THE EFFECT OF NON-ZERO CLOSED ORBIT ON ELECTRON-CLOUD PINCH DYNAMICS

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Abstract

A study on the pinch dynamics of electron cloud during a bunch passage under the effect of a single arbitrary-order multipole was presented at IPAC2011. The complexity of the pinch pattern is directly related to the order of the multipolar field. However, in a realistic situation, the proton beam will not be located in the center of the vacuum chamber. If the beam is offset a new pinch regime is encountered, where feed-down effects and asymmetry of pinch density render the dynamics more challenging. In this paper we discuss the pinch dynamics with orbit offset, including the resulting orbit variation along a bunch, and address their relevance for the incoherent effect of the electron cloud.

INTRODUCTION

The mechanism of incoherent effects of emittance growth or poor beam lifetime has been discussed in several papers. Typically a transverse detuning driven by a transverse longitudinal coupling gives rise to a periodic resonance crossing. For bunched beams, resonance crossing is of relevance when stable islands are driven into and out of the beam core. For the case of space charge the amplitude dependent detuning is created by the beam field itself and, therefore, normally shows a maximum at the beam center, which scales, for a transverse Gaussian distribution as $\Delta Q_x \sim \Delta Q_{x0}/(1 + I_x/(2\epsilon_{x0})]$, with I_x the Courant-Snyder (action) variable, ϵ_{x0} the rms beam emittance, and ΔQ_{x0} the maximum tune-shift. The detuning due to space charge fades away with the inverse square of the transverse oscillation amplitude. For the case of the electron-cloud pinch, discussed in this paper, the situation is more complicated. The electron pinch produces a complex structure of localized high density peaks, that change according to the longitudinal position along the bunch. The electron cloud structure resulting from the pinch process affects the proton dynamics, also in this case, by creating an amplitude dependent detuning (coupled with the longitudinal motion), and a web of structure resonances [1]. We here propose a characterization of the electron pinch for the incoherent effects, and, in particular, study the effect of a beam displacement.

ELECTRON-PINCH CHARACTERIZATION

From the point of view of beam degradation, the detuning at the beam core $(\Delta x_c, \Delta y_c)$ is of key relevance for the process of resonance crossing. $\Delta x_c, \Delta y_c$ are the horizontal,vertical displacement of the beam with respect to the center of the vacuum chamber. In other words, although the electron cloud structure is quite rich and complex, only the (maximum) detuning on the beam (z) axis is relevant for identifying the start of any diffusion process. In terms of proton dynamics the detuning is related to the gradient of the force created by the structure of electrons: a highly localized peak of electrons at $(\Delta x_c, \Delta y_c)$ and $z = 0$ will certainly produce a higher detuning than if located at $(\Delta x_c + 4\sigma_r, \Delta y + 4\sigma_r)$. These considerations suggest a criterion to quantify the relevance of the electron pinch for the beam dynamics. Namely we can use as indicator of the importance of the electron pinch the gradient of the electric field created by the pinched electron distribution at the transverse position $(\Delta x_c, \Delta y_c)$ (the beam closed orbit) for several z locations along the bunch. However, even by using this criterion it is not easy to compare the effect of beam mismatch because of the presence of several peaks along z. As a first approach we only consider the maximum gradient found along z at $(\Delta x_c, \Delta y_c)$, and compare it with the initial value due the unperturbed electron distribution from before the start of the pinch.

The force of the electrons is computed assuming each macro-electron to be an infinitely long thin wire so that the force scales as $1/r$. To prevent artificial effects a cut-off is implemented. The electric field on (x, y), $E_x(x, y)$, and $E_y(x, y)$ is computed by summing up all the forces exerted by all electrons except for those located inside a circle of radius $r_{\min} = 0.05\sigma_r$ centered at (x, y). For N macro-electrons uniformly distributed inside a cylinder of radius R we find a gradient $dE_x/dx \propto 2N/R^2$. We cut off all particles that can create a gradient larger than this value since the gradient field created by one macro-electron is $dE_x/dx \propto 2/r^2$. The minimum radius is $r_{\min} = R/\sqrt{N}$. For $R = 10\sigma_r$, and $N = 5 \times 10^5$ we find $r_{\min} = 0.014\sigma_r$, and for safety we take $r_{\min} = 0.045\sigma_r$. This procedure produces a systematic error in estimating the gradient of -$(r_{\min}/r_g)^2$ with r_g the distance from the beam center where the field is computed. The gradient is computed as $dE_x(x, y)/dx = [E_x(x + \Delta x, y) - E_x(x - \Delta x, y)]/(2\Delta)$ with $\Delta = 0.1\sigma_r$. A similar definition is used to compute $dE_y(x, y)/dy$. The systematic error in the estimation of the gradient is then $-(0.045/0.1)^2$, that is about -20%. (on the other hand, without the cut-off the fluctuations would be much larger).

Simulation Condition

In the following we consider the simulations of electron pinch under the passage of an LHC proton bunch with transverse rms size σ_r of 0.88 mm, an rms bunch length σ_z of 11.4 cm, with a bunch population $N_p = 1.15 \times 10^{11}$

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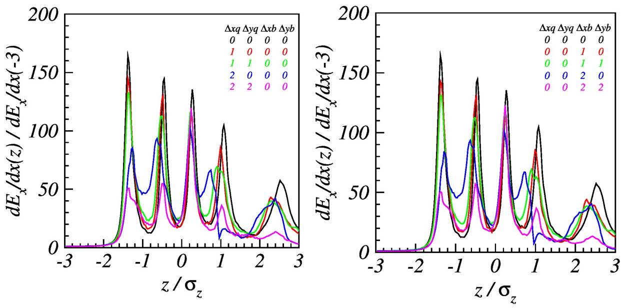

Figure 1: Left: Gradient of electron cloud induced electric field along the bunch. The colors refer to different degree of displacement of the quadrupole as indicated by the table in the picture. Right: Dependence of the pinch gradient on beam center as function of the beam displacement $\Delta x_b, \Delta y_b$.

protons, and a beam energy of 450 GeV. The initial electron distribution is uniform in a circle of radius $R = 10\sigma_r$, and it is always considered centered in the vacuum chamber, the number of macro-electrons is $N = 5 \times 10^5$. With respect to this reference frame we will consider displacement of the beam $\Delta x_b, \Delta y_b$, or displacement of an element, as a quadrupole, $\Delta x_q, \Delta y_q$.

CHARACTERIZATION IN A QUADRUPOLE

Displacing a Quadrupole

We consider here the effect of the displacement of a quadrupole of $\Delta x_q, \Delta y_q$, meaning that the center of the quadrupole is shifted with respect to the vacuum chamber which defines the location of the initial distribution of electrons co-axial with the beam. In Fig. 1 (left) we show the gradient at $(\Delta x_c, \Delta y_c)$ along z for 4 different displacements of the quadrupole (according to the table in the picture) in units of beam σ_r.

The bunch considered is the LHC type bunch, and for the case of the quadrupole on axis each spike represents the development of a consecutive electron pinch. We see that the strength of the pinch in terms of field gradient is 100 times larger than the effect produced by the (uniformly distributed) electrons at the beginning of the bunch passage. The picture shows that increasing the displacement of the quadrupole the first peak reduces in strength the more the quadrupole is displaced from the central position. The situation is complex: by shifting the quadrupole the field acting on the electrons contains a dipolar feed down.

Displacing the Beam

The shift of the beam with respect to the vacuum chamber center, is instead equivalent to the shift of the origin of the pinch: therefore, the evolving distribution of the electrons is now shifted off axis following the transverse center of the beam. The pinch process should in this case be affected by the asymmetry of the initial electron distribution, with respect to the displaced center of the beam. However, the conflicting effects of the quadrupole forces centered at the origin of the beam pipe and the Coulomb attraction towards the shifted bunch also play a crucial role. We expect that a shift of the beam axis will significantly reduce the effect of detuning experienced along the longitudinal beam direction. Figure 1 (right) presents this effect when displacing the beam by the same amounts as for considered for the quadrupoles in the left figure. The comparison shows that there is no significant difference in the electron-cloud gradient experienced on the beam axis when displacing either the beam or the quadrupole by the same amount. A similar finding is obtained for the case of dipole magnets.

COMPARING THE EFFECT OF SHIFT IN DIFFERENT ELEMENTS

We here discuss the effect of the beam displacement in several basic accelerator elements such as 1) drift, 2) dipole, and 3) quadrupole. The study is made by plotting the maximum gradient along z for several beam displacements. In Figs. 2 and 3 we show the result of the simulation study where $\mathrm{Grad} \equiv \max\{[dE_x(x, y, z)/dx]/[dE_x(x, y, -3)/dx] : -3 < z < 3\}$.

Discussion:

Drift. In Fig. 2 (left) we explore the maximum pinch for a beam a displacement in the range $-4\sigma_r, 4\sigma_r$ for x and y planes. We notice that for the pinch in a drift the maximum gradient is located in a circular region of radius $2\sigma_r$ where the relative gradient is $\sim$ 430 times larger than the initial gradient created by the uniformly distributed electrons. The circular region arises due to the shape of the initial electron distribution, a circular uniform distribution of radius $R = 10\sigma_r$. We conclude that a displacement within a radius of $2\sigma_r$ does not affect the electron pinch in the drifts. (In case we had a larger initial electron distribution this radius would be larger).

Dipole. Figure 3 (left) demonstrates that the normalized gradient is around $\sim$ 25 in the full region explored. This result is a consequence of the physical process creating the pinch in a dipole. As discussed in Ref. [2] the electrons are constrained to vertical motion by the strong dipolar magnetic field. Hence the electrons participating in the pinch on the longitudinal axis are mainly those in the neighborhood of a vertical slice of electrons (centered around $x = 0$) passing through the longitudinal axis. The number of these electrons is significantly smaller than those participating in the pinch for the drift case. This explains the relative weakness of the normalized gradient, which now is 25 compared with 430 for the case of the drift. Note that the constant value of the gradient in the full explored region suggests that the electrons contributing to the peak gradient are those coming from a height of $\sim \pm 1\sigma_r$. This was confirmed by a further simulation (Fig. 3 (right)) made in the range $-10\sigma_r, +10\sigma_r$ in x,y planes. In fact in this picture the black color correspondent to $\mathrm{Grad} = 27$ does not reach $x = 0$, $y = 10\sigma_r$, but stops at $y = 9\sigma_r$, because the process of pinch formation misses the electrons to form the proper localized pinch.

Quadrupole. The pinch in a quadrupole shows maximum gradients that are weaker with respect to those of the drift case; see Fig. 2 (right). The picture exhibits the symmetry imposed by the quadrupole, but the maximum gradient is now smaller because the effect of the quadrupole is absent only for electrons located on the diagonals [3]. For all other electrons the force of attraction towards the center of the bunch is affected by the force of the quadrupole that pushes the electrons away, thereby diminishing the pinch effect and hence the gradient on the beam axis.

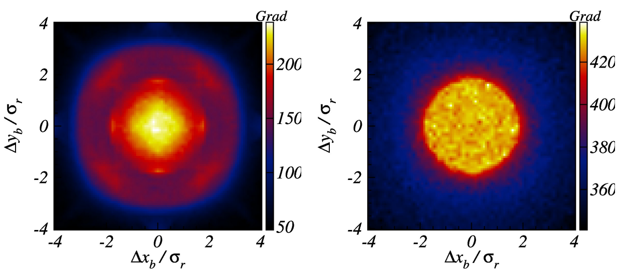

Figure 2: Maximum gradient for drift (left) and quadrupole (right) as function of the beam displacement $(\Delta x_b, \Delta y_b)$. The gradient is normalized to the gradient created by the electrons at the beginning of the bunch passage.

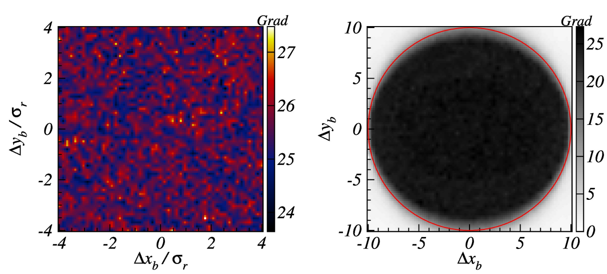

Figure 3: Maximum gradient for a dipole as function of the beam displacement $(\Delta x_b, \Delta y_b)$. The gradient is normalized to the gradient created by the electrons at the beginning of the bunch passage. The left and right pictures refer to different ranges of displacements $(\Delta x_c, \Delta y_c)$. Indicated in red is the initial edge of the electron distribution.

CONCLUDING REMARKS

This study shows that for displacements of the beam with respect to the vacuum chamber no significant effect is expected if the displacement is within a radius of $1\sigma_r$. For larger values the most significant reduction is found in the quadrupoles. The results in Figs. 2 and 3 also allow comparing the relative importance of localized electrons along the machine: They show that the gradient in a quadrupoles is significantly larger than in a dipole. However, a full comparison should also include the integrated effect over the total length of these elements, taking into account as well large differences in the initial electron density, which means that the higher gradient enhancement in the quadrupoles should be weighted by the (possibly shorter) total length of the quadrupoles and the (possibly lower) average electron density, so that the electron cloud in the dipoles may still play a significant role. The validity of our conclusions based on the proposed indicator (which is a "static" as the proton dynamics is not treated) should be validated in a future study versus full long term simulation.

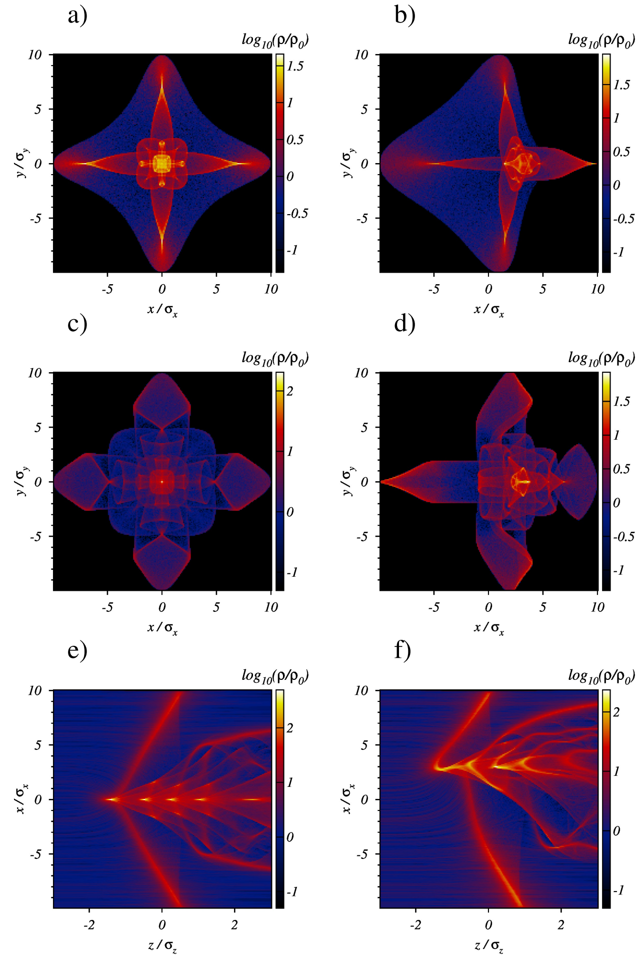

Figure 4: Electron density enhancement for the proton beam on axes (left column), and the same simulation with the beam shifted of $\Delta x_b = 3\sigma_r$ (right column). ab) the $x - y$ plane at $z = 0$; cd) the $x - y$ plane at $z = 1$; ef) the $z - x$ plane at $y = 0$.

For the sake of comparison and to show the complexity of the electron dynamics in Fig. 4 we present a comparison between the pinch in a quadrupole obtained for a beam on axis and for a beam shifted by $\Delta x_b = 3\sigma_r$.

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