A RECIPROCITY PRINCIPLE FOR WAKEFIELDS IN A TWO-CHANNEL COAXIAL DIELECTRIC STRUCTURE*

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Abstract

Here we report the employment of the reciprocity principle for testing wakefields set up by an electron bunch in a two-channel coaxial dielectric structure (CDWA). For numerical studies we take a ~1-THz fused silica coaxial structure which we plan to test at FACET/SLAC; it has dimensions: outer shell, OD=1600 μm, ID=1000 μm; inner shell OD=362 μm, ID=100 μm. The structure is energized by a 23GeV, 3nC bunch having axial RMS size = 20μm. Our analytical studies and numerical simulations prove that for the axial wakefield, an annular drive bunch can be replaced by a pencil-like bunch of the same charge travelling in the annular vacuum channel or in the central vacuum channel. The longitudinal electric field registered by a witness bunch moving along the opposite vacuum channel is the same as in the conventional structure of the CDWA.

INTRODUCTION

The reciprocity principle is often used in applications of classical electromagnetism. In classical electromagnetism, reciprocity refers to a variety of related theorems involving the interchange of time-harmonic electric current sources and the resulting EM fields, for time-invariant linear media under certain constraints [1]. Lorentz reciprocity states that the relationship between an oscillating current and the resulting field is unchanged if one interchanges the points where the current is placed and where the field is measured. The proof is based on the relation at a fixed frequency in linear media between the Fourier image of the current and the Fourier image of the field. Actually, from Maxwell's equations, assuming that a current is localized and there are no incoming waves from infinitely far away, one can easily obtain the next equality connecting two different sources and the electric fields produced by them:

$$\int \mathbf{j}_{ex}^{(1)} \mathbf{E}_2 dV_1 = \int \mathbf{j}_{ex}^{(2)} \mathbf{E}_1 dV_2, \qquad (1)$$

where a current density $\mathbf{j}_{ex}^{(1)}$ produces electric field $\mathbf{E}_1$ and a current density $\mathbf{j}_{ex}^{(2)}$ produces electric field $\mathbf{E}_2$, the integration in (1) is carried out over a volume of the first source V_1 and over a volume of the second source V_2.

If the dimensions of current sources are small compared with the wavelength and the distance between them, we can take $\mathbf{E}_1$ and $\mathbf{E}_2$ outside the integrals and can rewrite the equality (1) as [1]:

$$\mathbf{E}_2(1) \cdot \mathbf{P}_1 = \mathbf{E}_1(2) \cdot \mathbf{P}_2, \qquad (2)$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are dipole moments of the first and the second sources, and $\mathbf{E}_2(1)/\mathbf{E}_1(2)$ is the field from the second source at the location of the first/second source. If $\mathbf{P}_1 = \mathbf{P}_2$, i.e. the first source is identical to the second source, from (2), the projections of the electric fields on the direction of the dipole moment are the same:

$$\mathbf{E}_{2,\parallel}(1) = \mathbf{E}_{1,\parallel}(2). \qquad (3)$$

The equality (3) is written in the frequency domain, but it remains valid in the time domain with a single constraint: by causality, a source must be ahead of the observation point.

It turns out the reciprocity principle can be very useful in the investigation of the coaxial dielectric wakefield accelerator [2]. The CDWA structure consists of two coaxial dielectric tubes, enclosed one inside the other. The conventional CDWA structure is energized by an annular drive bunch traversing the annular vacuum channel; an accelerated witness bunch travels in the central vacuum channel. In addition to providing a high acceleration gradient ~ 1GeV/m for mm-scale structures [3], the two-channel structure has a larger transformer ratio than the single-channel DWA, the transverse forces acting upon the bunch to be accelerated are focusing, and the annular drive bunch has been shown to move appreciable distance without undergoing distortion or deflection [4]. Proof-of-principle experiments to test a mm-scale THz CDWA will be carried at FACET (SLAC) in June of current year. At present SLAC has no annular drive bunch. However, in what follows, we shall show that for the building up of wakefields a solid drive bunch can be used which will establish the same fields we wish to study. Furthermore, we shall show how we may obtain information from this study whereby the data can be compared with theoretical simulations obtained with the CST STUDIO code. This is possible because of the reciprocity principle recorded above.

We will consider three regimes of operation of the CDWA: 1) conventional CDWA[1], see above; 2) "inverse" CDWA when the structure excited by a solid drive bunch that moves along the central channel axis

while the witness bunch samples its wakefield as it moves along the annular channel; 3) the CDWA with a solid drive bunch that moves in the annular channel while the witness bunch accelerates along the central channel axis. Certain of these field components are simply related, as we shall establish in the next section. This allows us to study regimes 2 and 3 at FACET, and relate the measurements made there to the desired operation with an annular drive bunch, conventional regime 1.

RECIPROCITY IN A CDWA STRUCTURE

For the numerical studies (PIC Solver of the CST Studio Suite 2011 [5]) we choose one of the examples of CDWA structure that will be tested at SLAC; its parameters are listed in Table 1.

Table 1: Parameters used for CDWA (fused silica)

Frequency of dominant mode (E_{03})	475 GHz
External radius of outer coaxial cylinder	800 µm
Inner radius of outer coaxial waveguide	500 µm
External radius of inner coaxial cylinder	181 µm
Accl. channel radius (inner radius of inner coaxial cylinder)	50 µm
Relative dielectric constant ε	3.75
Bunch axial RMS dimension σ_z	20 µm
Full bunch length used in PIC simulation	100 µm
Outer drive bunch radius (annular bunch)	415 µm
Inner drive bunch radius (annular bunch)	265 µm
Bunch transverse RMS dimension σ_r (solid bunch)	10 µm
Cutoff beam radius (solid bunch)	25 µm
Bunch energy	23 GeV
Total bunch charge	3 nC

In Fig.1 are shown axial profiles of axial wakefields for the first regime of CDWA operation when the structure is excited by an annular drive bunch traversing the annular
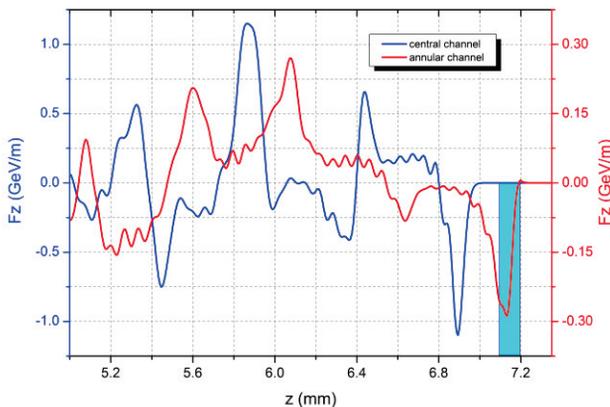

Figure 1: Longitudinal force axial profile at the center of the annular vacuum channel (red line) and at the central channel axis (blue line) in the case of excitation of conventional CDWA [1]. Cyan rectangle shows the location of the drive bunch particles.

vacuum channel. The maximum accelerating gradient in the central channel is 1.15 GeV/m, average decelerating force on drive bunch particles is 0.2 GeV/m, so the transformer ratio for this device is ~6.

The longitudinal component of force acting on a witness bunch for the three ways of excitation of the structure is given in Fig. 2. The component $F_z(z)$ is the same at corresponding locations where the witness bunch resides. The witness bunch in either of these cases would experience an accelerating force ~ 1.15 GeV/m if located at z ~ 5.85mm. This figure shows convincingly that an annular drive bunch can be replaced by a point drive bunch located in the annular channel, as we must do at FACET. Understanding is provided from analytic theory [2] where it is found that the on-axis longitudinal wakefield amplitude depends on an integral over the eigenfunctions $e_{zn}(r_0)$, where n is the radial index and r_0
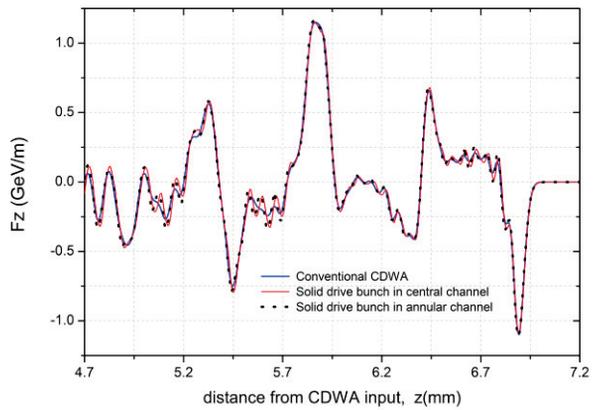

Figure 2: Axial profile of wakefield registered by witness bunch for three regimes of operation of the CDWA: 1) blue line - conventional CDWA; 2) red line - solid drive bunch is in the central channel; 3) black dots - solid drive bunch is in the annular channel.

is the location of an element of drive bunch charge. It turns out that the eigenfunction is nearly constant across the radius of the annular channel, so the integral is insensitive to the transverse bunch charge profile. Furthermore, each azimuthal mode amplitude is proportional to the modified Bessel function $I_m(\kappa r)$ where κ is the transverse wavenumber and m is the Bessel function order. For regions close to the axis with $\kappa r \ll 1$, $I_m(\kappa r) \sim (\kappa r)^m$. Thus the only contribution is from the symmetric monopole term which is valid for the on-axis field from a point bunch and generally true for an annular bunch.

Let's compare the results obtained for the excitation regime 3 with the excitation regime 2 ("inverse" CDWA) when the solid drive bunch is sent along the central channel. The simulations show that the axial field along the central channel is then approximately one order of magnitude larger (~10 GeV/m), but from Fig. 2 follows that the maximum wakefield in the annular space is again 1.15 GeV/m at z= 5.85mm, the same field that a point

drive bunch located in the annular channel sets up in the central channel. This provides evidence that a reciprocity principle [3] is governing "Green's function" excitation in a two-channel structure (for regimes 2 and 3, the source and the observation point are interchanged). This is key to linking the FACET experimental results to data that apply to the annular drive bunch operation.

We have studied the 2D distribution of wakefields for the three drive bunch cases. We point out an interesting situation that applies to the fields at the outer surface when the drive bunch moves in the central channel: the fields at the metallic surface of the unit are modest here, unlike the case of the single-channel device [6] where high fields occur that can destroy the metallic coating.

BUNCH STABILITY

We have shown [4] that an annular drive bunch in the case of the conventional regime of CDWA operation is very stable. Our studies have shown that small misalignments (~10%) of the dielectric tubes and the drive bunch, as well as azimuthal charge non-uniformity, are not dangerous in the microwave CDWA experiment. Unlike this operating regime, the off-axis location of the drive bunch that is specific to regime 3 (solid bunch in the annular channel) indicates potential trouble from deflecting forces. Also, the operating regime 2 when the drive bunch travels in the central channel is very similar to the single–channel case that is subject to drive bunch breakup [7]. A mitigating factor is the high drive bunch energy, but the fields are large and the structure is small. Therefore, we have simulated the bunch motion for regimes 3 and 2. We find (Figs. 3,4) that the drive bunch will move with tolerable distortion a distance ~6-8 cm. This distance will depend on the diameter of the central channel (100 μm here), as that mostly determines how large the wakefields will be. It should be noted that the length of the inner dielectric tube must be less than 10 cm, otherwise a sag of the inner tube becomes a problem.
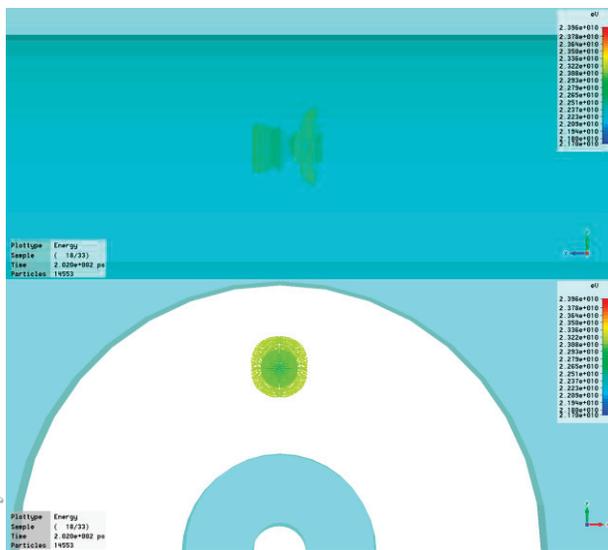

Figure 3: Side (top) and front (bottom) views of the point drive bunch located in the annular channel after it has moved 6.1cm. Note shape distortion shown at top.
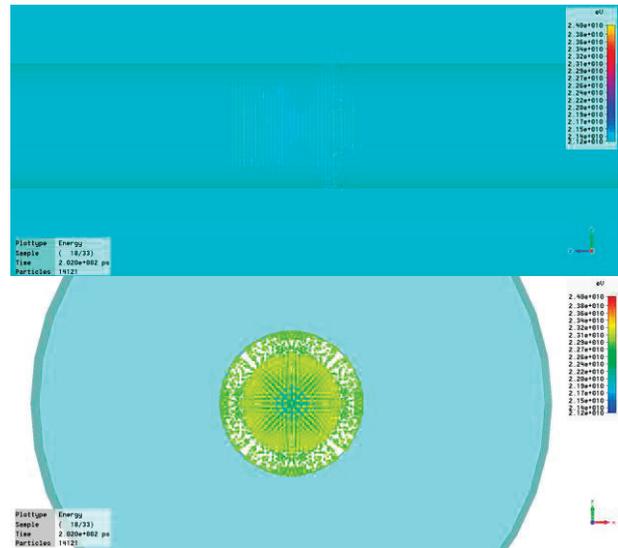

Figure 4: Side (top) and front (bottom) views of the point drive bunch located in the central channel after it has moved 6.1cm. However, particle losses here are negligible (less 3% of electrons are deposited on inner surface of the inner dielectric tube).

CONCLUSION

Our studies show that for the scheduled wakefield experiments with CDWA at FACET, an annular drive bunch can be replaced by a solid bunch that travels in
1) the annular vacuum channel, because the on-axis axial wakefield as registered by a witness bunch will be about the same owing to a flat transverse profile of the axial electric field across the annular drive channel and also to the vanishing of higher multipole modes ($m \geq 1$) on the structure axis;
2) the central vacuum channel, because the excited axial electric fields in the annular vacuum channel will be the same due to the reciprocity principle.

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