ENERGY MODULATION IN COHERENT ELECTRON COOLING*

G. Wang#, V. N. Litvinenko and M. Blaskiewicz, BNL, Upton, NY 11973, U.S.A.

Abstract

Coherent electron cooling (CeC) relies on Debye shielding to imprint information of the ion beam to an electron beam [1]. Apart from the density modulation, Debye shielding also modulates the energy of electrons, which provides additional seeding for the free electron laser (FEL) based CeC amplifier and serves as the major seeding for compressor based coherent electron cooling schemes such as micro-bunched electron cooling (MBEC) and enhanced electron cooling (EeC) [2].

In this work, we calculate the energy modulation of a longitudinal slice of the electrons, induced by an ion moving in electron plasma with κ -2 velocity distribution. The result is then applied to Genesis simulation for parameters of the CeC proof of principal experiment and the effects of energy modulation are investigated.

INTRODUCTION

Coherent electron cooling uses Debye shielding to pick up information about the ion beam. In the process of Debye shielding, an ion modifies the velocities of the surrounding electrons, which then result in their position changes. Unless the length of the cooling section is close to one-half plasma wavelength multiplied by an integer, the energy modulation always presents. For a FEL-based CeC system, the energy modulation contributes to the initial seeding of the FEL amplifier, which could affect the cooler performance. In addition, the electron density modulation usually requires a quarter to one half-plasma oscillation to develop and for hadron machine with ultra-high energy such as the LHC, the length of the modulator becomes prohibitively long. The velocity modulation, on the other hand, develops much faster than the density modulation, which makes cooling schemes based on the velocity modulation in combination with a compressor much more feasible for cooling ultra-high energy hadron beam.

For cold electron beam in a short modulator, the energy modulation of a longitudinal slice of electrons has been previously derived [3]. In this work, we study the energy modulation in a warm electron beam. The energy modulation of a longitudinal slice of electrons is directly related to the instantaneous current modulation in the beam frame. We show that, for ion moving in infinite anisotropic electron plasma, the instantaneous current modulation can be reduced to a 1D integral. Neglecting the effect of finite beam size to the electron velocity modulation and assuming that ion moves in the longitudinal direction, we obtained the instantaneous current modulation for a finite isotropic electron beam. Our result is then applied to calculate the energy-bunching factor for parameters of the CeC proof of principal experiment. The effect of the energy modulation is investigated by simulating the FEL amplifier with GENESIS.

ENERGY MODULATION IN INFINITE ELECTRON PLASMA

In beam frame, the phase space density modulation induced by a moving ion in infinite electron plasma reads

$$f_1(\vec{x},\vec{v},t)=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} f_1\left(\vec{k},\vec{v},t\right)e^{i\vec{k}\cdot\vec{x}}d^3k \ , \qquad (1)$$

where $f_1\left(\vec{k},\vec{v},t\right)$ is solved from integral equation

$$f_1\left(\vec{k},\vec{v},t\right)=-\frac{e^2}{m_e\varepsilon_0}\left(\frac{\vec{k}}{ik^2}\cdot\frac{\partial}{\partial\vec{v}}f_0\left(\vec{v}\right)\right) \times\int_0^t\left[\tilde{n}_1\left(\vec{k},t_1\right)-Z_i\right]e^{i\vec{k}\cdot\vec{v}(t_1-t)}dt_1 \ , \qquad (2)$$

with $\tilde{n}_1\left(\vec{k},t_1\right)\equiv\int_{-\infty}^{\infty}f_1\left(\vec{k},\vec{v},t\right)d^3v$. The Fourier components of the current density modulation is then calculated from

$$\vec{j}_1\left(\vec{k},t_1\right)=-e\int_{-\infty}^{\infty}\vec{v}f_1\left(\vec{k},\vec{v},t\right)d^3v \ . \qquad (3)$$

Inserting eq. (2) into eq. (3) and taking the background velocity distribution as

$$f_0\left(\vec{v}\right)=\frac{n_0}{\pi^2\beta_x\beta_y\beta_z} \times\left[1+\frac{\left(v_x+v_{0,x}\right)^2}{\beta_x^2}+\frac{\left(v_y+v_{0,y}\right)^2}{\beta_y^2}+\frac{\left(v_z+v_{0,z}\right)^2}{\beta_z^2}\right]^{-2} \qquad (4)$$

yield

$$\vec{j}_1\left(\vec{k},t\right)\equiv e\vec{v}_0\tilde{n}_1\left(\vec{k},t\right)+\vec{j}_d\left(\vec{k},t\right) \ , \qquad (5)$$

where $\beta_{x,y,z}$ are parameters describing the velocity spreads of the electron bean and $\vec{v}_0$ is the velocity of the ion. While the first term of eq. (5) is due to the fact that the solution is derived in the rest frame, the second term of eq. (5) corresponds to the energy modulation of electrons and is determined by the following equation:

$$\dot{\vec{j}}_d\left(\vec{k},t\right)=-iZ_ie\omega_p^2\cos\left(\omega_pt\right)\frac{\vec{k}}{k^2}e^{\lambda t}+iZ_ie\omega_p\sin\left(\omega_pt\right)e^{\lambda t} \times\frac{\partial}{\partial\vec{k}}\sqrt{\left(k_x\beta_x\right)^2+\left(k_y\beta_y\right)^2+\left(k_z\beta_z\right)^2} \ , \qquad (6)$$

* Work supported by Brookhaven Science Associates, LLC under Contract No.DE-AC02-98CH10886 with the U.S. Department of Energy.

#gawang@bnl.gov

where the dot representing the time derivative, $\omega_p \equiv \sqrt{n_0 e^2/(m_e\varepsilon_0)}$ is the plasma frequency and

$$\lambda(\vec{k}) \equiv i\vec{k}\cdot\vec{v}_0 - \sqrt{(k_x\beta_x)^2+(k_y\beta_y)^2+(k_z\beta_z)^2}\ . \quad (7)$$

The time derivative of the instantaneous current density is

$$\begin{aligned}\dot{I}_d(z,t) &= \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} d^3k \int_{-\infty}^{\infty} dx\,dy e^{i\vec{k}\cdot\vec{x}}\dot{\tilde{j}}_{d,z}(\vec{k},t) \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} dk_z \cdot e^{ik_z z}\dot{\tilde{j}}_{d,z}(0,0,k_z,t)\end{aligned}\ . \quad (8)$$

Inserting eq. (6) into eq. (8) and integrating both sides over time produces the instantaneous current modulation

$$I_d(z,t) = \frac{Z_i e\omega_p}{\pi}\left[\sin(\omega_p t)\arctan\left(\frac{v_{0,z}}{\bar{\beta}}+\frac{z}{\bar{\beta}t}\right) - \int_0^t d\tau \frac{v_{0,z}\bar{\beta}\tau\sin(\omega_p\tau)}{\bar{\beta}^2\tau^2+(z+v_{0,z}\tau)^2}\right] . \quad (9)$$

It is worth noting that eq. (9) is a general result applicable to ion moving with arbitrary velocity in infinite anisotropic electron plasma with velocity distribution of eq. (4). For ultra-relativistic electron beam, the total energy modulation of a slice of electrons in the lab frame is determined by the total longitudinal velocity modulation in the beam frame:

$$\sum_i \frac{\delta E_i}{E_0} = \sum_i \frac{v_{i,z}}{c} = -I_{d,z}(z,t)\cdot\frac{\Delta z_{slice}}{ec}\ , \quad (10)$$

where the summation is over all electrons in the slice, i.e. $z-\Delta z_{slice}/2 < z_i \le z+\Delta z_{slice}/2$, and Δz_{slice} is the longitudinal width of the slice.

ENERGYY MODULATION IN FINITE ELECTRON BEAM

For a transversely finite electron beam, if we assume the effects from the finite beam size is small such that eq. (6) remains valid in the regions of the beam, the beam frame current modulation can be obtained by integrating eq. (6) over the beam cross-section and consequently, instead of eq. (8), one has

$$\dot{I}_d(z,t) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} d^3k \int_S dx\,dy e^{i\vec{k}\cdot\vec{x}}\dot{\tilde{j}}_{d,z}(\vec{k},t)\ , \quad (11)$$

where S is the cross-section of the beam. Calculating eq. (11) for arbitrary ion velocities and anisotropic electron velocity distribution leads to multi-dimension integrals, which requires numerical approach. To proceed analytically, hereafter, we assume $\beta_x=\beta_y=\beta_z=\bar{\beta}$, and $v_{0,x}=v_{0,y}=0$. With above assumptions, the beam frame current density can be directly calculated from eq. (6) as

$$\vec{j}_d(\vec{x},t) = \frac{-Z_i e\omega_p^2 a_z}{2\pi^2}\int_0^t d\tau(\vec{x}+\vec{v}_0\tau)\left\{\frac{2\sin(\omega_p\tau)}{\left(\bar{\beta}^2\tau^2+|\vec{x}+\vec{v}_0\tau|^2\right)^2} + \frac{\cos(\omega_p\tau)}{|\vec{x}+\vec{v}_0\tau|^2}\left[\frac{\omega_p\tau}{\bar{\beta}^2\tau^2+|\vec{x}+\vec{v}_0\tau|^2} - \frac{\arctan\left(\frac{|\vec{x}+\vec{v}_0\tau|}{\bar{\beta}\tau}\right)}{a_z|\vec{x}+\vec{v}_0\tau|}\right]\right\}, \quad (12)$$

where $a_z=\bar{\beta}/\omega_p$ is the Debye radius in the beam frame. For $\bar{\beta}=0$ and $v_{0,z}=0$, eq. (12) reproduces the well known result,

$$\vec{j}_d(\vec{x},t) = \frac{Z_i e\omega_p}{4\pi}\sin(\omega_p t)\frac{\vec{x}}{|\vec{x}|^3}\ . \quad (13)$$

Integrating eq. (12) over a round beam cross section, $\sqrt{x^2+y^2}\le a$ yields

$$I_d(z,t) = -\frac{Z_i e\omega_p^2}{\pi}\int_0^t d\tau(z+v_{0,z}\tau)\left\{\frac{a_z\sin(\omega_p\tau)}{\left[\bar{\beta}^2\tau^2+(z+v_{0,z}\tau)^2\right]\left[1+\bar{\beta}^2\tau^2+(z+v_{0,z}\tau)^2/a^2\right]} - \cos(\omega_p\tau)\left[\frac{\arctan\left(|z+v_{0,z}\tau|/(\bar{\beta}\tau)\right)}{|z+v_{0,z}\tau|} - \frac{\arctan\left(\sqrt{(z+v_{0,z}\tau)^2+a^2}/(\bar{\beta}\tau)\right)}{\sqrt{(z+v_{0,z}\tau)^2+a^2}}\right]\right\}\ . \quad (14)$$

The average energy modulation of an electron at longitudinal location z_l reads

$$\left\langle\frac{\delta E(z_l)}{E_0}\right\rangle = \frac{\langle v_z\rangle}{c} = -\frac{1}{en_0\pi a^2 c}I_d\left(\gamma_0 z_l, \frac{L_{mod}}{\beta_0\gamma_0 c}\right), \quad (15)$$

where z_l is the longitudinal coordinate in the lab frame, L_{mod} is the modulator length and γ_0 is the Lorentz factor of the electron beam. For $\bar{\beta}=0$, $v_{0,z}=0$ and $L_{mod} << \beta_0\gamma_0 c/\omega_p$, eq. (15) reproduces the previous result, derived from a very different approach [3] :

$$\left\langle\frac{\Delta E}{E}\right\rangle \approx -2Z_i\frac{r_e}{a^2}\frac{L_{mod}}{\gamma}\cdot\left[\frac{z_l}{|z_l|} - \frac{z_l}{\sqrt{z_l^2+a^2/\gamma^2}}\right]\ . \quad (16)$$

Figure 1 plots the energy modulation induced by a rest ion for parameters in Table 1 and various electron

ISBN 978-3-95450-122-9

beam rms energy spreads. As a comparison, the result calculated from eq. (16) for a short modulator is also

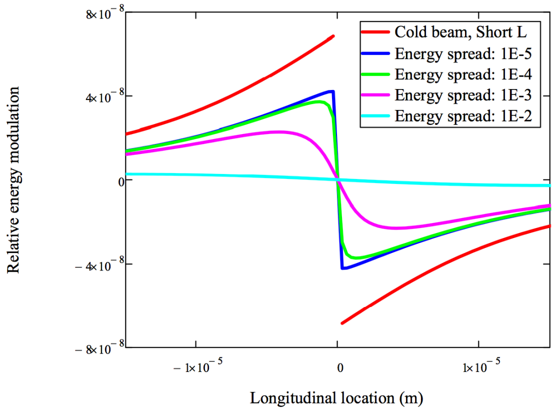

Figure 1: Energy modulation induced by a rest ion in isotropic electron plasma with various rms energy spread. The red curve is calculated from eq. (16) and other curves are calculated from eq. (15).

Table 1: Parameters Applied in the Simulation for the CeC Proof of Principal Experiment

Beam energy, γ	42.9
Modulator length	3 m
Emittances, rms, ε_n	$5\ \pi \cdot mm \cdot mrad$
Undulator length	7.5 m
Undulator period	4 cm
Undulator parameter, a_w	0.4
Peak current, I_{peak}	100 A
Energy spread, rms, $\delta\gamma/\gamma$	$1 \cdot 10^{-3}$
Beta function at modulator	4 m
Beta function at undulator	1 m

plotted, showing the maximal energy modulation for electron beam with rms energy spread of 10^{-3} is a factor of 3 smaller than that calculated from eq. (16)

EFFECTS OF ENERGY MODULATION TO AMPLIFIED WAVEPACKET

We use Genesis simulation to study the effects of the energy modulation to the electron density wave-packet at the exit of the FEL amplifier. The Debye length for the CeC proof of principal experiment is 1.1 μm in the lab frame, i.e. much smaller than the 13 μm optical wavelength, and hence we put initial seeding into the slice of electrons with maximal bunching in the simulation, neglecting contributions from other slices. Applying previously derived analytical formula for density modulation [4], the bunching factor at the position of the ion can be calculated as

$$b=\frac{1}{N_\lambda}\frac{Z_i}{\pi}\int_{-\pi/\alpha}^{\pi/\alpha} d\bar{z}\exp\left(i\alpha\bar{z}\right)\int_0^{\omega_p t}\frac{\tau\sin\left(\tau\right)}{\left(\bar{z}+v_{0,z}\tau/\bar{\beta}\right)^2+\tau^2}d\tau ,$$

where $\alpha\equiv 2\pi a_z/\left(\lambda_{opt}\gamma_0\right)$ and λ_{opt} is the optical wave length of the FEL amplifier. For parameters of Table 1, the bunching factor is 1.73×10^{-6} for a rest ion. As the first step, only the contribution from the slice with peak

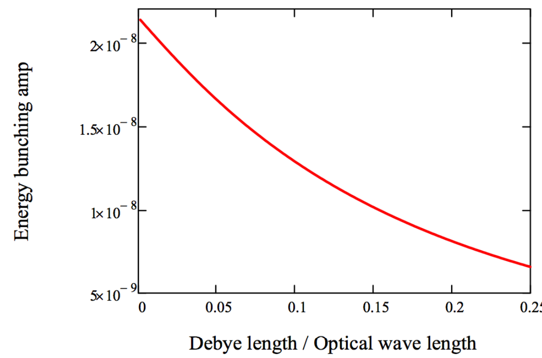

Figure 2: The amplitude of energy modulation factor at the slice of the ion as calculated from eq. (17).

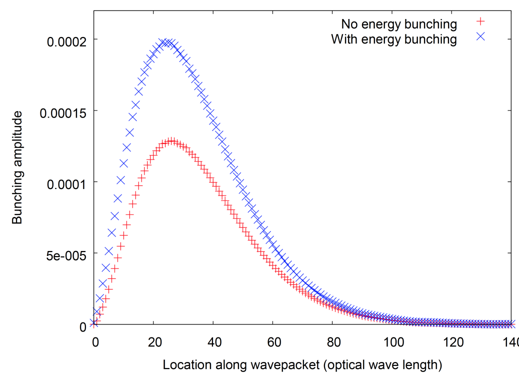

Figure 3: Simulation result of the wave-packet profile at the exit of the FEL amplifier. The red curve is simulated without energy modulation and the blue curve is simulated with the energy modulation factor.

energy modulation is considered in the simulation. The energy modulation factor at the slice of the ion is

$$b_\gamma=\frac{1}{\lambda_{opt}}\int_{-\lambda_{opt}/2}^{\lambda_{opt}/2} e^{i2\pi z_l/\lambda_{opt}}\left\langle\frac{\delta E\left(z_l\right)}{E_0}\right\rangle dz_l , \qquad (17)$$

with the energy modulation given in eq. (15). Figure 2 plots the energy modulation factor as a function of $a_{z,lab}/\lambda_{opt}$, where $a_{z,lab}=a_z/\gamma_0$ is the lab frame Debye length. For parameters of Table 1, the energy modulation factor is $-i1.38\times10^{-8}$. As shown in Fig. 3, the maximal bunching factor at the exit of the FEL amplifier increased by 54% with the calculated energy modulation factor.

REFERENCES

[1] V.N. Litvinenko, and Y.S. Derbenev, Physical Review Letters **102** (2009).
[2] D. Ratner, *SLAC note*, SLAC-PUB-15346, (2013).
[3] V.N. Litvinenko, and Y. S. Derbenev, in *29th International Free Electron Laser Conference* (Novosibirsk, Russia, 2007), p. 268.
[4] G. Wang, and M. Blaskiewicz, Phys. Rev. E **78** (2008).