VERY SHORT RANGE WAKE IN STRONGLY TAPERED DISK LOADED WAVEGUIDE STRUCTURES

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Abstract

Electron bunches are very short in linear colliders but even more in linac based X-FELs. Furthermore, typical disk-loaded waveguide structures used for particle acceleration are tapered. For example in CLIC, in order to achieve high accelerating gradient, the structure is short, 26 cells, which results in strong tapering. In this paper, very short range longitudinal wake is investigated in the regime where the number of cells needed to arrive at steady state is much larger than the number of cells in a single tapered structure. In this case the very short range wake is dominated by the wake from the smallest aperture. The results of an analytical model and numeric solutions are compared.

INTRODUCTION

In [1], a qualitative picture of the short range longitudinal wake in an infinitely long periodic structure of disk loaded waveguide has been described in details. It has been shown that the perturbation of the "pan cake"-like electromagnetic fields of an ultra relativistic particle moves away from the iris tip towards the structure axis as $\delta \approx \sqrt{2zs}$, where s is the distance between the drive and witness particles and z is the distance between the iris perturbing the fields and the witness particle. At the moment when perturbation reaches the witness particle $\delta = a$, where a is the iris radius, the wake starts to act on the witness particle. The distance covered by the witness particles until that moment from the moment when drive particle passed the iris is called critical distance. It is expressed by so-called critical number of cells of length L as following: $N_{crit} \approx a^2/2Ls$. In [2], the now widely used Karl Bane's model (KB model) has been proposed for calculation of the longitudinal short range wakefields for infinitely long periodic structures:

$$W_L = \frac{Z_0 c}{\pi a^2}\exp\left(-\sqrt{s/s_0}\right); \quad \text{where} \quad s_0 = 0.41\frac{a^{1.8}g^{1.6}}{L^{2.4}} \quad (1)$$

where g is the gap between the irises and Z_0 and c are the impedance and the speed of light in vacuum, respectively. Furthermore, the wakefields of the tapered NLC structure of 206 cells has been calculated by averaging Eq. (1) over all 206 cells. This was justified in this case since $N_{crit} = 14 \ll 206 = N_c$, number of cells in the tapered structure.

However, for example in CLIC main linac accelerating structure [3], the number of cells is only 26 and the bunch is shorter $\sigma_z = 44$ μm, so that N_{crit} is about 14 which is comparable to 25. Furthermore, in many linac based X-FEL projects which are now under operation or under development the bunch length is even shorter, i.e. $\sigma_z = 10$ μm or even $\sigma_z = 2$ μm. In these cases, N_{crit} would be 60 or

300, respectively, which is larger or even much larger than the number of cells in a high gradient X-band accelerating structure like the one for CLIC main linac. In this case, the wakefields in the tapered structure cannot be obtained correctly by simple averaging. Taking the extreme case of $s \to 0$ clearly indicates that averaging (on the right hand side) does not give the correct result, where $a_{min} = \min(a_n)$

$$W_L^{TS}(0) = \frac{Z_0 c}{\pi a_{min}^2} \neq \sum_{n=1}^{N_c}\frac{Z_0 c}{\pi a_n^2}$$

More sophisticated models or simulations are required in order to calculate very short range wakefields in strongly tapered structures.
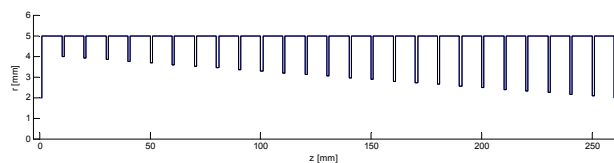
KB MODEL AND SIMULATIONS

Figure 1: Geometry of a sample structure.

In Fig. 1, a sample structure geometry is shown with $L = 10$ mm, $g = 9$ mm, and a is tapered from 4 to 2 mm over 26 cells. The short range longitudinal wakefields have been calculated in two ways using averaged KB model (<KB>) and numerically using ECHO2D code [4]. For numerical simulations the tapered structure geometry presented in Fig. 1 has been repeated N_s times in order to arrive to as critical length which gives 'steady-state' solution. In our case, the criterion was that the change of the total loss factor per meter length is below 2‰ when the total structure length changes by factor 2. Simulations have been performed for two different bunch lengths σ_z : 10 μm, and 5 μm. Critical numbers of structures were 32 and 200, respectively. Wake for $\sigma_z = 2$ μm was not possible to simulate due to practical limitations of computer resources. The results for $\sigma_z = 5$ μm are presented in Fig. 2 demonstrating clear difference between <KB> model and the ECHO2D simulations. A difference of 30% in the total loss factor is observed.

Qualitatively, the effect can be described as follows: As the witness particle comes closer and closer to the drive particle ($s \to 0$), the critical length increases and gets much longer than the tapered structure length. In this case the divergence angle of the perturbation of the 'pan cake'-like field of the drive particle becomes smaller and smaller leaving more and more irises of bigger radius unaffected. Expressed in the words of [5], the diffraction

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angle of the perturbed fields gets smaller as the frequency of the fields is getting higher at $s \rightarrow 0$ leading to the extreme case of diffraction only on the smallest irises spaced by the length of the tapered structure. All the other irises remain in the 'shadow' of the smallest iris.
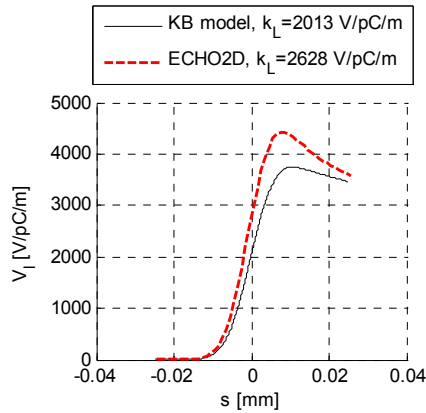

Figure 2: Short range wakefields for $\sigma_z = 5$ μm in the sample structure (Fig. 1) was calculated using <KB> model (solid black) and ECHO2D code (red dashed).

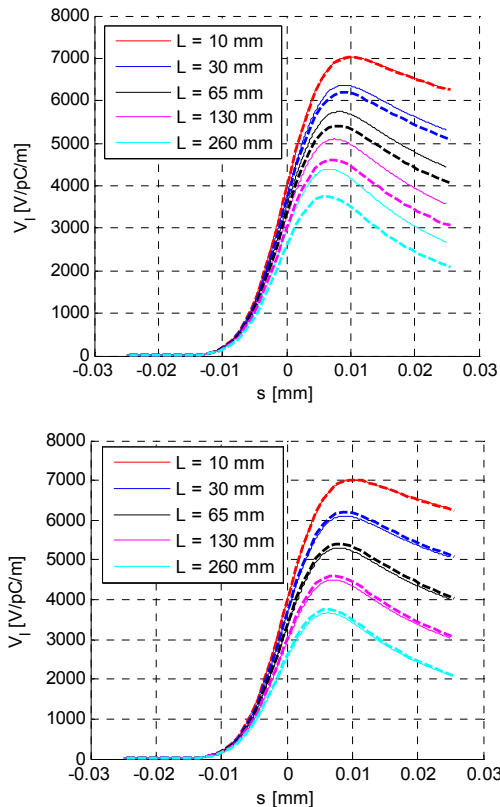## CORRECTED KB MODEL FOR PERIODIC STRUCTURES

Figure 3: Short range wakefields for $\sigma_z = 5$ μm in the periodic structure of radius $a = 2$mm was calculated using uncorrected KB (top) and corrected KB' (bottom) models (solid lines) and ECHO2D code (dashed lines).

The KB model is based on the semi-analytical approach fitting the model to a set of data obtained from numerical simulations [2]. It has its own applicability range: $0.34<a/L<0.69$; $0.54<g/L<0.9$ which is related to the NLC structure design parameters. In the case of very short range wake perturbed only by the smallest iris $a/(L'=LN_c)$ = 0.008 for our sample structure shown in Fig. 1. This is far off the range of applicability of KB model for periodic structures. Dedicated simulations have been performed to study the short range wake in periodic structures of $a = 2$ mm and $L - g = 1$ mm with very small a/L. The results for $\sigma_z = 5$ μm are presented in Fig. 3 (top) clearly demonstrating difference between KB model and the ECHO2D simulations for small a/L. Corrections to KB model have been found by fitting the ECHO2D simulation results at different iris radius a of 1 and 2 mm for two different bunch lengths of 10 and 5 μm. An expression with correction factors follows:

$$W_L' = \frac{Z_0 c}{\pi a^2} \exp\left(-[s/s_0']^{0.5 \cdot (a/L)^{0.03}}\right) \quad s_0' = 0.41 \frac{a^{1.8} g^{1.6}}{L^{2.4}} \cdot 1.5 \left(\frac{a}{L}\right)^{0.18} (2)$$

Comparison of the corrected KB model (KB') with the ECHO2D simulations is presented in Fig. 3 (bottom). In addition, in Fig. 4, wake functions calculated using KB and KB' models are shown for comparison.
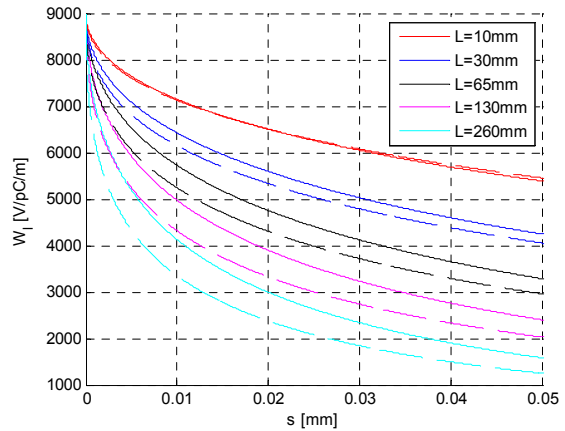

Figure 4: Short range longitudinal wake functions calculated using KB (solid) and KB' (dashed) model are shown for different a/L, a=2mm.

NEW MODEL FOR TAPERED STRUCTURES

Finally, a new model for very short range wake in strongly tapered structures is presented in this section. The idea is based on the quantitative picture elaborated in the beginning of the previous section. Assume that for each s there is an iris m, which divides tapered structure into two parts: the first part with larger irises remains in the 'shadow' of the other smaller irises, which constitute the second part of the structure. In this case, two functions are introduced with two terms corresponding to the

wakefields from the two parts of the tapered structure just described:

$$W_L^{a_{min}}(m) = \frac{1}{N_c}\left[m \cdot W_L'(a_{min}, g + m(N_c - 1), mL) + \sum_{n=m}^{N_c-1} W_L'(a_n, g, L) \right]$$

$$W_L^{a_n}(m) = \frac{1}{N_c}\left[m \cdot W_L'(a_m, g + m(N_c - 1), mL) + \sum_{n=m+1}^{N_c} W_L'(a_n, g, L) \right]$$

Using these two functions three models for the short range wake in a tapered structure have been investigated and compared to the ECHO2D simulation results:

Model 1: $W_L^{TS}{}' = \max\left\{ W_L^{a_{min}}(m = N_c), W_L^{a_n}(m = 0) \right\}$ (3.1)

Model 2: $W_L^{TS}{}' = \max_{m=1...N_c}\left\{ W_L^{a_{min}}(m) \right\}$ (3.2)

Model 3: $W_L^{TS}{}' = \max_{m=1...N_c}\left\{ \frac{2}{3}W_L^{a_{min}}(m) + \frac{1}{3}W_L^{a_n}(m) \right\}$ (3.3)
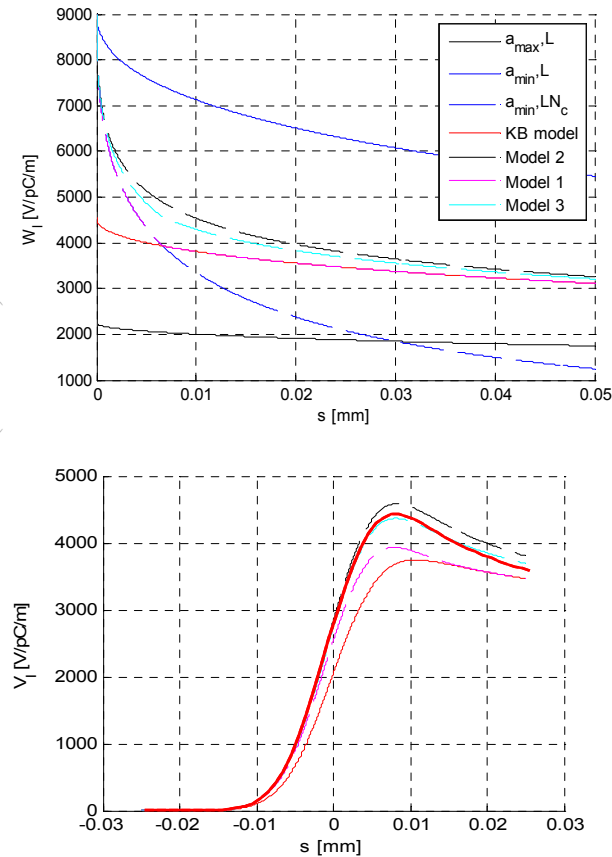

Figure 5: Top: short range wake functions calculated using KB' model for first cell (solid black), last cell (solid blue), last iris spaced by LN_c (dashed blue). Solid red: <KB'> model. M1 (magenta), M2 (black), M3 (cyan). Bottom: short range wakefields calculated for $\sigma_z = 5$ μm using averaged KB' (thin solid red), M1 (magenta), M2 (black), M3 (cyan) models. Thick solid red: ECHO2D

In Fig. 5 (top), short range wake functions are presented calculated using Models 1, 2 and 3, M1 Eq. (3.1), M2 Eq. (3.2), M3 Eq. (3.3), respectively. While M1 is clearly too simplified especially to adequately describe the wake function near the transition region (when the 'shadow' expands over the irises of the tapered structure, $0.005 < s < 0.02$ mm, in Fig. 5(top)), M2 and in particular, M3 provide a smooth transition and indeed better agreement with the ECHO2D simulations results presented in Fig. 5(bottom). Table 1 summarizes the total loss factor calculated using different models and compares to ECHO2D results if available. First it shows very good agreement between M2 and ECHO2D results. Second, the last row shows big difference between the averaged KB' model: <KB'> and M2.

Table 1: Total Loss Factor Calculated using Different Models and ECHO2D in our Example Structure

σ_z [μm]		2	5	10	44
ECHO2D [V/pC/m]		-	2630	2320	-
<KB'> [V/pC/m]		2100	2010	1920	1620
M2 [V/pC/m]		3070	2690	2400	1800
M3 [V/pC/m]		2990	2580	2300	1760
M2/<KB'>-1 [%]		46	33	25	11

CONCLUSIONS

Effect of tapering in the disk loaded waveguide structure on the very short range longitudinal wakefields is investigated. Depending on the bunch length and the structure parameters significant discrepancy has been between the widely used KB model [2] and numerical simulations using ECHO2D code [4]. A correction to the KB model for periodic structures with small a/L has been found and a new model for a very short range wakefields in a tapered structure has been developed. Both the correction and the new model show very good agreement with simulation results.

ACKNOWLEDGMENT

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