MULTIBUNCH TRACKING CODE DEVELOPMENT TO ACCOUNT FOR PASSIVE LANDAU CAVITIES

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Abstract

The MAX IV 3 GeV storage ring will achieve an ultra-low horizontal emittance of 0.24 nm rad by using a multi-bend achromat lattice. Passive harmonic cavities are introduced to relax the Touschek-lifetime and intrabeam scattering issues as well as fight collective beam instabilities via Landau damping. Since instabilities occur during injection, when the passive harmonic cavity potential is also time varying, it became important to simulate this transient process. The most promising approach was considered to be multibunch tracking which also allows for an arbitrary filling pattern. Since every bunch is represented by numerous macro-particles, internal motions as well as micro-structures in the charge distribution can be followed.

THE MULTIBUNCH TRACKING CODE

For multibunch instability studies, a 6D macroparticle tracking code, called *mbtrack*, was available, which proofed its ability already for the light sources SOLEIL[1] and DIAMOND[2]. In the present version, the effects of the geometric ring impedance and the wall resistivity are treated as intra-bunch effects. The impact of quantum excitation and radiation is optional and can also describe the effect of gradient dipoles and radiation from insertion devices. All transformations are performed turn-wise.

In addition, the long-range impact of the transverse resistive wall impedance can also be included. Therefor the center of mass of all bunches in both transverse planes are recorded over several turns, internal bunch structures are neglected for this effect. This reduced information is exchanged amongst all bunches and allows a fast computation of the resistive wall effect with a low demand of memory.

Following these lines, the modeling of a passive harmonic cavity (HC) was just a logical progression: The cavity is "powered" by the induced fields from the beam over about a hundred turns. The available center of mass information for each of those bunches had to be extended by few statistical moments of higher order to allow a sufficiently accurate modeling. For speed reasons, the code is parallelized and creates one master task which organizes the data exchange by using MPI between the slave tasks, each corresponding to one present bunch.

IMPACT OF A HARMONIC CAVITY

The passive higher harmonic cavity adds another potential to the rf-cavity one, which causes an energy change of U_{HC} for each electron. The total loss per turn $U = U_{rad} + U_{HC}$ includes the radiation losses U_{rad} and is compensated for by the rf cavities:

$$U_{rad} = eV(0) = eV_{rf}\left[\sin(\phi_s) + k\sin(n\phi_h)\right] \quad (1)$$
$$= U - U_{HC} \quad (2)$$

with the synchronous phase ϕ_s and the phase of the nth-harmonic cavity ϕ_h.

The evoking wake potential $V_{HC}(\tau)$, acting on a charge at the longitudinal positions τ, excited by the bunches of the last k buckets with the charge distributions ρ_k, depends on the wake field W of the harmonic cavity:

$$V_{HC}(\tau) = \sum_k^{\text{buckets}} \int_{-\infty}^{\infty} d\tau' \rho_k(\tau') W(k\Delta\tau_b + \tau - \tau'), \quad (3)$$

where $\Delta\tau_b$ is the bucket distance.

The HC wake field can be modeled as a resonator with a resonance frequency ω_r close to a multiple of the rf-frequency $n\omega_{rf}$, where the cavity detuning $\Delta\omega = n\omega_{rf} - \omega_r$ will have the biggest influence on the resulting potential:

$$W(\tau) = 2\alpha R_s e^{-\alpha\tau}\left(\cos\bar{\omega}\tau - \frac{\alpha}{\bar{\omega}}\sin\bar{\omega}\tau\right), \quad \tau > 0 \quad (4)$$
$$\alpha = \frac{\omega_r}{2Q}, \qquad \bar{\omega} = \sqrt{\omega_r^2 - \alpha^2} \quad (5)$$

with the shunt impedance R_s and the quality factor Q.
To achieve optimal bunch lengthening with the help of the HC, the first and second derivative of $V_{HC}(\tau)$ at $\tau = 0$ should be zero [4]:

$$\left(\frac{\partial V(0)}{\partial\tau}\right) = 0, \qquad \left(\frac{\partial^2 V(0)}{\partial\tau^2}\right) = 0 \quad (6)$$

In that case, the expected electron distribution is not Gaussian anymore but quartic with a bunch length of σ_τ:

$$\rho(\tau) = c \cdot \exp\left[-\left(\frac{\tau - \mu_1}{1.72\sigma_\tau}\right)^4\right], \quad (7)$$

Following [3], the induced voltage can expanded in Taylor-series and then well approximated by the statistic moments μ_j^k of the k electron distributions:

$$V_{HC}(\tau) = \sum_k^{\text{buckets}} \int d\tau' \rho_k(\tau') \sum_j^{\text{orders}} \frac{1}{j!} W^{(j)}(k\Delta\tau_b)(\tau - \tau')^j \quad (8)$$
$$= \sum_k \sum_j W^{(j)}(k\Delta\tau_b) \int \frac{(\tau - t)^j}{j!}\rho_k(t)dt \quad (9)$$
$$= \sum_k \sum_j W^{(j)}(k\Delta\tau_b)\text{Pol}(\tau, j, \mu_i^k) \quad (10)$$

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with $\mu_j^k = \int \rho_k(t) t^j dt$ and $i = 1, 2, \ldots, j$.

The statistic moments enter in the polynomials $\mathrm{Pol}(\tau, j, \mu_i^k)$, and in second order this polynomial is for example:

$$\begin{aligned}\mathrm{Pol}(\tau, 2, \mu_i^k) &= \frac{1}{2}\int \rho_k(t)(\tau - t)^2 dt \\ &= \frac{1}{2}(\tau^2 - 2\tau\mu_1^k + \mu_2^k) \qquad (11)\end{aligned}$$

On one hand, this method obliges us to determine not only the wake field but also its derivatives $W^{(j)}$, which can be done analytically for a resonator. On the other hand, neither the position of the macro-particles from earlier turns, nor their quantity in distinct bins are needed, but only a small number of statistical moments. In addition, the wake field information can be retrieved once in a preprocess and only the bunch information has to be updated after every turn. The resulting reduction of memory as well as computation time is significant.

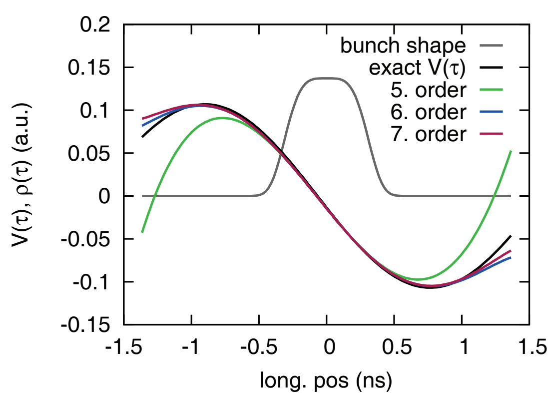

Figure 1: Beam induced voltage in the passive harmonic cavities from a quartic electron distribution. Comparison of the exact result with a Taylor-series expansion of the electron distribution up to different orders.

Figure 1 compares the retrieved wake voltages accounting for different orders of the expansion with the full numerical determination of the integral from equation (3). Over a wide range, the consideration of the 7th order gives no significant improvement so that always the first 6 orders were used for the following simulations. In both cases, wake fields up to a length corresponding to 10 damping times of their oscillation were considered.

THE MAX IV 3 GEV RING AS AN EXAMPLE

The default initial particle distribution $\rho(x, x', y, y', \tau, \epsilon)$ in *mbtrack* is Gaussian with the possibility to specify an offset and standard deviation in all six dimensions. τ is the longitudinal deviation from the synchronous particle and $\tau > 0$ if the test particle is leading, while ϵ is the relative energy deviation $(E - E_0)/E_0$ from the nominal energy E_0. The tracking of the horizontal and vertical planes are optional and were ignored for this study of the HC effect. *mbtrack* calculates the bunch center (e.g. $\bar{\tau} = \frac{1}{M}\sum \tau_i$, for $i = 1, \ldots, M$ particles) and the standard deviation (e.g. bunch length σ_τ) after every turn for all tracked variables. Every N turns these values are further analyzed to obtain the time dependent bunch statistics and the averages over all bunches.

Table 1 gives an overview of the MAX IV 3 GeV ring parameters for a bare machine, which were used for the following results. The parameters are optimized to achieve best bunch lengthening at an operating current of 500 mA [5]. As a first test, the initial Gaussians of all h bunches

Table 1: MAX IV parameters of the 3 GeV ring, bare lattice

Beam energy	E_0	3.0	GeV
Beam current	I	500	mA
Ring length	L	528.0	m
Harmonic number	h	176	
Bunch length w/o HC	σ_τ	40	ps
Bunch length at 500 mA	σ_τ	195	ps
Peak rf-voltage	V_{rf}	1.02	MV
rf- frequency	f_{rf}	99.931	MHz
Energy loss per turn	U_{rad}	360	keV
Higher harmonic of HC	n	3	
Quality factor HC	Q_f	21600	
HC detuning	Δf	48.1227	kHz
Total shunt impedance HC	R_s	2.36441	MΩ

in the ring had the nominal bunch length of $\sigma_\tau = 195$ ps and no longitudinal offset $\mu_1(\tau) = 0$, as Fig. 2 shows. After some hundred turns under the influence of the HC, the shape adopts the expected quartic electron distribution.

Figure 3 compares the voltages seen by the bunch and its shape of this equilibrium. The sum of the rf- and HC-potential is flat around the bunch center, where its first and second derivative should be zero in the ideal case. The average bunch length settles to 190±1 ps, which is a bit lower than expected.

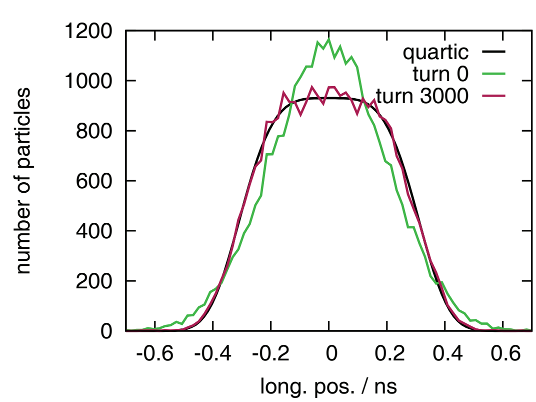

Figure 2: Initial Gaussian and final quartic charge distribution under the influence of the passive HC.

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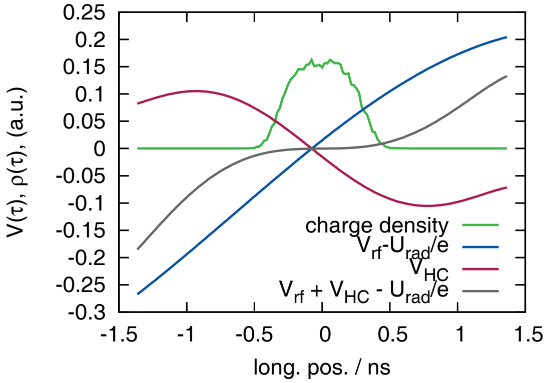

Figure 3: Rf-voltage, beam induced voltage and sum of the potential in comparison with a charge distribution snapshot from multibunch tracking.

To imitate the beam filling process of the ring, the runs were started with a low beam current which got increased by 25 mA every 50,000 turns (corresponding to three energy damping times). The bunch lengthening over current due to the increasing HC potential is shown in Fig. 4. The comparison of the bunch length of a single bunch with the averaged bunch length shows no discrepancy within the operation current range. The errorbars indicate the standard deviation of the averaged values over all 176 bunches during 1000 turns and the gray line shows the nominal value for 500 mA.

Figure 5 presents the averaged relative energy spread σ_E over the corresponding current. It stays constantly around the nominal value of 7.69×10^{-4} up to 550 mA. The coupled nature of electron distribution and potential needs several damping times to settle to an equilibrium before it is possible to decide if σ_E converges to the nominal value or if this is already an energy spread widening due to an instability. During the real operation, the possibility to tune the HC further exists, in order to achieve a stronger lengthening for low currents and less for hight currents and to dampen occurring instabilities.

CONCLUSION

The option of a passive harmonic cavity was implemented in *mbtrack* via its wake potentials produced by the actual electron distributions of all bunches. It is hence fully dynamic and able to simulate arbitrary filling pattern and current ramps.

From the self-consistent calculations [5], bunch length and electron distribution for some set of parameters were already known and served as first test of feasibility. The quartic electron distribution as well as bunch length and energy spread could be well reproduced. The algorithm seems to be robust and can also handle starting distribution far from equilibrium conditions.

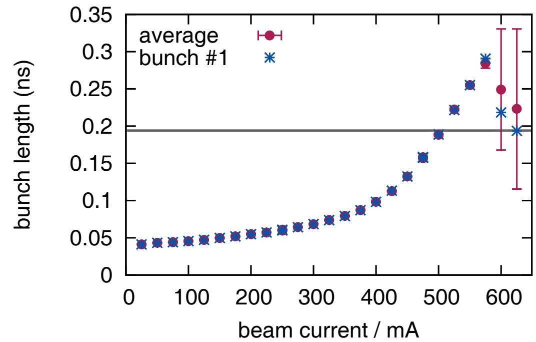

Figure 4: Bunch length under the influence of the HC. From the natural bunch length of 40ps, the beam induced voltages lengthens the bunches up to 190 ps for the nominal operation current of 500 mA, indicated by the gray line. The red points show the averaged values and its standard deviation over all bunches and the last 1000 turns before the current gets increased.

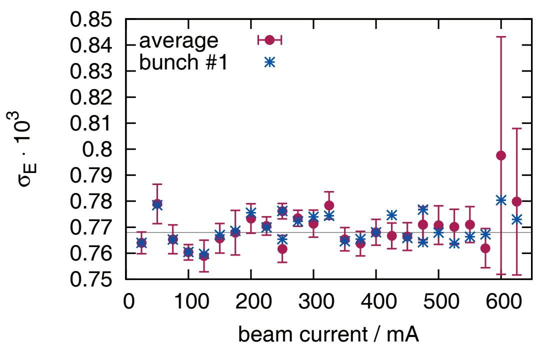

Figure 5: Energy spread development for increasing current: Every 50,000 turns the current is increased by 25 mA. The errorbars are the standard deviation of the value for all bunches and over 1000 turns.

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REFERENCES

[1] R. Nagaoka et al., "Beam Instability Observation and Analysis at SOLEIL", PAC'09 Vancouver, Canada

[2] R. Nagaoka et al., "Studies of Collective Effects in SOLEIL and DIAMOND Using the Multiparticle Tracking Codes SB-TRACK and MBTRACK", PAC'09 Vancouver, Canada

[3] G. Bassi e al., "Passive Landau Cavity Effects in the NSLS-II Storage Ring", IPAC'2012 San Sebastian, Spain

[4] A. Hoffman et al., "Beam Dynamics in a Double RF System", CERN-ISR-TH-RF-80-26

[5] P.F. Tavares et al., "Collective Effects in the MAXIV 3 GeV Ring", IPAC'2012 San Sebastian, Spain

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