EIGENMODE COMPUTATION FOR ELLIPTICAL CAVITIES SUBJECT TO GEOMETRIC VARIATIONS USING PERTURBATIVE METHODS*

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Abstract

Parametric studies of geometric variations are an essential part of the performance optimization and error estimation in the design of accelerator cavities. Using common eigenmode solvers the analysis of intentional and undesired geometric perturbations tend to be very extensive since any geometric variation involves an entire eigenmode recomputation. Perturbative methods constitute an efficient alternative for the computation of a multitude of moderately varying geometries. They require a common eigenmode computation of solely one (so called unperturbed) geometry and allow for deriving the eigenmodes of similar but modified (so called perturbed) geometries from these unperturbed eigenmodes. In [1], [2] the practicability of perturbative methods was proven by means of simple cavity geometries.

In this paper we investigate the applicability and efficiency for practice-oriented cavities. For this, basic geometric parameters of elliptical cavities are varied and the respective eigenmodes are computed by using perturbative as well as common methods. The accuracy of the results and the computational effort of the different methods are compared.

INTRODUCTION

Perturbative methods (PM) take advantage of the fact that the eigenmodes of a cavity form a system of mutually orthogonal functions. This entails that the modes of one (unperturbed) cavity can be used to expand the modes of another (perturbed) cavity presupposed that the unperturbed cavity shape completely includes the perturbed one. To determine the weighting factors for such a series expansion the mutual interactions of the known unperturbed eigenmodes inside the volume ΔV (volume that is removed from the unperturbed volume by the perturbation) have to be computed. The interaction of two modes is composed of integrals of the scalar products of their electric and magnetic fields respectively over the volume ΔV.

Currently, we use two different perturbative methods, a generalization of Slater's theorem (GST) [3] and a self-developed method [2]. The methods differ in the composition of interaction terms. In the following the results of GST are discussed.

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SIMULATIONS

For the unperturbed cavity a single elliptical cell according to Cornell's 7-cell SRF cavity design was chosen. Cylindrically symmetric perturbations were applied to it presenting typical modifications of a cavity optimization process (see fig. 1). A variation of the cell diameter was done by reducing the equator radius by 1%, 5% and 10%. The cell width was modified by simultaneously decreasing the longitudinal equator half axis and increasing the longitudinal iris half axis by 5% and 10%.

The unperturbed eigenmodes were simulated with the eigenmode solver of CST Microwave Studio 2012 (CST MWS) [5] using a tetrahedral mesh with curved elements. 620 modes in the range of 1.3 to 19.2 GHz were computed by using transversal ($H_{\text{tang}} = 0$) and longitudinal ($E_{\text{tang}} = 0$) symmetry planes. The simulated modes were exported as discrete values from CST MWS and all further computations were done in Wolfram Mathematica 8.0 [6].

The volume integrals forming the interactions of unperturbed modes significantly determine the accuracy of the perturbed results. The integration over ΔV is done as a summation of discrete values since a continuous integration turned out to be too expensive [4]. However, the volume ΔV cannot be adequately discretized by commonly used voxel elements since ΔV is mainly located in the curved boundary region of the cavity. Furthermore, the discrete field values may have zero values if their coordinates are located close to or outside the boundary. For this reason, a special algorithm is used for the volume partitioning [4] that precisely computes volume and center point of the elements (see fig. 1) thus allowing for accurate computation of the interactions terms.

To verify the results comparative eigenmode computations of the investigated perturbed cavity shapes were carried out with CST MWS.

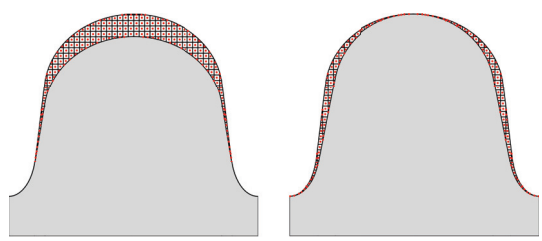

Figure 1: Shape of Cornell cell with perturbed diameter (left) and width (right). Partitioning of ΔV into discrete elements with centered coordinates for field allocation.

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RESULTS

The perturbative method yields accurate results for the frequency f_{Per} of the perturbed modes over wide frequency range. Regarding all investigated perturbations, the relative error of f_{Per} between 1.24 and 12.25 GHz is smaller than $6.7 \cdot 10^{-3}$. The majority of frequencies deviates even less from the comparative results as fig. 2 shows. For instance, the mean relative error (MRE) for a 10% cell width perturbation is only $1.6 \cdot 10^{-3}$. For a 1% cell diameter perturbation f_{Per} can be computed up to 14.25 GHz with a MRE of $3.9 \cdot 10^{-3}$.

The PM based fields of lower order modes coincide very well with the comparative simulation like fig. 3 shows for the accelerating π mode. By increasing frequency the deviations also start to increase. But the fundamental field patterns are maintained even for high frequencies and large perturbations as fig. 4 and particularly fig. 5 illustrate.

It turned out that the size of volume elements necessary to accurately compute the discrete volume integrals is mainly determined by the unperturbed mode with the shortest wavelength λ_{min}. The dimension of the perturbed volume part ΔV has only little impact. For the used 620 modes ($\lambda_{min} = 1.56$ cm) the integrals converge at a step size of 1.25 mm. Depending on the investigated perturbation this corresponds to 12,000 to 40,000 elements.

Using all simulated unperturbed modes, a number of 400 to 550 modes was needed to reach convergence of the results. It became apparent that only certain kind of modes actually interact with each other and affect the results. This arises from the fact that only cylindrically symmetric perturbations were investigated which entails that only modes with the same azimuthal dependency (e.g. monopole modes) interact inside ΔV. Splitting the complete mode set into respective subsets, only about 50 modes are necessary to reach an identical accuracy (fig. 6). Furthermore, the mode number depends on the dimensions of the perturbation. This means that the smaller the perturbation is the shorter has to be the minimal wavelength λ_{min} of the unperturbed modes to accurately expand a perturbed mode in terms of the unperturbed ones. Like previously mentioned, the results for a 1% cell diameter reduction are less accurate than the other results. Inspecting the convergence behavior proved that the use of further unperturbed modes may improve the accuracy.

The simulation of unperturbed modes is the most time consuming part of the PM implementation. Simulating 100 modes with a very high accuracy takes about three hours computational time (CST MWS, CPU: 2×3.30 GHz, RAM: 256 GB). Considering only a single perturbation this would involve a high effort. But considering a multitude of

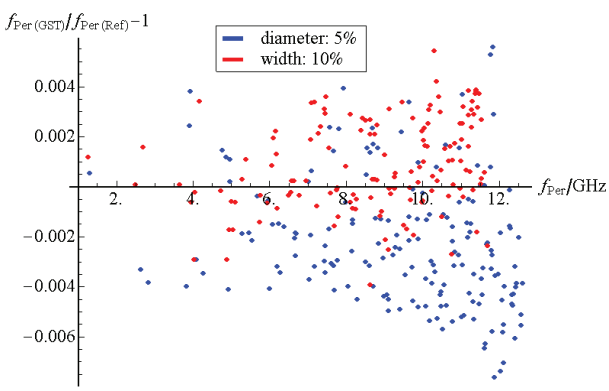

Figure 2: Relative error of f_{Per} of about 170 modes for 5% cell diameter (MRE: $2.9 \cdot 10^{-3}$) and 10% width perturbation (MRE: $1.6 \cdot 10^{-3}$).

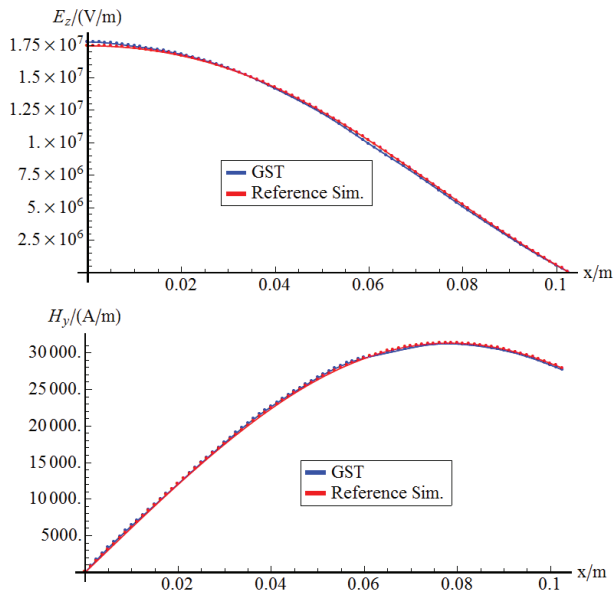

Figure 3: Perturbed field of the accelerating mode for 10% cell width reduction (y = z = 0 m).

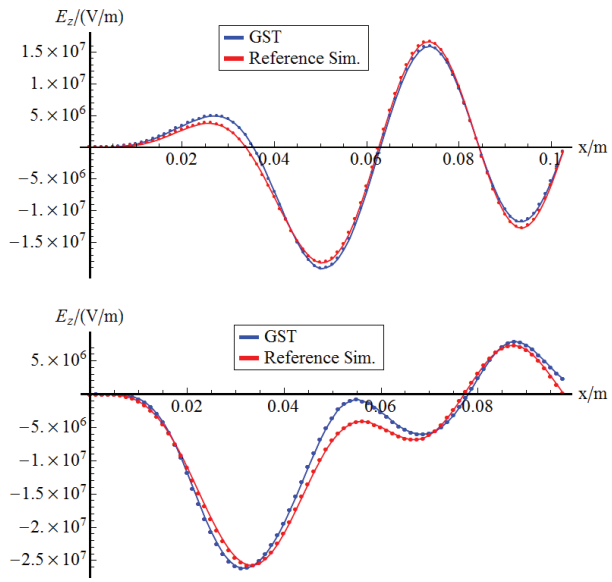

Figure 4: Longitudinal electric field of two higher order modes with $f_{\mathrm{Per}} = 8.94$ GHz at 10% width reduction (top) and $f_{\mathrm{Per}} = 8.69$ GHz at 5% diameter reduction (bottom).

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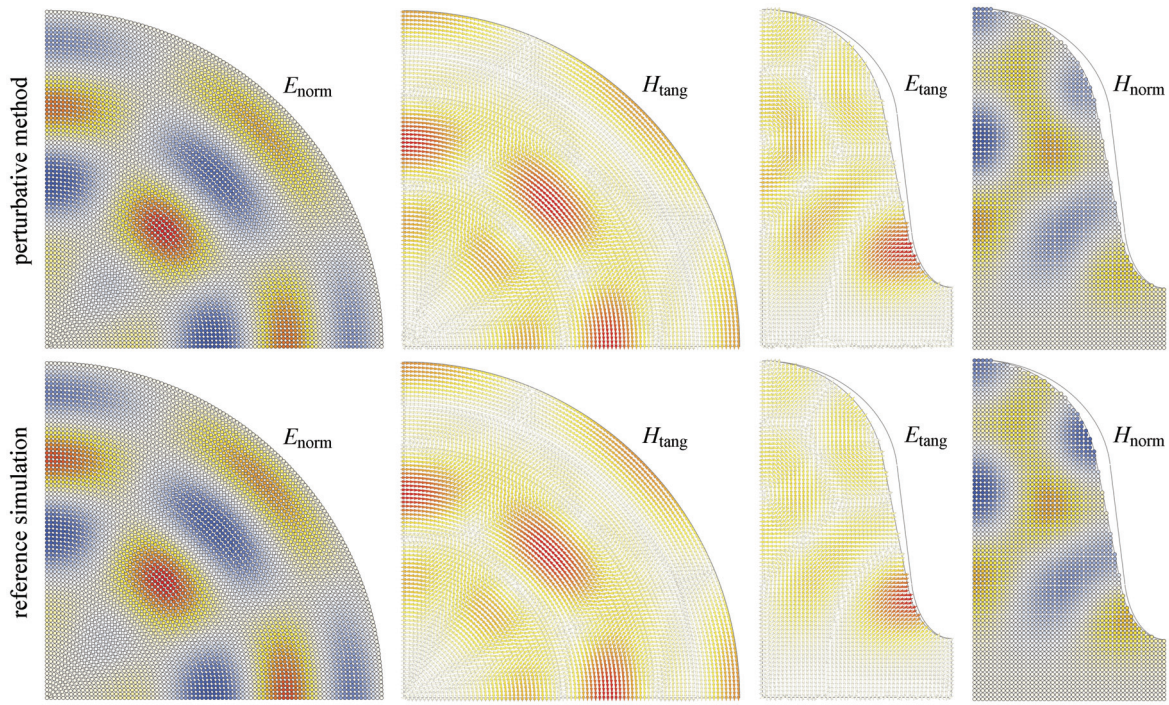

Figure 5: Comparison of field of perturbed mode with $f_{Per} = 8.94$ GHz in the cross cut (z = 0 m) and longitudinal cut (y = 0 m) of the cell (10% width perturbation). The unperturbed and perturbed cavity shapes are shown as gray curves.

perturbations (like it is common for parameter studies), this is a comparatively low effort since the unperturbed modes have to be computed only once. The computation of discrete volume integrals for a set of 50 modes takes between 50 s and 120 s (12,000 to 40,000 elements) while the final determination of the perturbed frequencies and series expansion factors lies in the range of some seconds (Mathematica, CPU: 2 × 2.66 GHz, RAM: 20 GB). Thus, the execution of perturbative methods algorithms is highly efficient.

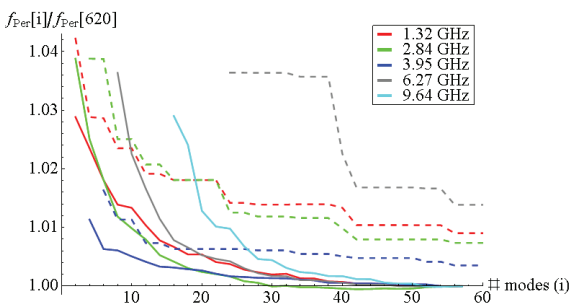

Figure 6: Change of perturbed frequencies f_{Per} depending on number of unperturbed modes. f_{Per} converge faster using a subset of interacting modes (solid lines) instead of the entire mode set (dashed lines).

CONCLUSIONS

Perturbative methods could be applied successfully to elliptical single cell cavities by investigating different forms of minor and moderate perturbations. The frequency as well as the field of perturbed modes can precisely be computed over a wide frequency range. Once the modes of the unperturbed cavity are simulated, the perturbative methods can be executed with a very low effort. Splitting the unperturbed modes into subsets of interacting modes (depending on symmetries of the cavity geometry and the perturbation) may further reduce the computational time since the effort quadratically scales with the number of used modes.

REFERENCES

[1] K. Brackebusch, H.-W. Glock, U. van Rienen, Calculation of High Frequency Fields in Resonant Cavities based on Perturbation Theory, Proceedings of 2nd IPAC, San Sebastian, Spain, September 4-9, pp. 2235-2237, 2011.

[2] K. Brackebusch, U. van Rienen, Eigenmode Computation for Cavities with Perturbed Geometry based on a Series Expansion of Unperturbed Eigenmodes, Proceedings of 3rd IPAC, New Orleans, USA, May 20-25, pp. 277-279, 2012.

[3] D. Meidlinger, A General Perturbation Theory for Cavity Mode Field Patterns , Proceedings of SRF2009, Berlin, Germany, THPPO005, 2009.

[4] K. Brackebusch, U. van Rienen, Implementational Aspects of Eigenmode Computation Based on Perturbation Theory, Proceedings of 11th ICAP, Warnemuende, Germany, August 19-24, pp. 48-50, 2012.

[5] Computer Simulation Technology AG, *Microwave Studio®*, 64289 Darmstadt, Germany.

[6] Wolfram Research, Inc., *Mathematica®Edition: Version 8.0*, Champaign, 2010.