

NUMERICAL STUDY OF SELF MODULATION INSTABILITY OF ATF ELECTRON BEAM*

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Abstract

In this paper, We investigate numerically the development of self-modulation instability (SMI) with electron bunch parameter available at Accelerator Test Facility (ATF) of Brookhaven National Laboratory (BNL). The results show that the long bunch drives wakefields with periods one to one fifth of the bunch length in plasmas with various densities. Both 50 pC and 1 nC electron bunches are investigated for the SMI development. All the simulations are performed with the 2D-cylindrically symmetric particle-in-cell code OSIRIS [1].

INTRODUCTION

Recently, using proton bunches to drive wakefield is becoming attractive for PWFA due to the large amount of energy they carry. The proton bunches produced by Large Hadron Collider (LHC) at CERN will have up to 7 TeV by particle and carry up to 136 kJ, much higher than that the current available lepton bunches (around 20 GeV/particle and 100 J/bunch). So instead of multiple plasma acceleration stages, one can in principle accelerate a single 10 GeV incoming lepton bunch to the TeV energy scale in a single PWFA stage driven by a relativistic proton bunch [2] rather than in multiple stages, as envisaged with electron bunch drivers.

In general, exciting large amplitude plasma waves requires a drive bunch with length on the order of plasma wavelength. In order to achieve high acceleration gradient, short drive bunches are needed. A. Caldwell et al. proposed the idea of a proton-driven PWFA and demonstrated the possibility of producing a TeV electron bunch in a single acceleration stage using a short (100 μm) proton bunch driver [2]. However, generating such short proton bunches is very challenging. Kumar et al. suggested that the self-modulation of a long (when compared with the relativistic plasma wave period) relativistic proton bunch in a plasma could result in the resonant excitation of large amplitude plasma wakefield [3], which in principle can be used to accelerate electrons. The self-modulation is a result of transverse two-stream instability, occurring through

the coupling of the transverse wakefield with the beam radius evolution. When a long bunch with the beam length $L^{beam} \gg \lambda_{pe}$ (λ_{pe} is the plasma wavelength) and transverse size σ_{r0} enters the plasma, it is radially modulated by the periodic focusing forces, and the beam density modulation ($n_b \propto 1/\sigma_r^2$) provides a positive feedback for the instability to grow. Consequently, this instability self modulates the long beam into ultra short bunches at the plasma wavelength scale, which resonantly drive the plasma wake. The instability is convective and grows both along the bunch (ξ) and along the plasma (z), as illustrated by the number of e-folding growth for a flat-top bunch [4, 5]:

$$N_{e-folding} \cong \frac{3^{3/2}}{4} \left(\frac{n_{b0} m_e}{n_e M_b \gamma} \right)^{1/3} (k_p \xi)^{1/3} (k_p z)^{2/3}, \quad (1)$$

where k_p is the wave number and $k_p = c/\omega_{pe}$, $\omega_{pe} = \left(\frac{n_e e^2}{\epsilon_0 m_e} \right)^{1/2}$, $\nu \approx 1 - \frac{k_p \sigma_{r0}^2}{6}$, r_0 the initial bunch radius, n_{b0} the initial beam density, M_b the bunch particles' mass.

However, another two-stream instability, the hosing instability, has also been reported to affect long bunches during the propagation in the plasma [7]. Originating from the interaction between the plasma electron sheath and the beam, this instability leads to spatiotemporally growing oscillations of the beam centroid at each axial slice, and therefore it may cause the bunch to break before SMI develops. Numerical simulations indicate that the seeding of the SMI may suppress the development of the hosing instability [6]. A number of methods could be used to seed the instability, including shaping the long bunch with a sharp (when compared to plasma wavelength) current rise at the front.

In the following sections, simulations are performed with the currently available ATF beam and plasma parameters to explore the development of SMI.

BEAM PARAMETERS AT ATF AND SIMULATION PARAMETERS

Table 1 shows the electron beam parameters that are available at ATF. The present experimental condition allows for the creation of a square bunch profile by using a rectangular mask [8, 9]. Therefore, in the simulation, the

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Table 1: ATF Electron Bunch Parameters

charge	50 pC-1 nC
current profile	"Square"
beam length L^{beam}	960 μm
σ_{r0}	120 μm
n_{b0}	3.59×10^{12} - $7.18 \times 10^{13} \text{ cm}^{-3}$
ϵ_N	13 mm-mrad
plasma length L_p	2 cm
mean energy	58.2 MeV

Table 2: Simulation Parameters

l	Plasma Densities	$\frac{n_b}{n_p}$ (50 pC)	$\frac{n_b}{n_p}$ (1 nC)
1	$1.21 \times 10^{15} \text{ cm}^{-3}$	2.96×10^{-3}	5.92×10^{-2}
2	$4.85 \times 10^{15} \text{ cm}^{-3}$	7.41×10^{-4}	1.48×10^{-2}
3	$1.09 \times 10^{16} \text{ cm}^{-3}$	3.29×10^{-4}	6.58×10^{-3}
4	$1.94 \times 10^{16} \text{ cm}^{-3}$	1.85×10^{-4}	3.70×10^{-3}
5	$3.03 \times 10^{16} \text{ cm}^{-3}$	1.18×10^{-4}	2.36×10^{-3}

electron density profile is assumed to be:

$$n_b = n_{b0} \cdot \Theta(L^{beam} - \xi) \cdot \exp\left(\frac{-r^2}{2\sigma_{r0}^2}\right), \quad (2)$$

where n_{b0} is the initial peak density, Θ is the Heaviside function. The charge of the bunch can be tuned from 50 pC up to 1 nC, and simulations are performed for both limits in order to explore the growth of SMI. As the low charge may be more appropriate to observe the energy modulation resulting from the occurrence of SMI, the high charge may be more suitable to directly observe the radial modulation [12]. The plasma density can also be varied between 10^{14} and 10^{18} cm^{-3} experimentally, therefore for simulation purpose, different plasma densities are chosen such that $l = L^{beam}/\lambda_{pe} = 1, 2, \dots, 5$, as shown in Table 2. Table 2 also shows that the beam density is much smaller than the plasma density ($n_b/n_p \ll 1$) for both the low charge bunch ($Q = 50 \text{ pC}$) and the high charge bunch ($Q = 1 \text{ nC}$) at various plasma densities, so the beam plasma interaction is in the linear regime. Although the plasma capillary length is limited to 2 cm at ATF [10], in the simulation, we propagate the bunch in plasmas for 20 cm in order to understand the physics involved in the development of the SMI.

INITIAL LONGITUDINAL AND TRANSVERSE WAKEFIELDS

According to 2D linear theory, for an electron bunch with density profile as in Eqn. (2), the initial ($z = 0 \text{ cm}$) longitudinal wakefield E_z on the beam axis ($r = 0$) and the transverse focusing field E_{foc} at $r = \sigma_r$ inside the bunch can be expressed as [11]:

$$E_z = \frac{en_{b0}}{\epsilon_0 k_p} \cdot \sin(k_p \xi) \cdot R(0) \propto n_{b0} \cdot n_p^{-1/2} \cdot R(0) \quad (3)$$

$$E_{foc} = \frac{en_{b0}}{\epsilon_0 k_p^2} \cdot (1 - \cos(k_p \xi)) \cdot \frac{dR(\sigma_r)}{dr} \propto \frac{n_{b0}}{n_p} \cdot R'(\sigma_r) \quad (4)$$

where $R(0)$ is the unitless transverse component. Both $R(0)$ and $dR(\sigma_r)/dr$ are increasing functions of $k_p \sigma_r$, the bunch transverse size σ_r relative to the plasma skin depth (c/ω_{pe}). For the electron bunch with low charge $Q = 50 \text{ pC}$, Figure 1 shows the lineout of the periodic E_z (Fig. 1a) and E_{foc} (Fig. 1b)), obtained from Eqn. (3) and Eqn. (4) respectively. The amplitude of longitudinal wakefield E_z decreases from ~ 4 to $\sim 2 \text{ MV/m}$ as the plasma density n_p increases, indicating the dominance of the decreasing term $n_p^{-1/2}$ over the increasing term $R(0)$. Similarly, due to the more rapid decrease of n_p^{-1} than the increase of $dR(\sigma_r)/dr$, the amplitude of the focusing field E_{foc} also decreases with increasing plasma density. Such transverse focusing fields modulate the bunch periodically and leads to the SMI. The wakefields at the plasma entrance ($z = 0 \text{ cm}$) obtained from the 2D OSIRIS simulation agrees very well with the formulas given above (as will be shown in the following section). When the bunch charge is increased to 1 nC, where linear theory is still valid, the amplitude of the wakefields are 20 times larger than the 50 pC bunch ($E_z, E_{foc} \propto n_b$) and the trend is the same as 50 pC for different plasma densities (not shown here).

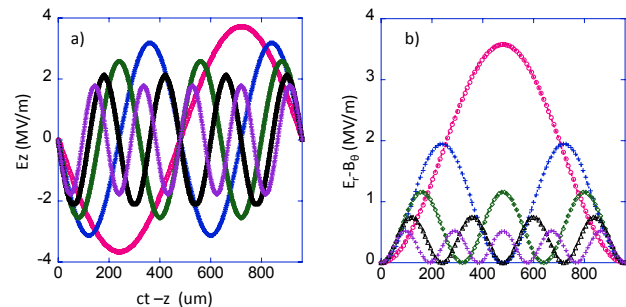


Figure 1: a) Initial longitudinal wakefield near the beam axis and b) initial transverse focusing field at $r = \sigma_r$ for various plasma densities such that L^{beam}/λ_{pe} is equal to: 1 (red line), 2 (green line), 3 (blue line), 4 (black line) and 5 (purple line), as shown in Table 2.

THE EVOLUTION OF PEAK ACCELERATING FIELD

Figure 2 shows the peak accelerating field E_z along the propagation distance z for the bunch charge of 50 pC (Fig. 2 a)) and 1 nC (Fig. 2 b)), respectively. In Fig. 2 a) where $Q = 50 \text{ pC}$, the initial values $E_z(z = 0)$ decreases from ~ 4 to $\sim 2 \text{ MV/m}$ as the plasma density increases, which is in good agreement with the prediction of the above 2D linear theory (shown in Fig.1 a)). As the SMI grows, E_z saturates and the saturation distances increases from 9 cm to 16 cm with increasing plasma densities. The simulation result also indicates that within the propagation distance of $z = 2 \text{ cm}$, E_z remains almost constant, and this is

confirmed by the estimation of $N_{e\text{-folding}}$ from Eqn. (1): $N_{e\text{-folding}} < 2$. The plasma capillary length of ATF is limited to 2cm and therefore no significant SMI growth is expected at the plasma exit in the 50 pC case.

In the case of $Q = 1\text{ nC}$ (Fig. 2 b)), as expected from the linear theory, the value of initial E_z ranges between ~ 80 to $\sim 40\text{ MV/m}$ and the trend is identical to the above 50 pC case. However, due to the higher growth rate of SMI ($N_{e\text{-folding}} \propto n_b$ and $N_{e\text{-folding}} > 30$) the propagation distances required to reach the SMI saturation decreases and lies between ~ 1.8 and $\sim 2.8\text{ cm}$. Such simulation results predict that the SMI is well developed at the plasma exit for the electron bunch with high charge $Q = 1\text{ nC}$.

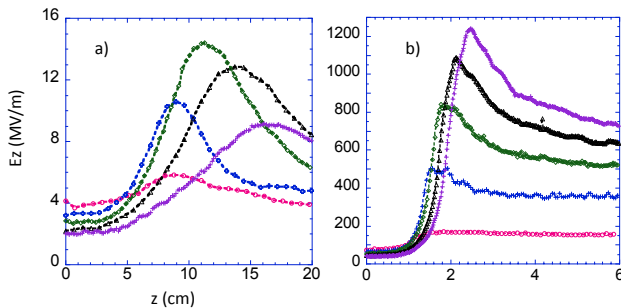


Figure 2: Peak acceleration wakefield E_z along the propagation distance z for the bunch charge of a) 50 pC and b) 1 nC at various plasma densities such that L^{beam}/λ_{pe} is equal to: 1 (red line), 2 (green line), 3 (blue line), 4 (black line) and 5 (purple line), as shown in Table 2.

THE RADIAL MODULATION OF THE ELECTRON BUNCH

Figure 3 a) shows the the electron beam density at the plasma exit for $Q = 50\text{ pC}$. As expected, no radial modulation is observed in the simulation result. Only modulation of the particles' transverse momentum has occurred [12]. On the contrary, for the high charge case where $Q = 1\text{ nC}$, the electron bunch is clearly modulated into two small beamlets on the scale of plasma wavelength (as shown in Fig. 2 b) for the case $L^{beam}/\lambda_{pe} = 2$), which drives the plasma wakefield resonantly. Such radial modulation is the direct evidence of the occurring of SMI.

HOSING INSTABILITY

The previous simulations demonstrate that 50 pC electron bunch does not have significant SMI growth at the plasma capillary exit, and therefore no hosing instability is expected since it has the similar growth rate as the SMI. However, whether hosing occurs or not is not clear for the 1 nC electron bunch. Further 3D simulations are needed to investigate this issue.

CONCLUSION

We have showed in initial simulations that the long electron bunch available at ATF drives wakefields with peri-

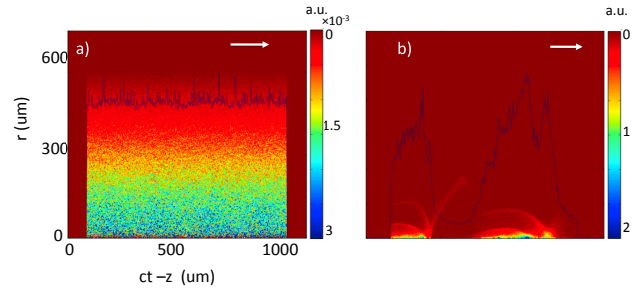


Figure 3: Electron beam density after the propagation distance of 2 cm for the bunch charge of a) 50 pC and b) 1 nC . The blue lines are the summed beam density over the r -direction. The white arrows indicate the propagation direction and the color bars show the normalized beam density n_b/n_p .

ods dependent on the plasma densities. Simulations indicate that the initial accelerating and focusing field amplitudes decrease with the increasing plasma density n_e . While 50 pC bunch does not experience significant SMI growth within the 2 cm -long plasma available for experiment. Simulation results suggest that performing the experiment with 1 nC bunch could lead to the radial modulation of the bunch at the plasma exit, which is the direct evidence of SMI. However, the availability of the experimental diagnostics to detect the radial modulation needs to be considered and the hosing instability needs to be evaluated through the 3D OSIRIS simulations.

REFERENCES

- [1] R. A. Fonseca et al., Lect. Notes Comp. Sci. vol. 2331/2002, (Springer Berlin/Heidelberg:2002).
- [2] A. Caldwell et al., Nature Physics 5 (2009) 363.
- [3] N. Kumar et al., Phys. Rev. Lett. 104 (2010) 255003.
- [4] A. Pukhov et al., Phys. Rev. Lett. 107 (2011) 145003.
- [5] K. Schroeder et al., Phys. Rev. Lett. 107 (2011) 145001.
- [6] J. Vieira et al., submitted to Physics of Plasmas (2012).
- [7] D. H. Whittum et al., Phys. Rev. Lett. 67 (1991) 991.
- [8] P. Muggli et al., Phys. Rev. Lett. 101 (2008) 054801.
- [9] P. Muggli et al., Phys. Rev. ST Accel. Beams 13 (2010) 052803.
- [10] W. Kimura et al., AIP Conference Proceedings 877, p. 534 (2006).
- [11] P. Chen et al., IEEE Transactions on Plasma Science, Volume PS-15, No. 2 (1987).
- [12] Y. Fang et al., to be submitted.