COMPUTED WAKE FIELD EFFECTS FROM MEASURED SURFACE ROUGHNESS FOR THE WALLS OF THE CORNELL ERL*

M.G. Billing#, G.H. Hoffstaetter, C.E. Mayes, K. Smolenski, H.A. Williams,
CLASSE, Cornell University, Ithaca, NY, USA

Abstract

Wake fields arise from the discontinuities in a smooth vacuum chamber and will cause energy spread in the passing bunch. In an Energy-Recovery Linac (ERL), the spent bunches are decelerated to reuse the beam's energy for the acceleration of new bunches. While the energy spread accumulated from wakes before deceleration is small compared to the beam's energy at full acceleration, it becomes more important relatively as the beam's energy decreases. Thus in an ERL wake fields can produce very significant energy spread in the beam as it is decelerated to the energy of the beam dump. We report on calculations of wake fields due to the roughness of the surface of the vacuum chamber walls as it affects the Cornell ERL design. These calculations include the effects from the measured roughness and it correlation lenght for a vacuum chamber wall surface.

LIMITS FOR WAKE FIELDS IN AN ERL

When a charged particle beam passes through the accelerator's vacuum chamber, its electro-magnetic (E-M) fields interact with discontinuities in the chamber's cross-section. A way to characterize this interaction is through the induced wake voltage felt by a reference particle traveling with the bunch displaced in time τ from a point moving along with the bunch. If this reference particle's position stays fixed with the respect to bunch of charge q_b, then the induced voltage for one traversal of the ERL is

$$V_{\|}(\tau)\Big|_{\text{entire ERL}} = q_b\, W_{\|}(\tau)\Big|_{\text{entire ERL}}$$

where $W_{\|}(\tau)$ is the longitudinal wake. The wake field increases the energy spread of particles within the bunch. At the highest energy beam in the ERL this is not serious; but, as the beam decelerates to the energy of the beam dump, its relative energy spread increases inverse to its energy. If the beam dump has a maximum energy acceptance $\max\{\Delta E|_{dump}\}$, then to allow for other sources of energy errors, the peak wake fields will be limited to ½ of the acceptance. This gives a limit for the maximum total wake field for the ERL operating with q_b=77 pC of

$$\max\left\{W_{\|}(\tau)\Big|_{\text{entire ERL}}\right\} = \frac{1}{q_b}\max\left\{eV_{\|}(\tau)\Big|_{\text{entire ERL}}\right\}$$
$$\leq \frac{1}{2}\frac{1}{q_b}\max\left\{\Delta E\big|_{\text{dump}}\right\} \approx \frac{1}{2}\frac{5\text{ MeV}}{77\text{pC}} \approx 32\text{ kV / pC}$$

where the 10 MeV beam dump has an energy acceptance of 5 MeV. This places an impedance budget limit on the total ERL wake field of 32 kV/pC. If the total wake fields from the actual vacuum system chamber components exceeds this impedance budget, then either some of the particles in the bunches will be lost before reaching the beam dump or the charge per bunch must be reduced.

One of the results found in an early study of the wake fields arising for typical vacuum chamber components for the Cornell ERL was that, since the bunch RMS length (σ_z=0.6 mm) is relatively short, a significant wake field contribution can arise from E-M fields scattering off of rough vacuum chamber surfaces [1]. An inconsistency that was found later in these calculations for the surface roughness wake fields and a desire to test some of the assumptions for the surface properties of typical vacuum chambers has caused us to reinvestigate this particular interaction.

WAKE FIELD DESCRIPTION

It is generally useful to describe an approximate form for the wake field of a single vacuum chamber component in terms of current distribution of the bunch and parameters, R, L, and C, which vary slowly as the scale of bunch length changes. One such description is [2]

$$W_{\|}(\tau) = -R\,\lambda(\tau) - L\frac{d\lambda}{d\tau} - \frac{1}{C}\int_{-\infty}^{\tau} dt'\,\lambda(t')$$

where $\lambda(\tau)$ is the unit normalized longitudinal distribution for the charge in the bunch. For the surface roughness wake field, the inductance L will dominate for short bunches. However, when the characteristic length, over which the bunch shape varies, becomes even much smaller, the resistive term R will begin to be important.

MODELS FOR SURFACE ROUGHNESS

Two different models for wake fields arising from the vacuum chamber wall roughness have been considered. In this paper we will refer to them as the "Dielectric Model" and "Scattering Model". The dielectric model [3] describes the rough surface as a thin region of dielectric due to the enhanced electric field strength arising from the field of sharp peaks within this surface layer. In a vacuum chamber, where b is the chamber radius of the round beam pipe, ε is the effective dielectric constant for the surface layer, and δ is thickness of the layer, the bunch's E-M fields will couple to a TM mode in the chamber with coupling impedance per unit length of R' and a propagation wave number of k_0 given by

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# mgb9@cornell.edu

$$R' = \frac{Z_0\,c}{\pi\,b^2};\qquad k_0^2 = \frac{2\,\varepsilon}{(\varepsilon-1)\,b\,\delta}$$

For Gaussian bunches (RMS bunch length of σ_z) the wake function per unit length of chamber may then be written in the form,

$$\frac{d\,W_\parallel}{dz}(\tau) = \frac{c}{\sqrt{2\pi}\,\sigma_z}\int_0^\infty dt\left\{-R'\cos[k_0 ct]\exp\left(-\frac{c^2\,(t-\tau)^2}{2\,\sigma_z^2}\right)\right\}$$

where τ describes time relative to the center of the bunch. Although this form appears to yield a resistive wake, because k_0 is much smaller than $1/\sigma_z$, the cosine-factor will vary rapidly and produce a wake field proportional to the derivative of the Gaussian bunch current.

The scattering model [4] describes the roughness layer as a bumpy surface, which deviates from the ideal cylindrical beam pipe by $\Delta b(x,z)$, where z is along the beam direction and x is transverse. The bunch's E-M fields generate vacuum chamber wall currents that travel along with the bunch. As these current encounter the bumps on the surface they deviate around the bumps in small current loops, which ultimately generate an inductive wake field. Following the derivation for this model, which assumes that the bumps on the surface are distributed randomly, the function Δb is used to generate the correlation function by averaging over a typical section of the surface,

$$K(\Delta x,\Delta z) = \left\langle \Delta b(x+\Delta x, z+\Delta z)\,\Delta b(x,z)\right\rangle_{\text{surface}(x,z)}$$

The spatial Fourier transform of the correlation function,

$$R(\kappa_x,\kappa_z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} d\Delta z\; d\Delta x\; K(\Delta x,\Delta z)\,\exp\left[-j(\kappa_x\Delta x + \kappa_z\Delta z)\right]$$

is used to calculate the wake field per unit length by summing over contributions from all TM and TE modes in the beam pipe

$$\frac{d\,W_\parallel}{dz}(\tau) = \frac{c}{\sqrt{2\pi}\,\sigma_z}\,L'_S\,\frac{c^2\,\tau}{\sigma_z^2}\exp\left(-\frac{c^2\,\tau^2}{2\,\sigma_z^2}\right)$$

$$\text{where } L'_S = \frac{Z_0}{4\,\pi^2\,c\,b}\int_{-\infty}^{\infty} d\kappa_z\int_{-\infty}^{\infty} d\kappa_x\,R(\kappa_z,\kappa_x)\,\frac{\kappa_z^2}{\kappa}$$

The author suggests treating the surface as a fractal distribution where δ_{RMS} is the RMS deviation of Δb, κ_0 is the correlation length of the features of the surface roughness and q is the inverse power law for the wave number for the fractal landscape model of the surface. In this case L_S' may be simplified to yield

$$L'_S = \frac{Z_0\,\delta_{RMS}^2\,\kappa_0}{8\,\pi^2\,c\,b}\left(\frac{q-2}{q-3}\right)$$

where the author also suggests using $\kappa_0=1/\delta_{RMS}$ for typical rough surfaces.

For typical parameters the dielectric model and scattering model may be directly compared. Examining the wake field of a possible undulator beam pipe having b=3 mm, σ_z=0.66 mm, $\delta=\delta_{RMS}$=0.5 µm (assumed to be electro-polished), we use parameter values in the ranges suggested by the respective authors, ε=1.515, κ_0=1/ δ_{RMS}, and the fractal exponent of q=4. These results for these two models are shown in Figure 1. Note that both wake fields have the same general time dependence and, although peak amplitudes for the wake fields are 0.57 V/pC/m and 0.83 V/pC/m for the dielectric and scattering models, respectively, a slight adjustment of the model parameters would bring these two into agreement.

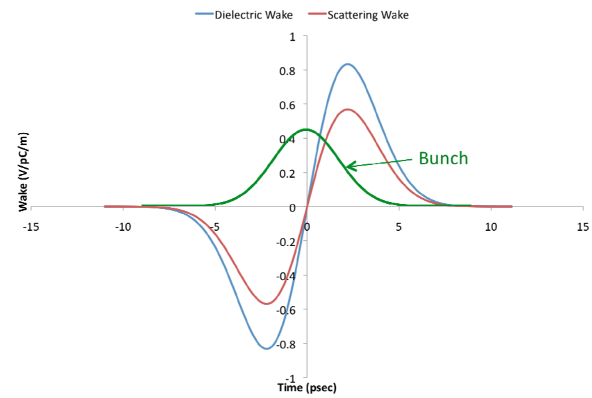

Figure 1: Comparison between models for the dielectric and the scattering wake for surface roughness.

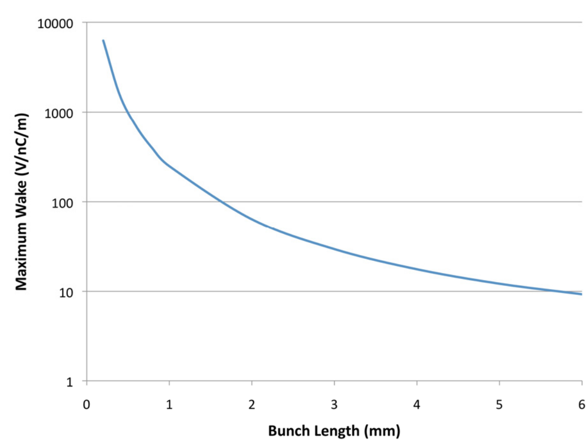

Figure 2: Peak value of the wake field from the dielectric model as a function of bunch length.

Over a modest range of bunch lengths inductive wake fields tend to produce a rapid variation of peak voltage. Figure 2 examines the results of a calculation for the dielectric model as a function of the bunch length of a Gaussian ERL bunch, using the same parameters for the possible undulator vacuum chamber. This implies the peak wake voltage varies as nearly an inverse square of the bunch length, implying that the surface roughness

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effects become vastly more important as the bunch length decreases for ultra-short bunch length ERL operations.

SURFACE ROUGHNESS MEASUREMENT

In order to check the assumptions, suggested by the authors of the dielectric model and the scattering model, optical measurements of vacuum chamber wall surface roughness were undertaken at Cornell. Two samples each of a copper extrusion and the aluminum CESR vacuum chamber walls were measured. We report here on the results of the preliminary analysis of one of the copper samples. The copper sample was scanned over surface with dimensions of 630 μm by 850 μm. The sample was prepared by sawing it from a larger section of a representative portion of pipe, then it was cleaned and solvent rinsed and maintained in a dust-free containment. Otherwise there was no special handling of the sample. Observations by eye show very slight approximately period undulations in the long direction of the extrusion and also the presence of small scratches running in various directions, likely originating from the machining and subsequent handling before cleaning.

The optical scans were performed using a MicroXAM Optical Scanner [5], which determined the elevation within the sampling area in 1.1 μm by 1.35 μm steps. The instrument provides elevation results, which are registered with respect to the planer surface, which best fits the entire scanned region. We have analyzed the data off-line within the central section of 560 μm by 690 μm. Within this region the surface has peak-to-valley deviations for 5-10 μm, the RMS deviation of the surface from a plane is approximately 0.4 μm. The 2-dimensional correlation function was computed, yielding an approximately exponential dependence with a correlation length of 45 μm in direction of the beam's motion and an initial exponential decay (correlation length=10 μm) followed by smaller undulations (from the extrusion process) in the direction transverse to the beam's motion. Relating this back to the assumptions made for the ERL wake field study [1], the 0.4 μm RMS surface roughness for an untreated surface is comparable to an assumed electro-polished surface (0.5 μm), taken for the 3 mm radius undulator beam pipes, and much better than the 3 μm roughness for normal untreated vacuum chambers of radius 12.7 mm. In addition the inverse to the correlation length is smaller than the $\kappa_0=1/\delta_{RMS}$ relation used for the ERL wake study. The combination of these effects is to significantly reduce the peak wake field anticipated to arise from surface roughness.

ESTIMATE OF ERL ROUGHNESS WAKE

The results of the preceding section imply that the wake field estimates performed for the earlier ERL study were overly conservative. One manner for proceeding with an updated estimate for the roughness wake fields is to use the data from the surface measurements, $\delta=\delta_{RMS}=0.5$ μm and $\kappa_0=1/45$ μm, in conjunction with the undulator beam pipe, b=3 mm, $\sigma_z=0.66$ mm, $\varepsilon=1.515$, and the same fractal exponent of q=4. Taking these parameters directly yields peak wake amplitude of 0.45 V/pC/m and 0.0059 V/pC/m for the dielectric and scattering models, respectively. Although these peak wake amplitudes are lower (and in the scattering case very much lower), they are not close to agreement. One possibility is that we continued to use $\varepsilon=1.515$ for the dielectric model, while it is likely that the longer coherence length implies a smoother surface, which would result in (ε-1) being much smaller. We hope to make a more realistic determination for the wake field after we integrate the spatial transform of the correlation function from the four beam pipe samples.

Nonetheless continuing to use the parameters for each model as specified in the preceding paragraph and this allows us to update the estimates for the roughness wake field's peak amplitudes for the Cornell ERL. These have been computed for two vacuum chamber radii for the normal beam pipe and the undulator beam pipe. Unlike in the earlier study, we take both types of chamber to have $\delta=\delta_{RMS}=0.5$ μm. The results are found in Table 1 and are much lower than earlier estimates of 14.0 KV/pC (normal chamber) and 3.60 KV/pC (undulator chamber).

Table 1: Roughness Wake Estimates for 2 Different Models

Component	Total Length	Total - Wake (KV/pC)	Total + Wake (KV/pC)
Scattering Model			
Normal Beam Pipe (b = 12.7 mm)	2500 m	-0.0035	0.0035
Undulator Beam Pipe (b = 3 mm)	144 m	-0.00085	0.00085
Dielectric Model			
Normal Beam Pipe (b = 12.7 mm)	2500 m	-0.27	0.27
Undulator Beam Pipe (b = 3 mm)	144 m	-0.066	0.066

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REFERENCES

[1] M. Billing, et al, "Effect of Wake Fields in an Energy Recovery Linac", PAC'09, Vancouver, BC, Canada, 4-8 May 2009.

[2] M. Billing, et al, "Simulation Measurement of Bunch Excited Fields and Energy Loss in Vacuum Chamber Components and Cavities", PAC1979, Chicago, IL, USA, June 1979.

[3] A. Novohatski, *et al*, "The Surface Roughness Wakefield Effect", TU-Darmstadt, TEMF.

[4] G. V. Stupakov, "Impedance of Small Obstacles and Rough Surfaces", PRST-AB, Vol. 1, 064401 (1998).

[5] MicroXAM Optical Scanner, ADE Phase Shift, 3470 Universal Way, Tucson, AZ 85706.