# NONLINEAR LATTICE FOR SPACE-CHARGE DOMINATED BEAM TRANSPORT WITH SUPPRESSED EMITTANCE GROWTH*

Yuri Batygin† and Alexander Scheinker‡, LANL, Los Alamos, NM 87545, USA

## Abstract

We present a feasible design for the implementation of a beam emittance growth suppressing lattice for space-charge dominated beams. It is based on a FODO focusing channel with quadrupole and duodecapole components which on average create the field required to match the high-brightness beam with the structure. Matched beam exhibits smaller emittance growth than that in regular quadrupole focusing channel. Numerical results demonstrate the ability of the proposed lattice to prevent halo formation of a nonuniform space-charge dominated beam.

## BACKGROUND ON MATCHING OF A NONUNIFORM SPACE-CHARGE DOMINATED BEAM

Nonuniform space-charge dominated beams cannot be perfectly matched with a linear focusing channel resulting in emittance growth and halo formation, as shown in Figure 1. Ref. [1] deals with the transport of an intense, nonuniform beam with suppressed halo formation. In this work we present a practical structure for the implementation of the stabilizing fields for beam transport with reduced emittance growth.

### Self-consistent space charge beam potential

The analysis in [1] starts with the simplified single-particle Hamiltonian in a focusing channel:

$$H = \frac{p_x^2 + p_y^2}{2m\gamma} + qU_{\text{ext}} + q\frac{U_b}{\gamma^2}, \qquad (1)$$

where $U_{\text{ext}}(x, y)$ is the scalar potential of the focusing field and $U_b(x, y)$ is the space charge potential of the beam. Next, in order to find a self-consistent particle distribution the following variables are introduced:

$$V_{\text{ext}} = \frac{qU_{\text{ext}}}{H_0}, V_b = \frac{qU_b}{H_0}, \xi = \frac{r}{a}, H_0 = \frac{mc^2}{4\gamma}\left(\frac{\epsilon}{R}\right)^2, \quad (2)$$

where $a$ is the radius of the channel, $R$ is the beam radius, and $\epsilon$ is $4\times$rms normalized beam emittance. The unknown potential $V_b$ is then expressed as a Fourier-Bessel series, which satisfies the Dirichlet boundary condition at the conductive surface of a round pipe $V_b(a) = V_0$. The constant $V_0$ is defined such that the total potential of the structure vanishes at the axis: $V_{\text{ext}}(0, \varphi) + \frac{\bar{V}_b(0,\varphi)}{\gamma^2} + \frac{V_0}{\gamma^2} = 0$. After making several approximations the analysis in [1] arrives
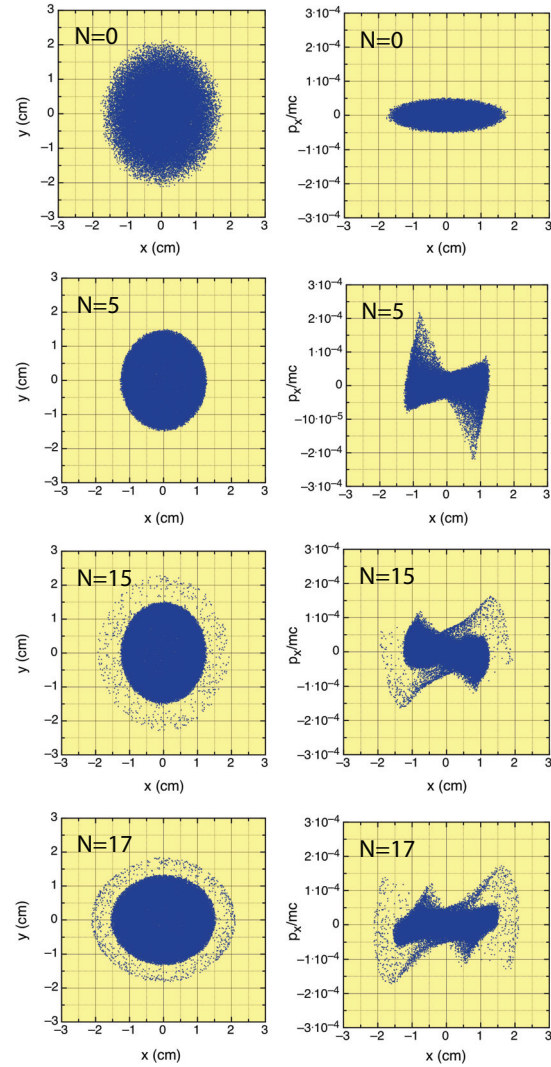
Figure 1: Emittance growth and halo formation of the 50 keV, 20 mA, 0.05 $\pi$ cm mrad proton beam in FODO quadrupole channel with the period of $L = 15$ cm, lens length of $D = 5$ cm and field gradient of $G_2 = 0.04278\frac{\text{T}}{\text{cm}}$. Numbers indicate FODO periods.

at the simplified approximate form of Poisson's equation: $V_0 + (1 + \delta)\bar{V}_b = \gamma^2(1 - V_{\text{ext}})$, where $\delta = \frac{1}{bk} \ll 1$, $b = \frac{2}{\beta\gamma}\frac{I}{\epsilon^2}\frac{R^2}{I_c}$ is the dimensionless beam brightness, $I_c = 4\pi\epsilon_0 mc^3/q$ is the characteristic value of beam current, and $k \approx 1$ is the coefficient depending on beam distribution. The self-consistent space-charge dominated beam potential $V_b$ and electric field $\vec{E}_b$ near axis are then

$$V_b = -\frac{\gamma^2}{1 + \delta}V_{\text{ext}}, \quad \vec{E}_b = -\frac{\gamma^2}{1 + \delta}\vec{E}_{\text{ext}}, \qquad (3)$$
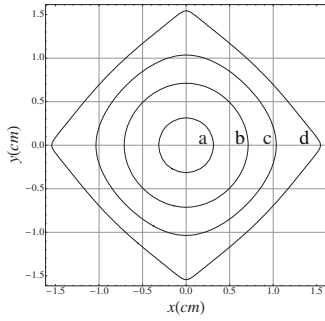
Figure 2: Lines of equal values of the function $C = \frac{1}{2}r^2 + \zeta r^6 \cos(4\varphi) + \frac{\zeta^2}{2}r^{10}$ for $\zeta = -0.03$: (a) $C = 0.05$, (b) $C = 0.25$, (c) $C = 0.5$, and (d) $C = 0.82$.

which imply that a space-charge dominated beam compensates for the focusing field in the beam core regardless of the applied external focusing potential, a phenomenon known as Debye shielding for nonneutral plasmas. The space-charge distribution required for matching is then derived from Poisson's equation as $\rho_b = -\epsilon_0 \Delta U_b = \frac{\epsilon_0}{1+\delta}\gamma^2 \Delta U_{\text{ext}}$.

*Matching channel for space-charge dominated beam*

In [1], a uniform four vane structure with field

$$
\begin{aligned}
\vec{E} = &\left[ -\vec{i}_r \left( G_2 r \cos(2\varphi) + G_6 r^5 \cos(6\varphi) \right) \right. \\
&\left. + \vec{i}_\varphi \left( G_2 r \sin(2\varphi) + G_6 r^5 \sin(6\varphi) \right) \right] \sin(\omega_0 t),
\end{aligned}
\tag{4}
$$

is considered, where $G_2$ is a quadrupole gradient, $G_6$ is a duodecapole component, and $\omega_0 = 2\pi c/\lambda$ is an operational frequency. This structure can be described by the effective potential

$$
U_{\text{ext}}(r, \varphi) = \frac{mc^2}{q} \frac{\mu_0^2}{\lambda^2} \left[ \frac{1}{2}r^2 + \zeta r^6 \cos(4\varphi) + \frac{\zeta^2}{2} r^{10} \right],
\tag{5}
$$

where $\mu_0 = \frac{q G_2 \lambda^2}{\sqrt{8\pi}mc^2}$ is a smooth transverse oscillation frequency [2] and $\zeta = \frac{G_6}{G_2}$ is the ratio of field components. Contour plots of Eq. (5) are shown in Figure 2. By applying Eq.(5), an expression is found for a self consistent space-charge distribution of the beam in the structure and thereby the required values of the focusing gradients $G_2$ and $G_6$ are found to be:

$$
G_2 = \frac{\sqrt{8\pi}mc^2}{q\lambda R}\sqrt{\frac{\epsilon^2}{R^2} + \frac{3I}{I_c \beta\gamma}},
$$

$$
G_6 = -\frac{G_2}{12\beta\gamma R^4}\frac{I}{I_c}\left(\frac{\epsilon^2}{R^2} + \frac{3I}{I_c\beta\gamma}\right)^{-1}.
\tag{6}
$$

## FODO CHANNEL WITH QUADRUPOLE-DUODECAPOLE FIELD

The focusing electric field, Eq. (4), can be realized by a uniform four-vane structure with specific pole-tip shape
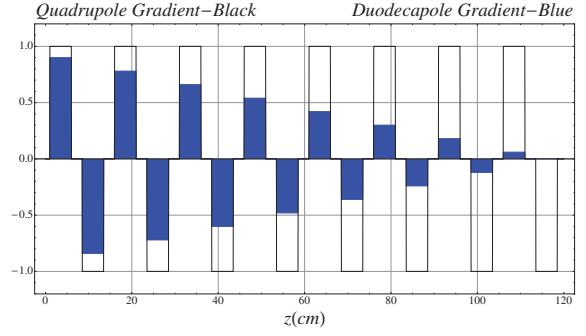


Figure 3: FODO quadrupole-duodecapole channel with combined lenses with the period of $L = 15$ cm, lens length of $D = 5$ cm, and adiabatic decline of duodecapole component to zero over a distance of 7 periods.

imposing duodecapole component in pure quadrupole. The construction of such a structure is mechanically complicated and expensive. We present a simpler and more practical structure, as shown in Figure 3. We consider a FODO lattice of lenses with combined quadrupole $G_2(z)$ and duodecapole $G_6(z)$ field components. Such magnets can be done as a combination of conventional quadrupoles with current sheet magnets [3]. The quadrupole field is kept constant along the structure while duodecapole component gradually decreases from nominal value to zero at a certain distance. It gives us the possibility to match initially non-unifom beam with the non-linear focusing channel and adiabatically transform it to the beam matched with quadrupole focusing structure. Magnetic field along the structure is represented by:

$$
\begin{aligned}
\vec{B} = &\left[ \vec{i}_r \left( G_2 r \sin(2\varphi) + G_6 r^5 \sin(6\varphi) \right) \right. \\
&\left. + \vec{i}_\varphi \left( G_2 r \cos(2\varphi) + G_6 r^5 \cos(6\varphi) \right) \right] G(z),
\end{aligned}
$$

where $G(z)$ is the longitudinal field dependence expanded in Fourier series:

$$
G(z) = \frac{4}{\pi} \sum_{\substack{n=1 \\ k=2n-1}}^{\infty} \frac{(-1)^{\frac{k-1}{2}}}{k} \sin\left(\frac{\pi k D}{L}\right) \sin\left(\frac{2\pi k z}{L}\right),
\tag{7}
$$

where $D$ is the length of the lens and $L$ is the period of the structure. According to the averaging method [4], particle trajectory in the fast oscillation field

$$
\ddot{\vec{r}} = \frac{q}{m\gamma}\vec{F}(\vec{r}, t), \quad \vec{F}(\vec{r}, t) = \sum_{k=1}^{\infty} \vec{F}_k(\vec{r})\sin(\omega_k t)
\tag{8}
$$

can be approximated by a Hamiltonian of averaged particle motion:

$$
H = \frac{1}{2}\dot{\vec{R}}^2 + \frac{q^2}{4(m\gamma)^2}\sum_{k=1}^{\infty}\frac{\vec{F}_k^2(\vec{R})}{\omega_k^2}.
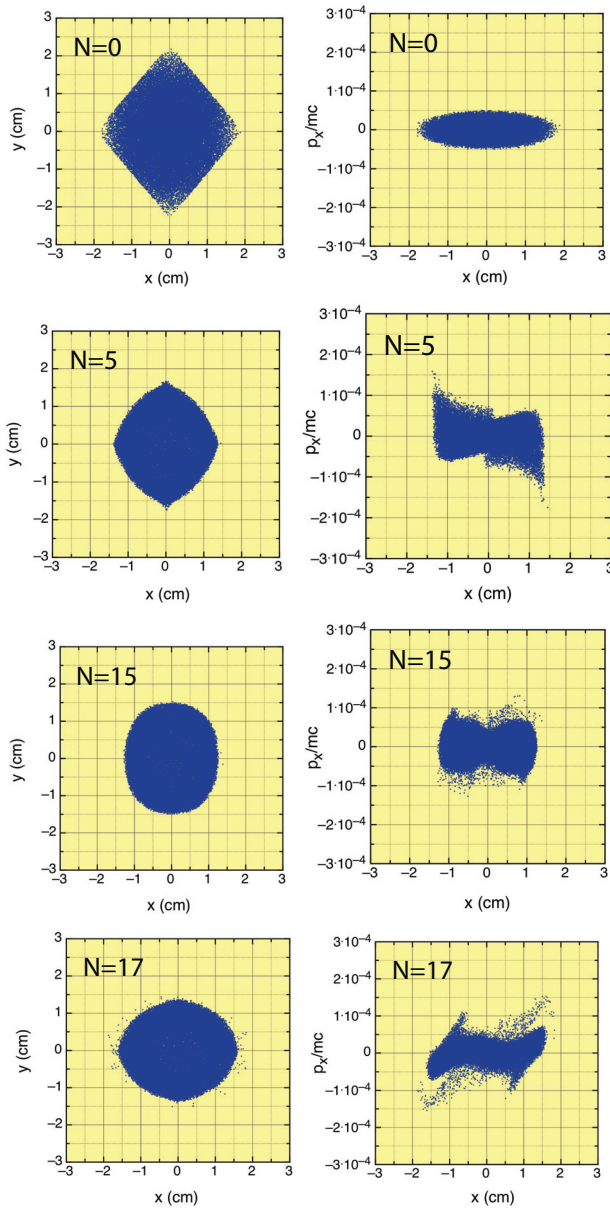\tag{9}
$$

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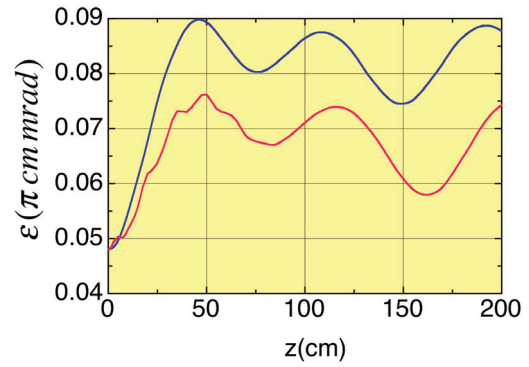**D04 High Intensity in Linear Accelerators**

Figure 5: Emittance growth in FODO quadrupole channel shown in blue, and in the quadrupole-duodecapole channel shown in red.

Calculation of potential part of Hamiltonian, Eq. (9), gives for effective potential of the structure:

$$U_{\text{eff}}(r, \varphi) = \mu_0^2 \left( \frac{\beta c}{L} \right)^2 \left( \frac{r^2}{2} + \zeta r^6 \cos(4\varphi) + \frac{\zeta^2}{2} r^{10} \right),$$
(10)

where $\mu_0$ is the phase advance of transverse oscillations per period of FODO channel [2]:

$$\mu_0 = \frac{L}{2D} \sqrt{1 - \frac{4}{3} \frac{D}{L}} \frac{e G_2 D^2}{mc\beta\gamma}.$$
(11)

Potential, Eq. (10), is similar to that derived earlier for the 4-rod structure, Eq. (5), with different value of transverse betatron tune $\mu_0$.

Figures 1, 4, and 5 contain results of BEAMPATH simulations of 50 keV, 20 mA, 0.05 $\pi$ cm mrad proton beam in the focusing structure presented in Fig. 3. Injected beam with parabolic distribution $f = f_0 \left( 1 - \frac{x^2+y^2}{2R^2} - \frac{p_x^2+p_y^2}{2p_0^2} \right)$ we truncated along equipotential lines of effective potential (see Fig. 2), to make the beam distribution close to the matched beam. Adiabatic change of duodecapole component along the structure results in gradual transformation of an initially non-uniform beam distribution into distribution matched with quadrupole channel with suppressed emittance growth and halo formation.

## ACKNOWLEDGEMENTS

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Figure 4: Adiabatic matching to avoid halo formation of the 50 keV, 20 mA, 0.05 $\pi$ cm mrad proton beam in FODO quadrupole-duodecapole channel with combined lenses with the period of $L = 15$ cm, lens length of $D = 5$ cm, quadrupole field gradient of $G_2 = 0.04278 \frac{\text{T}}{\text{cm}}$ and adiabatic decline of duodecapole component from $G_6 = -0.21 \times 10^{-3} \frac{\text{T}}{\text{cm}^5}$ to zero for the distance of 7 periods. Numbers indicate FODO periods.

Amplitude and frequency of field harmonics in Eq. (9) are determined by $\vec{B}$ and Eq. (7):

$$F_k^2(r, \varphi) = \left( \frac{4\beta c}{\pi} \right)^2 B^2(r, \varphi) \frac{\sin^2 \left( \frac{\pi(2k-1)D}{L} \right)}{(2k-1)^2},$$

$$\omega_k^2 = 4\pi^2 \left( \frac{(2k-1)\beta c}{L} \right)^2.$$

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