INVESTIGATION OF NUMERICAL PRECISION ISSUES OF LONG TERM SINGLE PARTICLE TRACKING*

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Abstract

Long-term single particle simplectic tracking is one of the most reliable tool to study the dynamic aperture of the circular accelerators. The present computational resources allow studying the long-term behaviour for an extended number of turns. In this paper we explore the possibility that artifacts of the double precision arithmetic may be visible when the number of turns is in the order of 10^6 to 10^7.

INTRODUCTION

Long-term tracking simulations are particularly sensitive to the intrinsic approximations of the floating point arithmetic due to the large number of operations that are accumulated on the same set of variables to obtain the final results. In addition, the motion under interest is always weakly chaotic, hence showing a high sensitivity to the choice of initial conditions, even if it leads to bounded motion when the number of turns is limited for most of the initial conditions. In this condition one needs to ensure that the results obtained are intrinsic to the problem under study and not related to the magnitude and/or the properties of the floating point errors.

SixTrack is a 6D tracking code used extensively at CERN to simulate the particle trajectories in the LHC [1]. Previous studies [2] indicate that even small changes, such as the type of CPU, lead to unreproducible results. However for global quantities, like the dynamic aperture as estimated using the typical procedures used for the nominal LHC [3], one expects to be accurate to 0.5σ resulting from the simplifications needed to obtain the results [4].

The approach used to assess whether the numerical results are stable consisted in perturbing slightly the model used in the numerical simulations. The small perturbation should not introduce any meaningful physical difference justifying a change in the numerical results. Hence, any non-negligible change should be induced by numerical effects. Three types of effects have been considered: different approximation of the computation of the beam-beam kick; inclusion of zero-intensity beam-beam kicks; perturbation of linear tunes and orbit (the linear tunes are perturbed by 10^{-12} and the orbit by 10^{-7} m). The first two will be discussed in the rest of the paper, while the last one confirmed that the results are indeed stable against small (numerical) differences. As far as the number of turns is concerned, values ranging from 10^6 and 10^7 have been considered.

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SIXTRACK FLOATING POINT FEATURES

Over the years SixTrack has been optimised to obtain reproducible results among a large range of combinations of architectures and compilers supporting IEEE 754 floating point arithmetic [5]. SixTrack also uses the `crlibm` [6] library in order to obtain numerically accurate values for the elementary functions. This point is crucial whenever distributed systems are considered for massive numerical simulations [7].

In the case of the inclusion of beam-beam effect in the weak-strong approximation, the complex error function

$$W(z) = \exp\left(-z^2\right)\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{-iz}\exp(-t^2)\mathrm{d}t\right], \quad (1)$$

is needed to compute the beam-beam kick [8] and it is evaluated around 400 times per turn for the case of the LHC. In SixTrack two alternative implementations are available, which give slightly different results close to the origin. The cases indicated in the figures with `ibtype=0` use a version derived from `wwerf` by K. Koelbig that uses an approximation around the origin. Alternatively (`ibtype=1`) uses an algorithm developed in collaboration with G.A. Erskine and E. McIntosh, which relies on a interpolation table computed only once, and has comparable speed and a precision better than 10^{-8}.

RESULTS

We base our analysis on the survival time defined as the number of turns that a particle circulates in the LHC ring, assuming top energy collision configuration, before undergoing transverse oscillations larger than 1 m. The trajectories of two close-by particles give an indication of the sensitivity of the motion with respect to the initial conditions and is, indeed, taken as an estimate of the maximum Lyapunov exponent of the system [10]. The main assumption in this paper is that a quantity that depends too strongly on the difference of the initial conditions has no useful physical meaning. Hence we try to observe the global dynamics by probing the whole phase space within the limitations of finite computational time. This approach is the one used in the past, when fast indicators of stability have been defined with the aim of finding globally-defined quantities linked with global stability properties [11] and when the survival time, with suitable post-processing, has been linked with the asymptotic behaviour of the dynamic aperture [12],

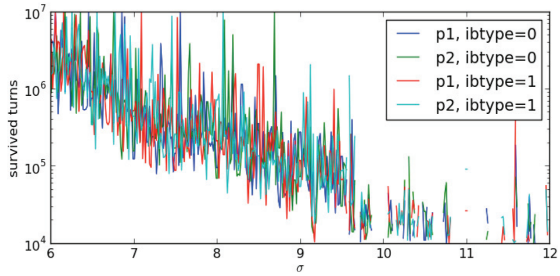

Figure 1: Survival time of two close-by particles (p_1, p_2) for a given seed and angle as a function of amplitude for the two beam-beam algorithms. The overall picture is clearly insensitive to the details (particle or algorithm).

thus providing useful and quantitative information on the system under study.

The simulations are carried out by choosing the initial conditions in a polar grid in the J_x-J_y plane and a momentum deviation corresponding to $2/3$ of the RF bucket. The horizontal and vertical phases are set to zero in the hypothesis that over few thousands of turns the particles have probed enough phases such that this information has no global impact. This hypothesis is well justified in case of weakly perturbed stable linear motion far from lower order resonances [4]. For each study, 60 different models of magnetic imperfections, called seeds, are generated from a statistical description of the field quality of the LHC magnets. Each seed represents a different physical system, but the set of seeds is considered an equally good approximation of the same real machine.

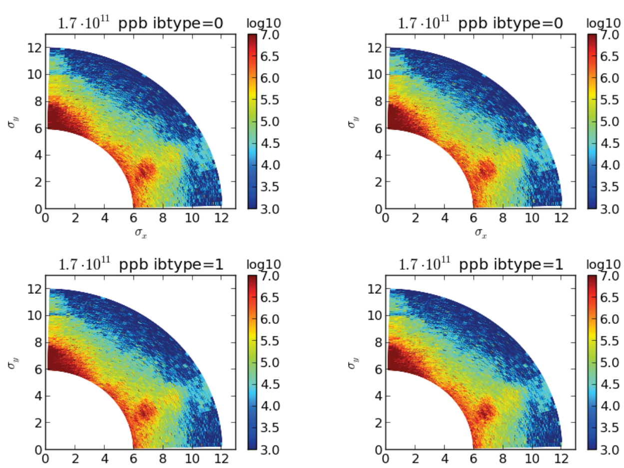

Figure 2: Survival time for a given seed and two close-by particles (left/right columns) and beam-beam algorithm type (upper/lower) as a function of amplitude and angle in phase space. Albeit details are different, the global dynamics is the same. The bunch intensity is 1.7×10^{11} p.

Figure 1 shows an example of the survival plot for two close-by particles for one angle and one seed as a function of amplitude. The details of the plots are completely uncorrelated although the overall picture is rather stable. A similar effect is generated when changing the algorithm type. One can conclude that differences in the algorithm for

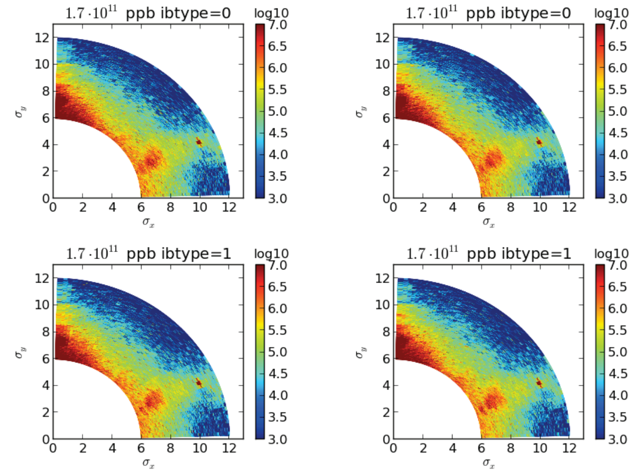

Figure 3: As Figure 2, but for a different seed. In this case a small island of stability disconnected from the main stable region appears at 30°. No difference is visible between the numerical approaches or the close-by particles.

the complex error function perturb the motion as much as a small difference in the initial conditions. However in both cases the global dynamics is preserved, see Fig. 2, where the information for all angles and amplitudes is plotted in a logarithmic scale. Similar conclusions can be drawn from Fig. 3.

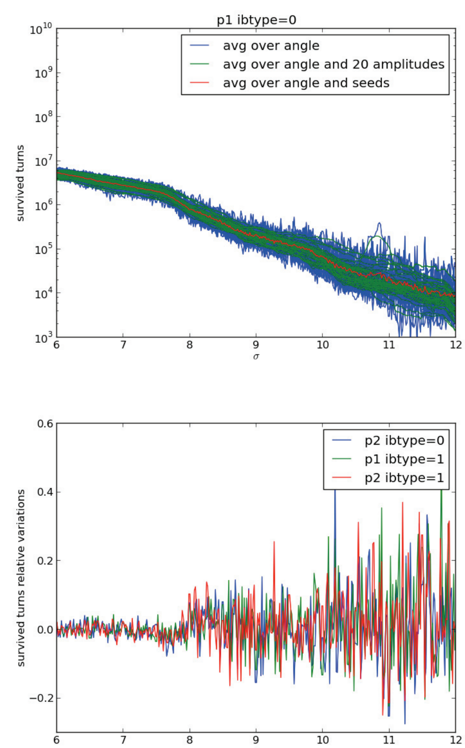

Figure 4: Top: survival time for one algorithm type as a function of amplitude. The curves show: an average over the angles for all the seeds (blue); as before but with a running average over $1/3\,\sigma$ (green); average over angles and seeds. Bottom: the relative variation of the averages between the cases shown on the top.

In order to be more quantitative an averaging on the data

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is performed to identify possible trends as shown in Fig. 4. The survival time is rather linear with a sudden change of slope around 8 σ, corresponding to a bit less than 10^6 turns, i.e., the limit selected for the standard LHC simulations including weak-strong beam-beam effects. No special difference appears beyond 10^6 turns, hinting to no particular numerical issue. The spread due to the presence of various seeds is also decreasing at lower amplitudes, which is compatible with the observation that the size of resonance islands is exponentially small close to the origin and such islands are generating the weakly chaotic behaviour that distinguish the various seeds.

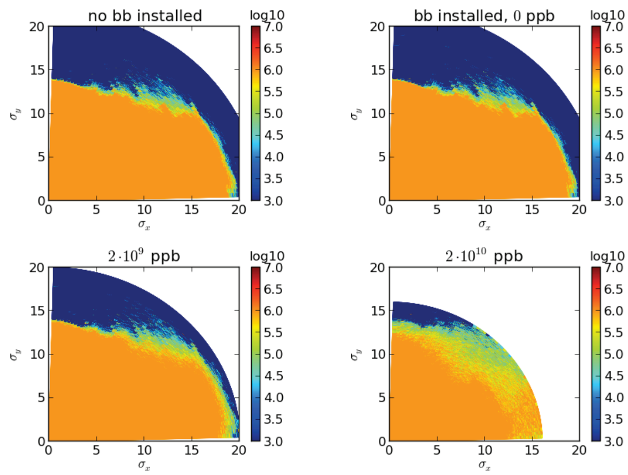

Figure 5: Survival time of one seed for several beam-beam settings. On the top a case with no beam-beam lens installed and a case with zero bunch intensity. On the bottom a case with 2×10^9 p (left) and 2×10^{10} p (right). The maximum number of turns is different.

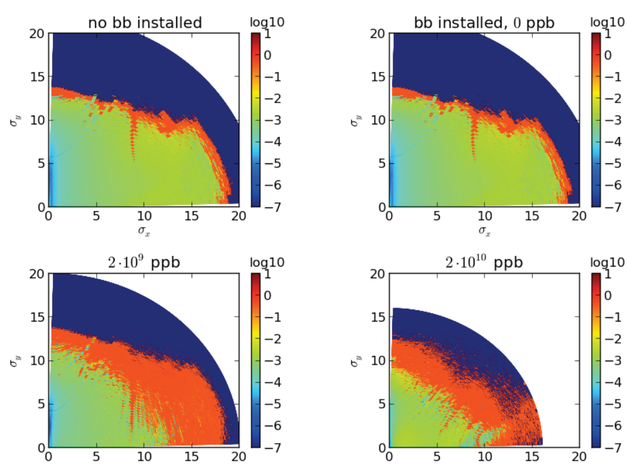

Figure 6: Maximum distance in phase space for two close-by particles and same parameters as Fig. 5.

In Fig. 5 a summary of the results obtained is shown. The inclusion of beam-beam lens with zero intensity does not generate any macroscopic difference in the numerical results. Furthermore, the process of reduction of the stable area in phase space with increasing bunch intensity is clearly visible. Figure 6 shows the maximum distance in phase space for close-by particles in the conditions of Fig. 5. The stable region does not seem affected by the small numerical differences due to the presence of zero-intensity beam-beam lenses (first row). As expected, the distance in phase space reflects the reduction of stable area with bunch intensity, although providing a rather pessimistic estimate [11].

CONCLUSIONS

The evaluation of numerical artifacts on long term tracking simulations are particularly challenging due to the sensitivity to small variations of direct observables that depend on individual trajectories. A first analysis of long term tracking simulations for 10^6 and 10^7 turns did not show any relevant numerical issues affecting the global dynamics observed from the survival time of an ensemble of particle trajectories. Nevertheless, this topic has not yet exhaustively explored. In particular simulations with time reversal and/or with extended numerical precision may give independent indications on numerical effects.

As a by-product, this analysis confirmed that, according to selected observables, the computationally faster algorithm for the evaluation of the complex error function can replace the slower, even if more accurate one.

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