

TO THE POSSIBILITY OF BOUND STATES BETWEEN TWO ELECTRONS

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Abstract

We analyze the possibility to compress dynamically the polarized electron bunch so that the distance between some electrons in the bunch comes close to the Compton wavelength, arranging a bound state, as the attraction by the magnetic momentum-induced force at this distance dominates repulsion by the electrostatic force for the appropriately prepared orientation of the magnetic moments of the electron-electron pair. This electron pair behaves like a boson now, so the restriction for the minimal emittance of the beam becomes eliminated. Some properties of such degenerated electron gas represented also.

OVERVIEW

Generation of beams of particles (electrons, positrons, protons, muons) with minimal emittance is a challenging problem in contemporary beam physics. With low emittance beam in hand one can arrange interactions between the beams of particles with minimal population.

For an electron gas in a volume V the total number of electrons in all states can be estimated for uniform distribution as [1]

$$N = \int dn \approx 2 \frac{p_x p_y \Delta p_{\parallel} S_{\perp} l_b \gamma}{(2\pi\hbar)^3} = 2 \frac{\gamma \mathcal{E}_x \gamma \mathcal{E}_y \gamma \mathcal{E}_z}{(2\pi\lambda_C)^3}, \quad (1)$$

where $\gamma \mathcal{E}_z = \gamma l_b (\Delta p / p_0)$ is an invariant longitudinal emittance, l_b is the bunch length, $\gamma \mathcal{E}_x$ and $\gamma \mathcal{E}_y$ are the transverse horizontal and vertical invariant emittances. Again, if N is close to the number of the particles in the bunch, then the particles in the bunch close to generation condition. The beam with the number of particles N cannot have emittances lower than the one defined by (1),

$$\gamma \mathcal{E}_x \gamma \mathcal{E}_y \gamma \mathcal{E}_z \geq \frac{1}{2} (2\pi\lambda_C)^3 N.$$

So the every state is occupied by a pair of electrons (positrons) with oppositely oriented spins. As this pair represents the pure state, its full wave function is asymmetric with respect to spatial coordinates. As the spin-part of the wave function is asymmetric, then the coordinate factor is symmetric, i.e. two electrons have nonzero probability to be identified in the same spatial point. In a view of the former comment, there is no contradiction with uncertainty condition.

One interesting circumstance here is that the fully degenerated beam has zero polarization, as each state occupied by two electrons having oppositely oriented spins.

In [2] and [3] there was proposed to consider the bound states between two electrons with oppositely oriented

spins for reduction of minimal emittance restriction arisen from Eq. 1. In some sense it is an attempt to prepare the pure quantum mechanical state between just two electrons. What is important here is that the distance between two electrons should be of the order of the Compton wavelength. We attracted attention in [2,3] that attraction between two electrons determined by the magnetic force of oppositely oriented magnetic moments. In this case the resulting spin is zero. Another possibility considered below.

In [4], it was suggested a radical explanation of structure of all elementary particles caused by magnetic attraction at the distances of the order of Compton wavelength. In [5], motion of charged particle in a field of magnetic dipole was considered. In [4] and [5] the term in Hamiltonian responsible for the interaction between magnetic moments is omitted, however as it looks like

$$H = \sqrt{c^2(\vec{p} - e\vec{A}/c)^2 + mc^2} - e^2/r \quad (2)$$

with vector potential $\vec{A} = \vec{m}(\vec{\sigma} \times \vec{r})/r^3$, where \vec{m} describes the magnetic momenta of the center.

In [6] a three body system was considered. The third heavy particle- proton served for partial compensation of electrical repulsion.

Meanwhile the magnetic dipole \vec{m}_1 sets magnetic field around as

$$\vec{H} = (3\vec{n} \cdot (\vec{m}_1 \cdot \vec{n}) - \vec{m}_1)/R^3, \quad (3)$$

where \vec{n} is unit vector from the center of magnetic momenta to the point of interest. In c.m. system the energy of another magnetic moment \vec{m}_2 in a magnetic field of the first one is

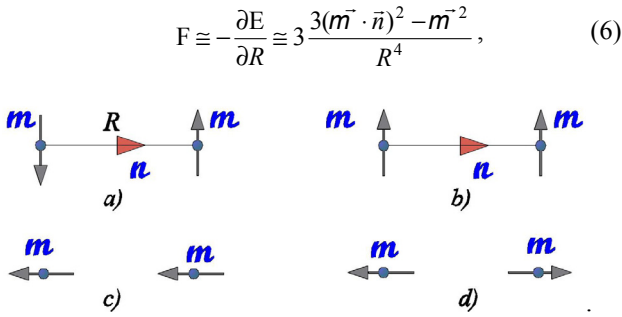
$$H = \vec{m}_2 \cdot \vec{H} = (3(\vec{m}_2 \cdot \vec{n}) \cdot (\vec{m}_1 \cdot \vec{n}) - \vec{m}_1 \cdot \vec{m}_2)/R^3. \quad (4)$$

Namely, this term should be added to Eq. 2. There are few basic orientations of each momentum with respect to the other one, shown in Fig. 1.

In cases a) and b), Fig. 1, the unit vector \vec{n} is orthogonal to the magnetic moment \vec{m} associated with the electron, so the magnetic field at the location of the second electron is $\vec{H} = -\vec{m}/R^3$. In cases c) and d) the magnetic field is $\vec{H} = 2\vec{m}/R^3$, so this case is twice more effective for the field value. As the energy Eq. 4 associated with the magnetic moment of a pair of electrons $|\vec{m}_2| = |\vec{m}_1|$ is now

$$H = \vec{m} \cdot \vec{H} = (3(\vec{m} \cdot \vec{n})^2 - \vec{m}^2)/R^3, \quad (5)$$

so the attracting force acting between two electrons due to theirs magnetic moments is



$$F \cong -\frac{\partial E}{\partial R} \cong 3 \frac{3(\vec{m} \cdot \vec{n})^2 - m^2}{R^4}, \quad (6)$$

Figure 1: Relative orientation of electron's magnetic moments. In cases a) and c) there is attraction by the magnetic force; in cases b) and d)–repulsion.

which should be compared with the repulsive electric force $F \cong -e^2 / R^2$, so the balance is

$$3 \frac{3(\vec{m} \cdot \vec{n})^2 - m^2}{R^4} \cong \frac{e^2}{R^2}. \quad (7)$$

Substitute for the magnetic moment of electron $|\vec{m}| = e\hbar / 2mc = e\tilde{\lambda}_C / 2$, one can obtain, that the distance for which these two forces equalize each other is

$$R \cong \hbar / 2mc = \sqrt{3} (2) \tilde{\lambda}_C, \quad (8)$$

where the factor 2 in brackets corresponds to the case c). This distance was identified in [4]-[6] as well.

One should agree that the orientation of spins a) and c) is stable, which could be concluded from behavior of ordinary magnets known to everyone.

The energy required for bringing two electrons to the distance equal to the Compton wavelength is

$$\frac{e^2}{\tilde{\lambda}_C} \cong \frac{e^2}{r_0} \cdot \alpha \cong \alpha \cdot mc^2 \cong 3.73kV, \quad (9)$$

i.e. pretty small compared with the energy of transverse motion at IP especially. One should remember that the energy associated with magnetic attraction

$$\frac{m^2}{\tilde{\lambda}_C^3} \cong \frac{e^2}{4\tilde{\lambda}_C} = \frac{e^2}{4r_0} \alpha \cong \frac{\alpha \cdot mc^2}{4} \cong 0.9kV \quad (10)$$

might make the minimal energy required for bonding particles even lower.

DAMPING RING FOR GENERATION THE BOUND STATES

The average distance between particles in the bunch as it is seen by the observer in a Lab system is

$$\delta \cong (V / N)^{1/3} \cong (\sigma_x \sigma_y \sigma_z / N)^{1/3}, \quad (11)$$

where N stands for the bunch population, σ_x, σ_y -are the transverse bunch sizes for corresponding directions.

Let us consider one example first. Substitute for estimations in Eq. 11 the parameters an electron bunch available for ILC at IP,

$\sigma_x \cong 500nm, \sigma_y \cong 3.5nm, \sigma_z \cong 200\mu m, N \cong 2 \cdot 10^{10}$, one can obtain from Eq. 8 $\delta \cong 10^{-8}cm$, meanwhile the Compton wavelength is

$$\tilde{\lambda}_C \cong r_0 \cdot \alpha \cong 3.8 \cdot 10^{-11}cm.$$

One peculiarity here is that these dimensions established dynamically, i.e. the particles cross the IP point in their transverse motion with angles up to

$$\sigma'_y \cong \sqrt{(\gamma \epsilon_y) / \gamma / \beta_y} \cong 10^{-4}rad \quad (12)$$

for $(\gamma \epsilon_y) \sim 2 \cdot 10^{-8}m \cdot rad, \beta_y \sim 0.2mm, \gamma \sim 10^5$ [7].

The angles like Eq. 12 provide collision transverse momenta as big as

$$p_{\perp c} \cong pc \sigma'_y \sim 5MeV \quad (13)$$

for 50GeV beam. One can see, that the transverse momenta is big enough compared with the minimal one Eq. 8, so the repulsion could be to overpassed in a moving frame, so the bound states could be generated here, Fig. 2.

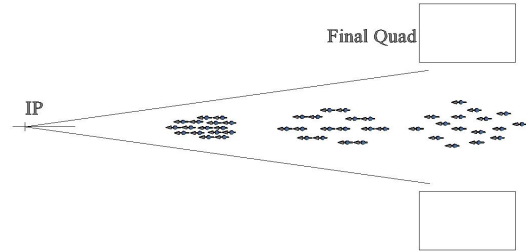


Figure 2: Dynamical process for compression of polarized bunch while moving towards the crossover.

So the idea of our proposal is very simple: one should arrange collisions between polarized beams of electrons with energy more than Eq. 8 and hope that in some collision the bonded state will emerge. As this probability is small, one should hope that accumulation of such pairs might be effective. One positive moment here is the pair of electrons bounded carries the doubled charge together with doubled mass, so the motion of such pair is possible in the ring without any problem. To register such pair one can hope with registering the SR as the pair should radiate coherently $\sim (2e)^2$.

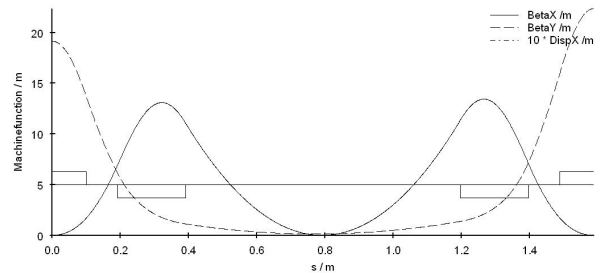


Figure 3: The envelope functions in the colliding section of the ring.

Values of envelope functions at crossover in Fig. 3 are $\sim \beta_x \cong \beta_y \sim 1cm$.

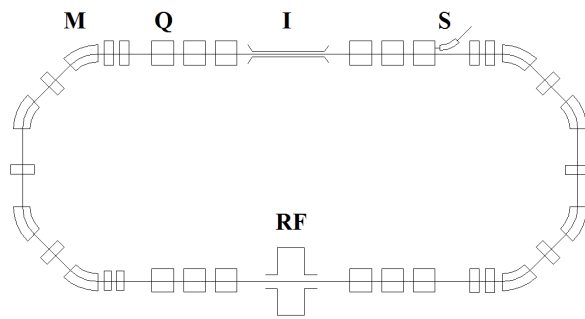


Figure 4: Damping ring. Q stands for a quadrupole, M is a bending magnet, I is inflector, S is a septum magnet, RF is a RF cavity.

The number of focusing sections from Fig. 4 in a straight section of the damping ring is chosen minimal (one in each straight section) as the rate of collisions in places with minimal beta-function practically weakly depends on perimeter of the damping ring if the only single bunch is circulating in the ring. If, however he number of bunches is raised also, then perimeter increase is desirable, Fig. 5.



Figure 5: Damping ring with increased amount of collision sections.

DISCUSSION

Material considered in this publication is not obvious, however it promises some positive yield in terms of low emittance beams and finally increased luminosity of any collider using this installation.

In conclusion let us speculate on possible “clasterization” of electrons in the beam. Some possibilities represented in Fig. 6.

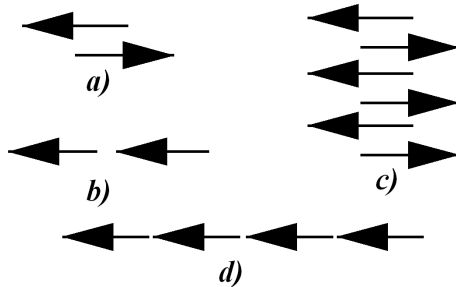


Figure 6: Possible orientation of spin in clusters.

Here the cases marked *a*) and *b*) were already mentioned. Resulting momentum of state *a*) is zero, but the momentum of case *b*) is one. Meanwhile the case *c*) represents a cluster of electrons with zero momentum also, but the number of electrons in such cluster can vary. Finally the case *d*) describes the cluster with integer momentum, equal to four etc.

Once again if someone registers the clusters by SR one should immediately register the splashes associated with coherent radiation of bounded electrons.

SUMMARY

The way considered opens a possibility of supercondensation of Fermion gas and opens possibilities in investigating the states with minimal emittance.

It is interesting to consider the similar possibilities for the bounded states of protons and muons in a future. The first type might be interesting for the LHC and the second one to the muon collider

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