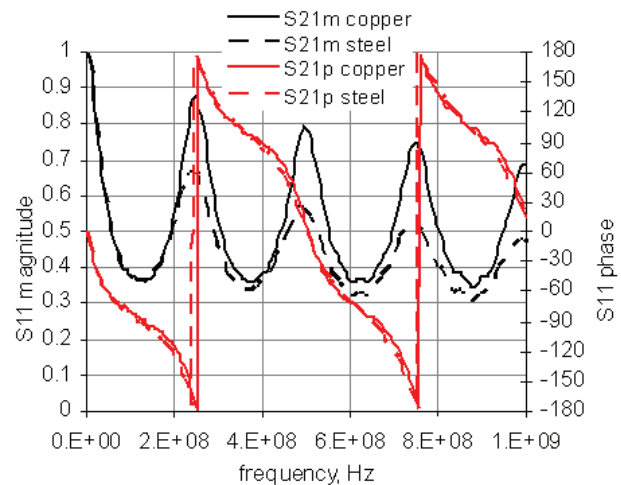

Figure 2: Comparison between S-parameters of copper and steel micro-strip lines at zero magnetic field.

RESULTS

In order to determine magnetic permeability, we need to know the steel conductivity σ. Since our frequencies are lower than the optical frequencies by several orders of magnitude, we can consider the conductivity being independent on the frequency. In this case it can be measured by usual DC techniques. The four-point measuring method revealed a steel conductivity of about $\sigma = 2.3 \cdot 10^6 \left(\Omega \cdot m\right)^{-1}$.

One can see the difference in S_{21} between the copper and steel micro-strips in Figure 2. Lower transmittance in the steel strip is caused by higher resistive losses.

Following the above described procedure we obtained magnetic permeability of soft steel as a complex function of frequency. The dependence of both real and imaginary parts of μ on frequency in magnetic field normal to the surface of steel micro-strip is presented in Figure 3. As expected, both real and imaginary parts are decreasing with magnetic field. Several irregularities on this picture are related to measuring system errors and don't convey a useful information. Our analysis showed that these irregularities would be much bigger, if we would not use the reference copper data at each value of the magnetic field.
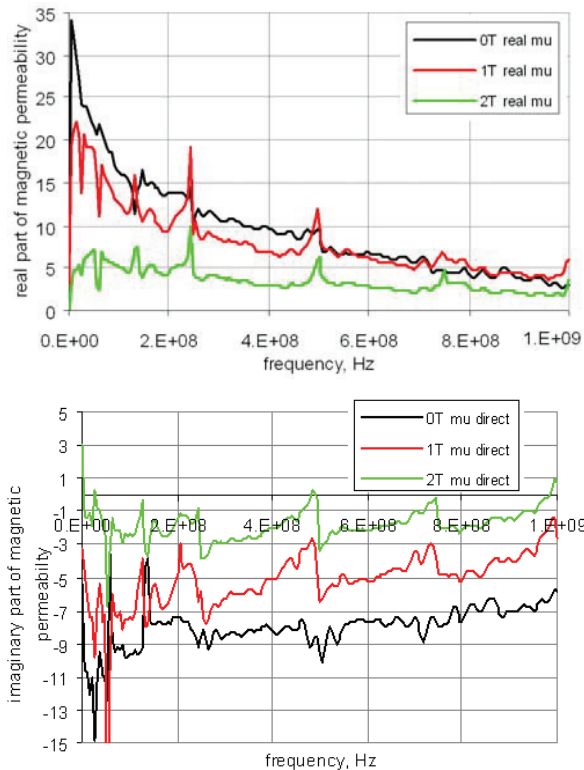

Figure 3: Dependence of magnetic permeability of steel on frequency for different magnetic fields for the case of magnetic field normal to the strip plane.

Figure 4 demonstrates the magnetic permeability of steel in magnetic field, parallel to the micro-strip plane. The data show a dramatic decrease in magnetic permeability. No difference between 2T and 1T data means that steel is already in saturation.

The reason for such big difference between values of magnetic permeability, calculated from data, taken in different orientations of magnetic fields, lies in different boundary conditions. Assuming infinitely wide strip, in the case of magnetic field, normal to the strip plane, the values of B-field are equal inside and outside of the strip in the vicinity of its surface. But in the case of magnetic field, parallel to the strip plane, the same situation appears for H-field. Since $\vec{B} = \mu\mu_0\vec{H}$, the steel reaches saturation

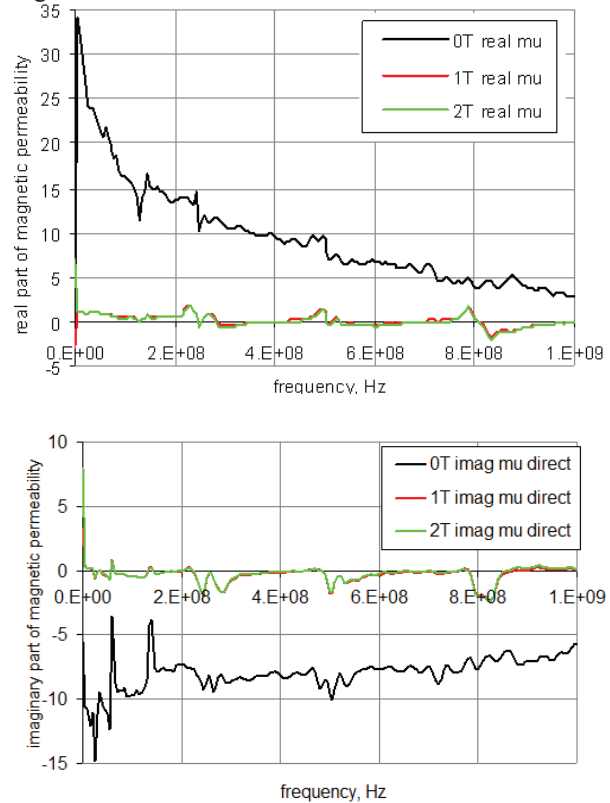in parallel magnetic field much earlier than in normal magnetic field.

Figure 4: Calculated magnetic permeability of steel in magnetic field, parallel to strip plane.

CONCLUSIONS

A method for measuring magnetic permeability of soft steel as a complex function of frequency was developed and applied to steel, which properties matches the properties of steel laminations in Fermilab Booster dipoles. Measured values of magnetic permeability agree with previous estimates, based on the theoretical and experimental work towards understanding impedances of Booster magnets.

ACKNOWLEDGMENTS

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