APPROXIMATE METHOD OF CALCULATION OF A BUNCH RADIATION IN PRESENCE OF COMPLEX DIELECTRIC OBJECT*

Andrey V. Tyukhtin#, Ekaterina S. Belonogaya, Sergey N. Galyamin
Physical Faculty of St. Petersburg State University, St. Petersburg 198504, Russia

Abstract

Cherenkov radiation is widely used for detection of charged particles and can be also applied for particle bunch diagnostics. As a rule, dielectric objects applied for these goals have complex forms. Therefore development of methods of calculation of bunch radiation in presence of complex dielectric objects is now of a great interest. The approximate method developed by us allows to take into account influence of the object boundaries closed to the charge trajectory as well as "external" boundaries of the object. The case of the charge crossing a dielectric plate was considered as a test problem. The exact solution of this problem is in a good agreement with our approximate solution. Next, the cases of more complex objects were analyzed. One of them is a dielectric cone with a vacuum channel. Particularly, it was shown that radiation can be convergent under certain conditions, that is the field outside the cone can be more intensive than on the cone boundary. Radiation of the bunch in the case of dielectric prism was considered as well.

BASIS OF THE METHOD

Problems of radiation of charged particles in the presence of dielectric objects are of interest for some important applications in accelerator and beam physics [1]. It can be mentioned, for example, that a new method of bunch diagnostics was offered recently [2]. For realization of this method, it is necessary to calculate the field of Cherenkov radiation outside a dielectric object (such radiation can be named “Cherenkov-transition radiation” (CTR)).

In a number of simple specific cases, an exact solution for the field has been obtained [3]. However, in a majority of cases of practical interest, the complex geometry of the problem does not allow obtaining rigorous expressions for the radiation field. Therefore development of approximate methods for analyses of radiation is very actual.

It should be noted that some of problems with dielectric objects were considered in series of papers, where certain approximate methods were elaborated (see, for example, [4-7]). The basic idea of these methods is to find so called polarization current using various simplifying assumptions. Along with such approach, it is reasonable to develop other methods which can be based on the ray optics laws. They can afford opportunities for analysis of problems with objects having complex shape without any restriction on refraction index.

We have offered the method based on combination of exact solution of problem without “external” boundaries of the object and on accounting of these boundaries using the ray optics [8]. This method concerns problems that are characterized by the following large geometric parameter: the object size is much larger than wavelengths under consideration. It should be emphasized that the other geometric parameters (such as the distance from the object’s border to the charge trajectory) can be arbitrary.

Under such conditions, the following approach can be applied [8]. At first, the field of the charge in an infinite medium without “external” borders is calculated. The second step is the approximate calculation of the radiation going out of the object. (This calculation is related to Fock’s method for analyzing reflection of waves from an arbitrary surface [9]; analogous calculations are applied to elaborate different optical systems [10].) At the second step, the incident field is multiplied by the Fresnel transmission coefficient, and then propagation of radiation is calculated using the ray optics technique. Thus, the first of the refracted rays is obtained. Probably, this ray will be satisfactory for the majority of applied problems. If necessary, multiple reflections and refractions from the object’s borders can be also taken into account.

For testing the method, we used the problem about the field of point charge flying through the dielectric layer with the thickness d and permittivity ε. Such a problem has exact solution [3]. We compared computations performed with use of exact formulas and approximate ones [8]. In the case when $d \geq 10\lambda_d$ (where λ_d is a wavelength in dielectric), the obtained approximate solution is in a very good agreement with exact one. This result is very encouraging, and it stimulates applying the method under consideration for more complex object where the exact solution cannot be obtained.

APPROXIMATE SOLUTIONS FOR CONE AND PRISM

We applied the method under consideration to three cases. In two cases we assume that the dielectric objects have a vacuum channel with a radius a, and the charge moves along the axis of the channel (z-axis). One of these objects is a cone [8] and the other is a prism (the channel is situated in the middle, see Fig.1). In the third case, the charge moves along the prism at distance a from one of boundaries (the “lower” part of the object is absent in Fig.1). For short, the prism with the charge in the channel will be called the “prism-I”, and the prism

*Work supported by the Grant of the President of Russian Federation (MK-273.2013.2), the “Dynasty” Foundation and the Russian Foundation for Basic Research (Grant No. 12-02-31258).

#tyukhtin@bk.ru

ISBN 978-3-95450-122-9

with the charge moving along the boundary will be called the “prism-II”.

The permittivity and permeability of the object material are ε and μ, respectively. The channel and the region outside the object are vacuum. A point charge moves along the z-axis. The value a can be both smaller and larger than the typical wavelengths. It is also assumed that the Cherenkov radiation travels the distance in the dielectric which is much longer than the wavelength under consideration.

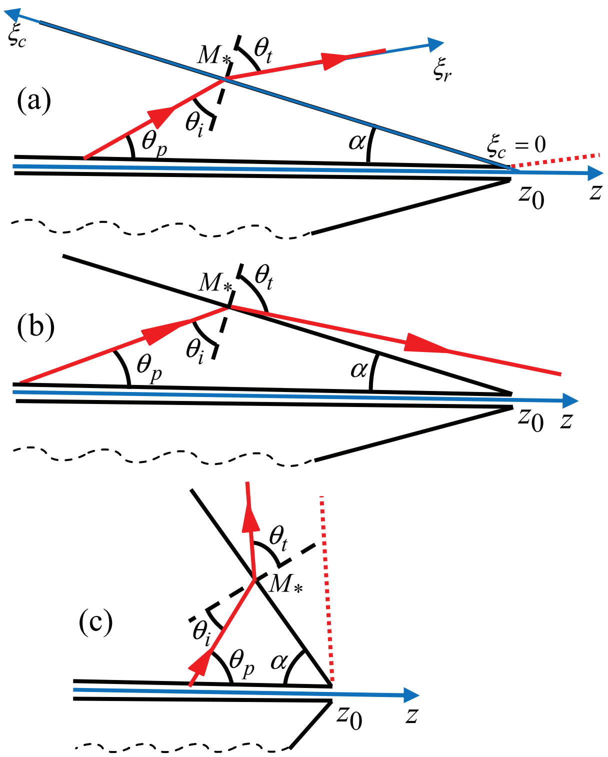

Figure 1: The section of the cone or prism for different ray dispositions; $\theta_{i,t} > 0$ for (a) and (b), and $\theta_{i,t} < 0$ for (c). The “prism-I” has the “lower” part, but the “prism-II” doesn’t have this part.

Note that, in the case of the prism, Fig. 1 shows the section which is orthogonal to the prism surfaces. We will consider the field in this plane only, because the transmitted radiation is maximum here.

The angles θ_i and θ_t have the same sign. Figures 1a and 1b illustrate the case $\alpha + \theta_p < \pi/2$ ($\theta_{i,t} > 0$); the opposite case $\alpha + \theta_p > \pi/2$ ($\theta_{i,t} < 0$) is characterized by another positional relationship of the rays and the normal to the boundary (Fig. 1c).

First, the problems should be solved for the case of the infinite medium with the vacuum channel and for the semi-infinite medium. The solutions of these problems are known [11].

Further, it is necessary to determine the point of incidence M_* for the Cherenkov wave on the cone (prism) surface. The coordinates of the incidence point ρ_*, z_* are functions of the coordinates of the observation point ρ, z situated in vacuum (we use cylindrical coordinates). Analysis gives the following results [8]:

$$\rho_* = (z_0 - z_*)\tan\alpha\,,\;\; z_* = \frac{z_0 \tan\alpha + z\cot(\alpha+\theta_t) - \rho}{\tan\alpha + \cot(\alpha+\theta_t)}\,,$$

$$\sin\theta_t = \sqrt{\varepsilon\mu}\sin\theta_i\,, \theta_i = \frac{\pi}{2} - \alpha - \theta_p\,, \cos\theta_p = \left(\sqrt{\varepsilon\mu}\beta\right)^{-1}.$$

Note that the volume wave of CTR exists outside the object only if the total reflection does not occur, i.e. $\sqrt{\varepsilon\mu}\sin|\theta_i| < 1$.

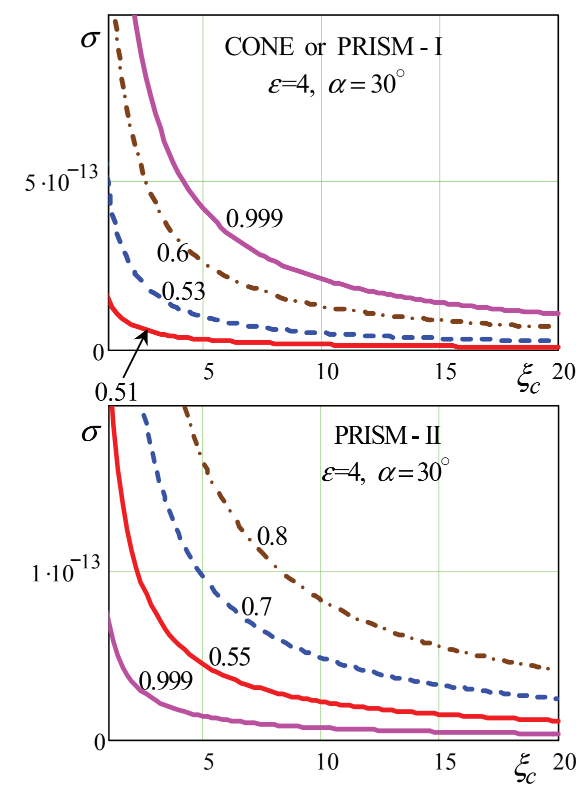

Figure 2: The spectral density $\sigma\left(\mathrm{J\cdot s/m^2}\right)$ as a function of the distance ξ_c(cm) from the cone (prism) vertex along the cone or prism surface; values of β are given near the curves.

The expression for the Fourier transform of magnetic component of radiation field outside the object can be written in the form

$$H_{\phi\omega} \approx H_{\phi\omega}^{(2)*}\sqrt{D(\rho_*)/D(\rho)}\,T\exp(i\omega l/c),$$

where $H_{\phi\omega}^{(2)*}$ is the incident field at the point M_*, T is the Fresnel transmission coefficient, l is the ray path in vacuum, and the value $D(\rho)$ is a square of cross-section of the ray tube. In the case of the cone

$D(\rho_*)/D(\rho) = \rho_*/\rho$, that is the cylindrical divergence (or convergence) takes place. For the prism, this expression is more complex.

The obtained result describes the CTR field outside the cone in the zone where the transmitted wave exists. If $\alpha + \theta_t < \pi/2$, then this zone is bounded:

$$z < z_0 + \rho \tan(\theta_t + \alpha \operatorname{sgn} \theta_t)$$

(dotted lines in Fig. 3a,c).

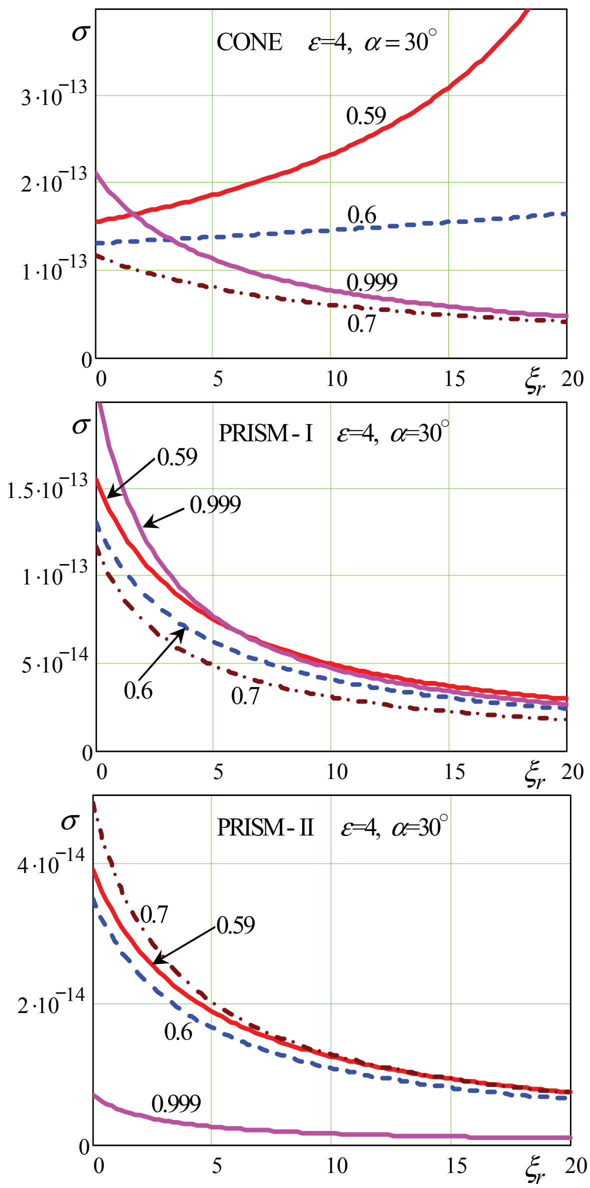

Figure 3: The spectral density $\sigma\left(\mathrm{J \cdot s/m^2}\right)$ as a function of the distance ξ_r(cm) along the ray from the cone or prism surface; the initial point M_* is situated at $\rho_* = 5\text{cm}$; values of β are given near the curves.

However, if $\alpha + \theta_t > \pi/2$ then the transmitted wave exists everywhere outside the object (Fig. 3b). In this case rays are convergent, therefore, for the cone, the wave amplitude increases with distance along the ray. For both prisms the wave amplitude always decreases along the ray, however the rate of change of amplitude depends on the problem parameters.

NUMERICAL RESULTS

Typical results of computations are presented in Fig. 2 and 3 where we used the following parameters: $\varepsilon = 4$, $\mu = 1$, $a = 2$ mm, $q = -1$ nC, and $\omega = 2\pi \cdot 3 \cdot 10^{10}$ s^{-1}. The value of σ shown in the figures is a spectral density of energy flowing through a unit square: $\sigma = c\left|H_{\phi\omega}\right|^2$.

Figure 2 illustrates the typical dependences of the spectral density of the radiation energy on the distance ξ_c from the object vertex (the observation point is situated on the border of the object). In any case, the CTR on the surface monotonically decrease with ξ_c, but it essentially varies depending on the charge velocity.

The typical dependences of σ on the distance ξ_r from the cone boundary along the ray are shown in Fig. 3. In the case of the cone, there are some values of the parameters when the radiation energy increases with ξ_r. It means that the radiation is a convergent cylindrical wave. As a rule, this effect takes place for velocities close to the Cherenkov threshold. In the case of prism such effect is impossible: radiation always decreases with distance from the border. Note also that radiation is more intensive for the "prism-I" than for the "prism-II".

REFERENCES

[1] V.P. Zrelov, VavilovCherenkov Radiation in High-Energy Physics (Israel Program for Scientific Translations, 1970).
[2] A.P. Potylitsyn, Yu.A. Popov, L.G. Sukhikh, G.A. Naumenko, M.V. Shevelev, J. Phys.: Conf. Ser. 236 (2010) 012025.
[3] V.L. Ginzburg, V.N. Tsytovich, Transition Radiation and Transition Scattering (Hilger, London, 1990).
[4] A.A. Tishchenko, A.P. Potylitsyn, M.N. Strikhanov, Phys. Rev. E 70 (2004) 066501.
[5] M.I. Ryazanov, M.N. Strikhanov, and A.A. Tishchenko, JETP 99 (2004) 311.
[6] M.I. Ryazanov, JETP 100 (2005) 468.
[7] D.V. Karlovets, JETP 113 (2011) 27.
[8] E.S. Belonogaya, A.V. Tyukhtin, S.N. Galyamin, Phys. Rev. E 87 (2013) 043201.
[9] V.A. Fok, Electromagnetic diffraction and propagation problems (Pergamon Press, 1965).
[10] S. Solimeno, B. Crosignani, P. DiPorto, Guiding, Diffraction, and Confinement of Optical Radiation (Academic Press, 1986).
[11] B.M. Bolotovskii, Sov. Phys. Usp. 4 (1962) 781.