REPRODUCTION OF CERAMIC CHAMBER IMPEDANCES WITH ELECTRIC AND MAGNETIC POLARITIES OF THE CERAMICS

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Abstract

In proton synchrotrons, ceramic chambers are used as vacuum chambers to avoid the effect on magnetic fields from eddy current excited by the magnetic fields. One of the standard methods of the derivation of the impedances of the ceramic chamber is the field matching technique. In this report, we reproduce the formulae of the ceramic chamber impedance in terms of electric and magnetic polarities. When the beam passes through the chamber, the impedance is mainly excited by the electric polarity of the ceramic.

INTRODUCTION

In proton synchrotrons, ceramic chambers are used to avoid effects on magnetic fields in the vacuum chamber from eddy currents excited by the magnetic fields [1, 2]. In order to accomplish the high intensity beam, it is important to evaluate the impedance due to the ceramic chamber correctly.

One of the standard methods of the derivation of the impedance of the ceramic chamber is the field matching technique [3], while it is difficult to imagine the impedance source in the calculation. It is known that the impedance is written by electric and magnetic polarities of the impedance source [4], which is useful to understand the excitation mechanism of the impedance source.

In this report, we reproduce the formulae of the ceramic chambers by explicitly calculating the electric and magnetic polarities of the ceramics. The derivation of the impedance reveals that the impedance is mainly caused by the excitation of the electric polarity of the ceramic.

In the next section, let us first derive the impedance of the ceramic chamber by using the field matching technique.

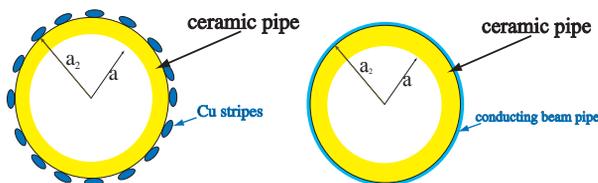## DERIVATION OF THE IMPEDANCES WITH THE FIELD MATCHING TECHNIQUE

Figure 1: A schematic picture of the ceramic chamber.

As shown in the left figure of Fig.1, copper stripes attach on the ceramic chamber to shield radiation effects from beams. In order to calculate the impedance of the chamber in an analytical way, the configuration is simplified as in the right figure of Fig.1, such that the ceramic pipe is surrounded by perfectly conductive pipe instead of copper stripes. It is known that the approximation well describes the impedance of the ceramic chamber except non-relativistic beams [5].

Longitudinal Impedance

Let us derive the longitudinal impedance when a relativistic beam passes through the chamber. A cylindrical coordinate (r, φ, z) is used in the following calculations. From now on, we omit the factor $e^{-j\tilde{k}z}$ in the expression of the fields. General solutions for the monopole mode $(m = 0)$ are expressed as

$$E_z = A(\tilde{k}), \tag{1}$$

$$H_\varphi = \frac{c}{2\pi r} + \frac{j\tilde{k}rA(\tilde{k})}{2Z_0}, \tag{2}$$

inside the chamber $(r < a)$, and

$$E_z = B(\tilde{k})[J_0(\tilde{\alpha}r) - \frac{J_0(\tilde{\alpha}a_2)}{Y_0(\tilde{\alpha}a_2)}Y_0(\tilde{\alpha}r)], \tag{3}$$

$$H_\varphi = \frac{j\epsilon'B(k)}{Z_0\sqrt{\epsilon'-1}}[J_1(\tilde{\alpha}r) - \frac{J_0(\tilde{\alpha}a_2)}{Y_0(\tilde{\alpha}a_2)}Y_1(\tilde{\alpha}r)], \tag{4}$$

in the ceramic $(a < r < a_2)$, where $\tilde{\alpha} = \tilde{k}\sqrt{\epsilon'-1}$, c is velocity of light, $\tilde{k} = \omega/c$, $Z_0 = 120\pi$, ϵ' is the relative dielectric constant of the ceramic, $A(\tilde{k})$, $B(\tilde{k})$ are arbitrary coefficients that will be determined by using the matching conditions on the surface specified by $r = a$. The longitudinal impedance is given as

$$Z_L = -\frac{A(\tilde{k})}{c}\mathcal{L}, \tag{5}$$

where $\mathcal{L}$ is the length of the chamber and

$$A(\tilde{k}) = \frac{-\frac{cZ_0}{2\pi a}}{-\frac{j\epsilon'[J_1(\tilde{\alpha}a) - \frac{J_0(\tilde{\alpha}a_2)}{Y_0(\tilde{\alpha}a_2)}Y_1(\tilde{\alpha}a)]}{\sqrt{\epsilon'-1}[J_0(\tilde{\alpha}a) - \frac{J_0(\tilde{\alpha}a_2)}{Y_0(\tilde{\alpha}a_2)}Y_0(\tilde{\alpha}a)]} + \frac{j\tilde{k}a}{2}}. \tag{6}$$

For low frequency, it is approximated to

$$(Z_L)_{ceramic} = j\frac{\omega Z_0(\epsilon'-1)\mathcal{L}}{2\pi\epsilon'c}\log\frac{a_2}{a}, \tag{7}$$

which is identical to the previous Tsutsui's results.

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Transverse Impedance

When a non-relativistic beam with the azimuthal dependency of $j_z = q\beta c\delta(r - r_b)\cos(\varphi - \varphi_b)e^{-jkz}/\pi r_b$ is running inside the chamber, general solutions for the dipole mode ($m = 1$) are expressed as

$$E_z = i_1(E_z^S + A(k)I_1(\bar{k}r)\cos(\varphi - \varphi_b)), \tag{8}$$

$$H_\varphi = i_1 H_\varphi^S$$
$$+ i_1\frac{j\gamma\left(\frac{B(k)I_1(\bar{k}r)}{r} + \frac{\beta\bar{k}A(k)}{Z_0}I_1'(\bar{k}r)\right)\cos(\varphi - \varphi_b)}{\bar{k}}, \tag{9}$$

$$H_z = i_1 B(k)I_1\left(\bar{k}r\right)\sin(\varphi - \varphi_b), \tag{10}$$

$$E_\varphi = i_1 E_\varphi^S - i_1\frac{j\beta\gamma Z_0}{\bar{k}}$$
$$\times \left(\bar{k}B(k)I_1'(\bar{k}r) + \frac{A(k)}{Z_0\beta r}I_1(\bar{k}r)\right)\sin(\varphi - \varphi_b), \tag{11}$$

inside the chamber ($r < a$), and

$$E_z = C_3(k)[J_1(\alpha r) - \frac{J_1(\alpha a_2)}{Y_1(\alpha a_2)}Y_1(\alpha r)]\cos(\varphi - \varphi_b), \tag{12}$$

$$H_\varphi = \frac{j\beta\epsilon'}{Z_0 k(1 - \epsilon'\beta^2)}[\frac{Z_0 C_1(k)(J_1(\alpha r) - \frac{J_1'(\alpha a_2)}{Y_1'(\alpha a_2)}Y_1(\alpha r))}{\beta\epsilon' r}$$
$$+ C_3(k)\alpha(J_1'(\alpha r) - \frac{J_1(\alpha a_2)}{Y_1(\alpha a_2)}Y_1'(\alpha r))]\cos(\varphi - \varphi_b), \tag{13}$$

$$H_z = C_1(k)[J_1(\alpha r) - \frac{J_1'(\alpha a_2)}{Y_1'(\alpha a_2)}Y_1(\alpha r)]\sin(\varphi - \varphi_b), \tag{14}$$

$$E_\varphi = \frac{1}{jk(1 - \beta^2\epsilon')}[\alpha\beta Z_0 C_1(J_1'(\alpha r) - \frac{J_1'(\alpha a_2)}{Y_1'(\alpha a_2)}Y_1'(\alpha r))$$
$$+ \frac{C_3}{r}(J_1(\alpha r) - \frac{J_1(\alpha a_2)}{Y_1(\alpha a_2)}Y_1(k\sqrt{\epsilon' - 1}r))]\cos(\varphi - \varphi_b), \tag{15}$$

in the ceramic ($a < r < a_2$), where

$$E_z^S = \begin{cases} \frac{jkcZ_0 I_1(\bar{k}r_b)}{\pi r_b\gamma^2}K_1(\bar{k}r)\cos(\varphi - \varphi_b) & \text{for } r > r_b, \\ \frac{jkcZ_0 K_1(\bar{k}r_b)}{\pi r_b\gamma^2}I_1(\bar{k}r)\cos(\varphi - \varphi_b) & \text{for } r_b > r, \end{cases} \tag{16}$$

$$E_\varphi^S = \begin{cases} \frac{cZ_0 I_1(\bar{k}r_b)}{r\pi r_b}K_1(\bar{k}r)\sin(\varphi - \varphi_b) & \text{for } r > r_b, \\ \frac{cZ_0 K_1(\bar{k}r_b)}{r\pi r_b}I_1(\bar{k}r)\sin(\varphi - \varphi_b) & \text{for } r_b > r, \end{cases} \tag{17}$$

$$H_\varphi^S = \begin{cases} \frac{\beta kc I_1(\bar{k}r_b)}{2r r_b\gamma}(K_0(\bar{k}r) + K_2(\bar{k}r))\cos(\varphi - \varphi_b) \\ \qquad\qquad \text{for } r > r_b, \\ -\frac{\beta kc K_1(\bar{k}r_b)}{2\pi r_b\gamma}(I_0(\bar{k}r) + I_2(\bar{k}r))\cos(\varphi - \varphi_b) \\ \qquad\qquad \text{for } r_b > r, \end{cases} \tag{18}$$

the factor e^{-jkz} is omitted, β is Lorentz-β, $\alpha = k\sqrt{\epsilon' - 1}$, $k = 2\pi f/\beta c$, $\bar{k} = k/\gamma$, $K_m(z)$ and $I_m(z)$ are

the modified Bessel functions, respectively, $i_1 = qr_b$, $A(k), B(k), C_1(k)$ and $C_3(k)$ are arbitrary coefficients, which are determined by boundary conditions at $r = a$, as well.

Using the Panofsky-Wenzel theorem, the expression of the transverse impedance for the relativistic beam in the low frequency as

$$Z_T = j\frac{\mathcal{L}Z_0(a_2^2 - a^2)}{2\pi a^2 a_2^2}. \tag{19}$$

When the thickness of the chamber $a_2 - a$ is so small compared to a, we reproduce the empirical formula:

$$(Z_T)_{ceramic} = j\frac{Z_0\mathcal{L}}{\pi a^2}\log\frac{a_2}{a} \simeq \frac{2}{ka^2}(Z_L)_{ceramic}. \tag{20}$$

DERIVATION OF THE IMPEDANCES WITH THE ELECTRIC AND MAGNETIC POLARITIES IN THE CERAMICS

In this section, we reproduce the impedances of the ceramic chamber in terms of the electric and magnetic polarities in the ceramic. Let us confine our discussion on the chamber whose longitudinal length $\mathcal{L}$ is sufficiently longer than the radius of the chamber a. First, we explain how to derive the magnetic polarity α_m in the ceramic. The assumption was made that the uniform magnetic dipole moment per a volume $\vec{m}$, which is equal to $m(-\sin\theta, \cos\theta, 0)$, exists at $(r_0\cos\theta, r_0\sin\theta, \xi)$. The vector potential $\vec{A}$ at $(r\cos\varphi, r\sin\varphi, z)$ can be described as,

$$\vec{A} = \int\frac{\mu_0}{4\pi}\frac{m(r') \times \vec{r}}{r^3}dV'$$
$$= \frac{\mu_0 m}{4\pi}\int_{-\infty}^\infty d\xi \oint d\theta \int_a^{a_2} dr_0 r_0 \times$$
$$\frac{[(z - \xi)\cos\theta, (z - \xi)\sin\theta, -r\cos(\varphi - \theta) + r_0]}{((r\cos\varphi - r_0\cos\theta)^2 + (r\sin\varphi - r_0\sin\theta)^2 + (z - \xi)^2)^{\frac{3}{2}}} \tag{21}$$

where both the upper and the lower bounds of ξ-integration approximately go to infinity. After the ξ-integration, only A_z component is not zero. Then, we obtain

$$A_z = \frac{\mu_0 m}{2\pi}\oint d\theta \int_a^{a_2} dr_0 r_0 \frac{-r\cos(\varphi - \theta) + r_0}{r^2 + r_0^2 - 2rr_0\cos(\varphi - \theta)}$$
$$= \mu_0 m(a_2 - \max\{r, a\}). \tag{22}$$

The vector potential gives the magnetic field as

$$H_\theta = \begin{cases} m & \text{for } r > a, \\ 0 & \text{for } r < a. \end{cases} \tag{23}$$

Since the magnetic field in the ceramic is given by the superposition of Eq.(23) and

$$H_\theta = \frac{I_B}{2\pi r}, \tag{24}$$

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where I_B is the beam current, it is described as

$$H_\theta = m + \frac{I_B}{2\pi r}. \quad (25)$$

In vacuum, the magnetic field is given by

$$H_\theta = \frac{I_B}{2\pi r}. \quad (26)$$

Since the azimuthal component of magnetic field should be continuous, the magnetic dipole moment m is equal to zero. In other words, the magnetic polarity α_m is identical to zero.

Secondly, we derive the electric polarity α_e in the ceramic. For this purpose, let us calculate the radial component of the electric field. When a longitudinally uniform charge, whose line density is equal to λ, exists at the center of the chamber, the radial component of the electric field E_r is given by

$$E_r = \begin{cases} \frac{\lambda}{2\pi\epsilon_0 r} & \text{for } r < a, \\ \frac{\lambda}{2\pi\epsilon'\epsilon_0 r} & \text{for } r > a. \end{cases} \quad (27)$$

Since there is perfectly conductive pipe at the outer surface of the ceramic a_2, the electric potential Φ is expressed as

$$\Phi = - \begin{cases} \frac{\lambda}{2\pi\epsilon_0} \log \frac{r}{a} + \frac{\lambda}{2\pi\epsilon'\epsilon_0} \log \frac{a}{a_2} & \text{for } r < a, \\ \frac{\lambda}{2\pi\epsilon'\epsilon_0} \log \frac{r}{a_2} & \text{for } r > a. \end{cases} \quad (28)$$

When the beam runs in the chamber, the negative charge is induced on the inner surface of the chamber and the positive charge is done on the outer surface of the chamber. Then, we can imagine that there is the electric dipole moment per a volume $\vec{p}$, which is equal to $p_0(\cos\theta, \sin\theta, 0)$, along the radial direction.

The electric potential Φ_e at $(r\cos\varphi, r\sin\varphi, z)$ due to the electric dipole is given by

$$\Phi_e = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{p}(r') \cdot \vec{r}}{r^3} dV'. \quad (29)$$

If the dipole moment exists at $(r_0\cos\theta, r_0\sin\theta, \xi)$, the electric potential Φ_e is calculated as

$$\Phi_e = \frac{1}{4\pi\epsilon_0} \int_{-\infty}^{\infty} d\xi \oint d\theta \int_a^{a_2} dr_0 r_0 \times$$

$$\frac{p_0[\cos\theta(r\cos\varphi - r_0\cos\theta) + \sin\theta(r\sin\varphi - r_0\sin\theta)]}{((r\cos\varphi - r_0\cos\theta)^2 + (r\sin\varphi - r_0\sin\theta)^2 + (z-\xi)^2)^{\frac{3}{2}}}$$

$$= -\frac{p_0}{\epsilon_0}(a_2 - \max\{r, a\}), \quad (30)$$

where the both the upper and the lower bounds of ξ-integration approximately go to infinity.

Here the assumption was made that the electric potential in the ceramic is given by the superposition of the potential Φ without the ceramic, which is equal to $-\frac{\lambda}{2\pi\epsilon_0} \log \frac{r}{a_2}$ and Φ_e, which is given by Eq.(30). It is given by

$$-\frac{\lambda}{2\pi\epsilon_0} \log \frac{r}{a_2} - \frac{p_0}{\epsilon_0}(a_2 - \max\{r, a\}). \quad (31)$$

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Since Eqs.(28) and (31) are continuous at $r = a$, we obtain

$$-\frac{\lambda}{2\pi\epsilon_0} \log \frac{a}{a_2} - \frac{p_0}{\epsilon_0}(a_2 - a) = -\frac{\lambda}{2\pi\epsilon'\epsilon_0} \log \frac{a}{a_2}. \quad (32)$$

Equation(32) is rewritten, and the electric dipole moment per a volume p_0 is expressed as

$$p_0 = \frac{\lambda}{2\pi(a_2 - a)}(\frac{1}{\epsilon'} - 1) \log \frac{a}{a_2}. \quad (33)$$

Since the volume of the ceramic pipe is equal to $\mathcal{L}2\pi a(a_2 - a)$, the total electric dipole moment P is calculated as

$$P = p_0 \mathcal{L}2\pi a(a_2 - a) = \mathcal{L}a\lambda(\frac{1}{\epsilon'} - 1) \log \frac{a}{a_2}. \quad (34)$$

Since the electric polarity α_e is defined as

$$P = \alpha_e \epsilon_0 \frac{\lambda}{2\pi\epsilon_0 a}, \quad (35)$$

it is expressed as

$$\alpha_e = 2\pi\mathcal{L}a^2(\frac{1}{\epsilon'} - 1) \log \frac{a}{a_2}. \quad (36)$$

We have already known that both the longitudinal and the transverse impedances are given by [4]

$$Z_L = j\frac{\omega Z_0}{c} \frac{\alpha_e + \alpha_m}{4\pi^2 a^2}, \quad (37)$$

$$Z_T = jZ_0 \frac{\alpha_e + \alpha_m}{2\pi^2 a^4}, \quad (38)$$

by using the electric and the magnetic polarities. Substituting Eq.(36) and $\alpha_m = 0$ into Eqs.(37) and (38), we obtain

$$Z_L = j\frac{\omega Z_0}{c} \frac{1}{4\pi^2 a^2} 2\pi\mathcal{L}a^2(\frac{1}{\epsilon'} - 1) \log \frac{a}{a_2}$$

$$\simeq j\frac{\omega Z_0 \mathcal{L}}{2\pi c} \log \frac{a_2}{a}, \quad (39)$$

$$Z_T = jZ_0 \frac{1}{2\pi^2 a^4} 2\pi\mathcal{L}a^2(\frac{1}{\epsilon'} - 1) \log \frac{a}{a_2}$$

$$\simeq jZ_0 \frac{\mathcal{L}}{\pi a^2} \log \frac{a_2}{a}. \quad (40)$$

Eqs.(39) and (40) are identical to Eqs.(7) and (20). The derivation of the impedance reveals that the dominant contribution to the impedances of the ceramic chamber comes from the electric polarity rather than the magnetic one.

REFERENCES

[1] M. Kinsho et al, J. Vac. Sci. Technol. A**20**, (3), 829, (2002); M. Kinsho et al, in Proceeding of PAC 2005, Knoxville, Tennessee, 2604, (2005)

[2] Y. Shobuda et al, PRST Accel. Beams **12**, 032401, (2009)

[3] B. W. Zotter and S. A. Kheifets, *Impedances and wakes in high-energy particle accelerators*, (World Scientific, Singapore, 1998).

[4] S. S. Kurennoy, PA **39**, 1, (1992); PA **50**, 167, (1995); R. L. Gluckstern, PR A **46**, 1106, 1110 (1992); S. S. Kurennoy, R. L. Gluckstern and G. T. Stupakov, PRE **52**, 4354, (1995);

[5] Y. Shobuda et al, in Proceedings of PAC 2005, Knoxville, Tennessee, 1898, (2005)