FOCUSING OF ACCELERATED PARTICLES BY WAKEFIELDS OF A DRIVE BUNCH IN A PLASMA-DIELECTRIC WAVEGUIDE*

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Abstract

The excitation of wake waves by the electron bunch in an isotropic plasma-dielectric waveguide is studied. It is shown that the excited field consists of two components: the field of the Langmuir wave and the field of eigenmodes of a dielectric waveguide. At a certain plasma density, the longitudinal component of Langmuir wave becomes significantly lower than the longitudinal component of the dielectric waves and transverse field component of the Langmuir wave is much higher than transverse component of the dielectric waves. The periods of these two types of waves differ significantly. This allows to provide the acceleration of the test bunch by a field of the dielectric wave with its simultaneous focusing by the plasma wave field.

INTRODUCTION

Acceleration of charged particles by wakefields is perspective, thriving direction in a high-energy physics. One of wakefield acceleration methods excited by relativistic electron bunches, as a slowing medium uses the plasma [1] created by the same electron bunches [2] or an external source. In particular, as an external source recently it was proposed to use a capillary discharge [3, 4]. The capillary tube is a slowing medium, therefore at propagation in its channel of a laser impulse or electron bunches along with plasma wakefields will be excited an eigen waves of dielectric structure modified by presence of plasma in the transport channel. Till now influence of electrodynamic properties of a material of capillary tubes on excitation of plasma wakefields is not investigated. On an example of a cylindrical waveguide of terahertz operation frequency range, we investigate excitation of wakefields by relativistic electron bunches in a dielectric waveguide with the accelerating channel filled with isotropic plasma. It is shown that the excited field consists of two items: Langmuir wave fields (LW) and fields of eigen waves of dielectric waveguide (DW). It turns out at certain density of plasma a longitudinal component of LW it is significantly less than a longitudinal component of the DW waves , and transverse components of the LW field is significantly higher than transverse component of the DW waves. The periods of these two types of waves generally do not coincide. By numerical calculations the range of density of plasma at which probably to provide acceleration of a test bunch with its simultaneous focusing by LW is defined.

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ANALYTICAL SOLUTIONS FOR THE WAKE FIELDS

For the investigation of the influence of a dielectric medium on the excitation of the plasma wakefields we find the wakefield of an electron bunch moving in a plasma waveguide with a dielectric ring insert. The plasma waveguide is a homogeneous plasma cylinder of radius a, surrounded by a perfectly conducting casing of radius b. The dielectric insert fills all the space between the casing and the plasma. The excitation of the waveguide will be considered in the approximation of a linear isotropic plasma with density n_p.

Let's begin from the determination of wake field of the bunch, having a shape of an endless thin axisymmetric ring radius r_0. The azimuthally symmetric wakefield excited by the bunch, it is described by the following system of Maxwell's equations:

$$\begin{cases} \dfrac{\partial E_r}{\partial z}-\dfrac{\partial E_z}{\partial r}=-\dfrac{1}{c}\dfrac{\partial H_\varphi}{\partial t}, \quad -\dfrac{\partial H_\varphi}{\partial z}=\dfrac{1}{c}\dfrac{\partial D_r}{\partial t}, \\ \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(rH_\varphi\right)=\dfrac{1}{c}\dfrac{\partial D_z}{\partial t}+\dfrac{4\pi}{c}j_z \end{cases} \tag{1}$$

where E_r, E_z are radial and longitudinal components of electric field, D_r, D_z are radial and longitudinal components of electric induction, H_φ is azimuthal magnetic field component, current density is equal

$$j_z=\frac{Q}{2\pi r}\delta(r-r_0)\delta(\tau-t_0) \tag{2}$$

$\tau=t-z/v_0$, t_0 is the time when the bunch crosses the plane z=0, v_0 is its velocity; Q is charge of the bunch, δ is the Dirac delta function. Expending the Maxwell's equations (1) in Fourier integral and at using the well-known boundary conditions at the dielectric surface and at the bunch surface we find the expression of Fourier component of electromagnetic field. Having performed the inverse Fourier transform, we obtain expressions for the components of the wakefield excited by the bunch, having the form of a thin ring of radius r_0.

By integrating these expressions over a time of flight t_0 and over a transverse coordinate of the ring particles r_0 of the bunch, we obtain expressions for the wakefields excited by a bunch of a finite longitudinal and a finite transverse dimensions. For a solid cylindrical bunch of the radius r_b and the length of L_b with a homogeneous distribution of the particle density

$$n(r_0,t_0)=\frac{v_0}{L_b r_b^2}\left[\Theta(t_0)-\Theta(t_0-L_b/v_0)\right]\Theta(r_b-r) \quad (3)$$

The final expressions for the components of the wakefield have the form of:

$$E_{z,r}=\begin{cases} I_{z,r}, r<r_b \\ II_{z,r}, r_b\le r<a \\ III_{z,r}, a\le r<b \end{cases},\quad H_\varphi=\begin{cases} I_\varphi, r<r_b \\ II_\varphi, r_b<r<a \\ III_\varphi, a<r<b \end{cases}, \quad (4)$$

where

$$I_z=-\frac{4Q\Theta_p}{L_b r_b}\left[\frac{1}{r_b k_p}-\frac{I_0(k_p r)}{I_0(k_p a)}\Delta_1(k_p r_b,k_p a)\right]-\frac{8Q\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{I_0(\kappa_p r)}{I_0(\kappa_p a)}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}, \quad (5)$$

$$II_z=-\frac{4Q\Theta_p}{L_b r_b}\frac{I_1(k_p r_2)}{I_0(k_p a)}\Delta_0(k_p a,k_p r)-\frac{8Q\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{I_0(\kappa_p r)}{I_0(\kappa_p a)}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}, \quad (6)$$

$$III_z=-\frac{8Q\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}\frac{F_0(\kappa_d r,\kappa_d b)}{F_0(\kappa_d a,\kappa_d b)}, \quad (7)$$

$$I_r=\frac{4Q\Theta_p}{L_b r_b^2}\frac{1}{I_0(k_p a)}\left[I_1(k_p r)\left[r\Delta_1(k_p r,k_p a)-r_b\Delta_1(k_p r_b,k_p a)\right]-\right.$$
$$\left.-\Delta_1(k_p a,k_p r)rI_1(k_p r)\right]+\frac{8Q\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{1}{\sqrt{1-\beta_0^2\varepsilon_p(w_s)}}\frac{I_1(\kappa_p r)}{I_0(\kappa_p a)}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}, \quad (8)$$

$$II_r=-\frac{4Q\Theta_p}{L_b r_b}\frac{I_1(k_p r_b)}{I_0(k_p a)}\Delta_1(k_p a,k_p r)+\frac{8Q\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{1}{\sqrt{1-\beta_0^2\varepsilon_p(w_s)}}\frac{I_1(\kappa_p r)}{I_0(\kappa_p a)}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}, \quad (9)$$

$$III_r=-\frac{8Q\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{1}{\sqrt{\beta_0^2\varepsilon_d-1}}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}\frac{F_1(\kappa_d r,\kappa_d b)}{F_0(\kappa_d a,\kappa_d b)}, \quad (10)$$

$$I_\varphi=II_\varphi=\frac{8Q\beta_0\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{\varepsilon_p(w_s)}{\sqrt{1-\beta_0^2\varepsilon(w_s)}}\frac{I_1(\kappa_p r)}{I_0(\kappa_p a)}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}, III_\varphi=-\frac{8Q\beta_0\Theta_s}{aL_b r_b w_s \kappa_p D'(w_s)}\frac{\varepsilon_d}{\sqrt{\beta_0^2\varepsilon_d-1}}\frac{I_1(\kappa_p r_b)}{I_0(\kappa_p a)}\frac{F_1(\kappa_d r,\kappa_d b)}{F_0(\kappa_d a,\kappa_d b)}, \quad (11)$$

$$\Theta_{p,s}=\Theta(\tau)\sin w_{p,s}\tau-\Theta(\tau-L_b/v_0)\sin w_{p,s}(\tau-L_b/v_0), \quad (12)$$

$\Theta(x)$ is the Heaviside step function, $F_n(x,y)=(-1)^n J_n(x)N_0(y)-(-1)^n N_n(x)J_0(y),\ n=0,1$, $J_0(x),J_1(y)$ and $N_0(x),N_1(y)$ are Bessel and Neumann functions of zeroth and of first order, respectively, $\beta_0=v_0/c$, $\varepsilon(w)=\varepsilon_p(w)=1-w_p^2/w^2$, if $r<a$ and $\varepsilon(w)=\varepsilon_d$ if $a\le r<b$; $w_p=\sqrt{4\pi e^2 n_p/m}$ is plasma frequency, $-e$ and m are charge and mass of electron; ε_d is relative permeability of the dielectric insert, which we supposed to be independent of frequency; $\Delta_n(x,y)=I_n(x)K_0(y)-(-1)^n K_n(x)I_0(y),\ n=0,1$ $I_0(x),\ I_1(x)$ and $K_0(x),\ K_1(x)$ are modified Bessel and MacDonald functions of the zeroth and of the first order, respectively, $k_p=w_p/v_0$, $\kappa_p^s=\kappa_p(w=w_s)$, $\kappa_d^s=\kappa_d(w=w_s)$, $D'(w_s)=\left.\frac{dD(w)}{dw}\right|_{w=w_s}$,

$$D(w)=\frac{\varepsilon_p(w)}{\sqrt{1-\beta_0^2\varepsilon_p(w)}}\frac{I_1(\kappa_p a)}{I_0(\kappa_p a)}+\gamma_d\frac{F_1(\kappa_d a,\kappa_d b)}{F_0(\kappa_d a,\kappa_d b)}, \quad (13)$$

where $\gamma_d=\varepsilon_d/\sqrt{\beta_0^2\varepsilon_d-1}$.

Dispersion function and the eigen frequencies w_s are determined by solving the dispersion equation:

$$D(w_s)=0. \quad (14)$$

THE NUMERICAL CALCULATIONS

Below we present the results of numerical calculations of wake field. For numerical calculations we choose the dielectric waveguide with transverse dimensions $a = 0.2$ mm, $b = 0.5$ mm, permeability $\varepsilon_d = 3.75$ (quartz), the energy of the electron bunch is equal to 5 GeV (the speed $v_0 = 2.998 \cdot 10^{10}$ cm / sec), $Q = -3$ nC bunch charge, the radius of the bunch, $r_b = 0.1$ mm, $L_b = 0.2$ mm bunch length.

The figures 1 and 2 show the results of calculations for the plasma density $n_p = 10^{14}\,\mathrm{cm}^{-3}$ (the frequency of plasma wave $w_p = 5{,}64 \cdot 10^{11}$ rad / sec, the wavelength is $\lambda_p = 2\pi v_0 / w_p = 0.334$ cm). Fig. 1 shows the axial distribution of the longitudinal and transverse forces acting on a test particle located at a distance of 0.01 cm from the axis of the waveguide. From comparison of these dependences, it follows that by placing a test bunch at some distance from the head of the drive bunch it is possible to provide an acceleration of charged particles and their simultaneous radial focusing.

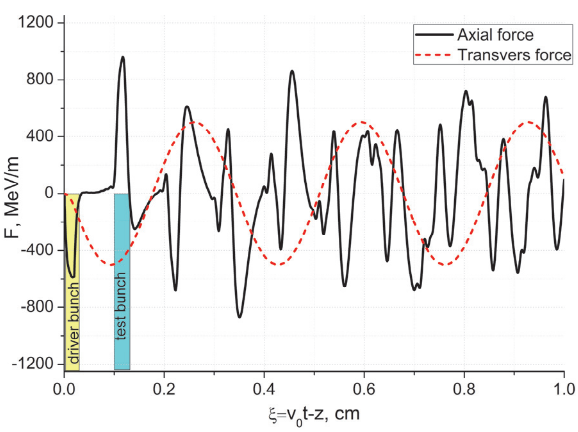

Figure 1: Axial profile of the longitudinal (solid line) and transverse forces (dotted line), acting on a test particle located at a distance of 0.01 cm from the axis of the waveguide. Here $\xi = v_0 t - z$, the head of the drive bunch is at $\xi = 0$.

As can be seen from the figure 1, the radial force has nearly harmonic dependence on the longitudinal coordinate with a period equal to ~ 0.33 cm, i.e. Langmuir wave makes the dominant contribution to the radial force. At the same time, its contribution to the longitudinal force, accelerating test particles are predominantly small. Longitudinal force is mainly determined by the eigen modes of a dielectric waveguide, its complex dependence on the longitudinal coordinate associated with the excitation of several radial modes of a dielectric waveguide.

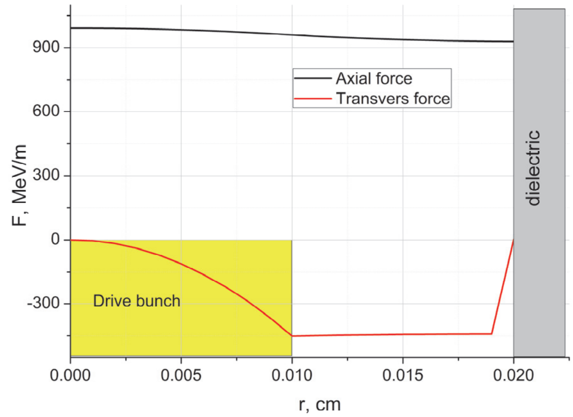

Figure 2: Transverse profile of the longitudinal (dotted line) and transverse forces (solid line), acting on a test particle, located at a distance of 0.01 cm from the head of the leading bunch.

Fig. 2 shows the radial dependence of the longitudinal and lateral forces acting on a test particle located at the first maxima of the accelerating field at a distance of 0.1 mm behind the head of the drive bunch. Longitudinal force slightly varies in the cross section of the transport channel, and the radial force is focusing on the entire cross section of the channel.

CONCLUSIONS

At studying the excitation of wake fields by an electron bunch in a plasma-dielectric waveguide we found that the filling of the vacuum transport channel dielectric waveguide by an isotropic plasma of a certain density leads to a focusing wake field acting on the accelerated bunch. Optimization of the focusing mechanism of a test bunch over plasma density and over other parameters of the dielectric waveguide will be conducted in further studies.

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