ACCURACY OF MEASUREMENTS OF ε AND μ FOR LOSSY MATERIALS *

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Abstract

Measurements of samples of lossy ceramics and ferrites for Higher Order Mode (HOM) loads are performed routinely in our Lab. Some difference of results for different batches of materials can be explained not only by technological deviations in the material production but also by errors in the dimensions of the measured samples. Analytical evaluations and simulations with MicroWave Studio for samples in the form of coaxial washers in the frequency range from 1 to 12.4 GHz helped to define the main sources of errors and to improve accuracy of measurements.

INTRODUCTION

Measurements of dielectric permittivity ε and magnetic permeability μ of samples presenting materials for HOM loads are performed in our Lab with help of a coaxial line or rectangular waveguides in which a sample of a measured material is inserted, Fig. 1. Dielectrics and ferrites were measured at room and cryogenic temperatures at a frequency up to 40 GHz [1]. In point of fact, S-parameters of the tested insertion, a holder with a sample inside, were measured, and from these two complex parameters, $S11$ and $S21$, two other complex parameters, ε and μ, were found.

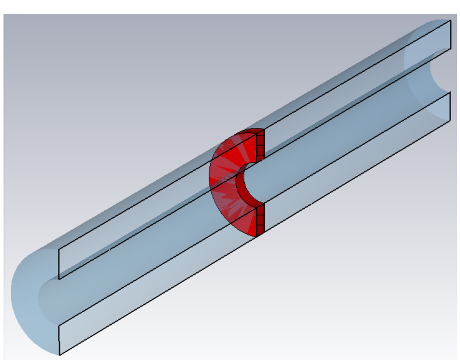

Figure 1: Cross-section of a coaxial line with a sample.

As it was pointed out at the HOM workshop [2], accuracy of measurements of dielectric permittivity ε of samples presenting materials for HOM loads was insufficient. Data for the samples made of the same material in different frequency ranges and with different transmission lines did not butt together at the boundaries of the frequency ranges.

We will show that in the case of dielectric/ferromagnetic washers used in the coaxial line measurements, even small gaps between the conductors and measured washers lead to big errors in the values of ε. Errors in μ are not so significant but can also be accounted for big μ if the values of gaps are known.

Because of small dimensions of the coaxial line used in the measurements, diameters 7.01/3.05 mm (these values will be used for further calculations), it appeared easier to precisely measure the inner and outer dimensions of the washers than accurately machine them with a needed tolerance. If the size of the gap between the washer and the surface of the coaxial line is known, a correction can be calculated and taken into account. This correction was estimated analytically and verified with CST Microwave Studio (MWS) [3].

For a practical implementation of the correction method described below, geometrical sizes of several dozens of silicon carbide washers were measured. These measurements were done using a Zeiss Accura Coordinate Measuring Machine, employing a VAST XT probe head. The measurements were accomplished using a 1 mm diameter ruby probe at 100 mN of force with accuracy of 2.5 μm. Washers were secured for measurements by an in-house built vacuum chuck. These measurements revealed not only deviations of the inner and outer radii but also disturbed roundness of both circles and the radial runout, i.e. shift between centres of both circles, and other defects.

We assume that eccentricity and ellipticity of the shape have an effect of the second order on the values of electromagnetic parameters and will concentrate on the errors due to symmetric gaps. Other defects can be also taken into account with help of simulations if needed.

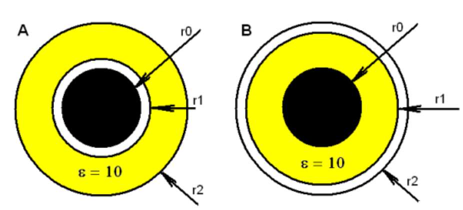

Figure 2: A dielectric washer with gaps in a coaxial line.

ANALYTICAL ESTIMATION OF CORRECTIONS

In the theory of transmission lines, a coaxial line is presented as a chain of small capacitors and inductances so

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that the propagation constant and the impedance are defined by the values of these parameters. So, it is logical to present the measured washer as one of the elements of such a line having its own capacitance and inductance. Values of ε and μ found for this element will be changed compared to the ideal shape if there is a gap between the washer and the inner or outer conductor. For example, deviation of capacitance in the cases shown in Fig. 2 is

$$\frac{\Delta C}{C_0} = -\frac{\varepsilon - 1}{\ln(r_2/r_0)} \cdot \frac{\Delta r}{r_0}, \qquad (1)$$

where $\Delta r = r_1 - r_0$ in the case A, and

$$\frac{\Delta C}{C_0} = -\frac{\varepsilon - 1}{\ln(r_2/r_0)} \cdot \frac{\Delta r}{r_2}, \qquad (2)$$

where $\Delta r = r_2 - r_1$ in the case B.

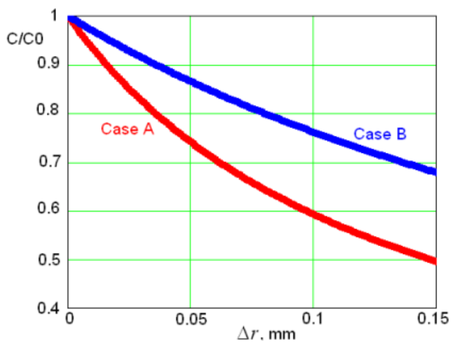

Figure 3: Errors in capacitance due to the gaps from Fig. 2.

If the thickness of the washer is small compared to the wave length we can believe that the measured ε is proportional to the capacitance of the washer. Then the absolute error of measurement in the case A is

$$\Delta\varepsilon = -\frac{\varepsilon(\varepsilon - 1)}{\ln(r_2/r_0)} \cdot \frac{\Delta r}{r_0}, \qquad (3)$$

and analogously in the case B.

For a 50 Ohm line and $\varepsilon = 10$, the multiplier in formulas before $\Delta r/r$ is close to 10. For a 1 % error in the inner radius of the sample, $r_0 = 1.52$ mm, the error of capacitance is 13 %, 10 % of error (0.15 mm) lead to 50 % in capacitance, Fig. 3. The curves in this figure are not straight lines as follows from (1) and (2): for their calculation the exact formula for two capacitors in series is used instead of the linearized expressions (1) and (2). This deviation from a straight lines becomes even more pronounced for bigger ε: for example, for $\varepsilon = 12$ and $\Delta r = 0.05$ mm, the actual correction is only 60% compared to the linearized formula (1). The fringe fields should be also taken into account (Fig. 4), these fields increase the gap capacity by a factor of $1 + \kappa p$ where $p = \Delta r/t$, t is thickness of the washer. The value of $\kappa = 5$ was found for $\varepsilon = 12$ by fitting the analytical solution to the results obtained with MWS. κ is smaller for bigger ε, for example, $\kappa = 3.5$ for $\varepsilon = 20$, because in this case the stray fields become less intense with distance from the gap.

A smaller value of capacitance will be interpreted as a smaller value of ε. This is just what is seen in the measurements for the samples with bigger gaps.

Deviation of inductance is not magnified as much as in the case of capacitance, for the case A it is:

$$\frac{\Delta L}{L_0} = -\frac{\mu - 1}{\mu \ln(r_2/r_0)} \cdot \frac{\Delta r}{r_0},$$

and analogously in the case B, so that errors in measurement of μ are not as big as in ε.

For a simplified guess of errors in the imaginary part of ε let us calculate the Poynting vector through the dielectric and through the gap using the real part of ε only. For example, in the case of the gap between the inner conductor and the washer (Fig. 2, A), the power fluxes through the gap and the dielectric are, respectively,

$$P_g = \varepsilon \int_{r_0}^{r_1} \frac{E_0 r_0}{r} \cdot \frac{H_0 r_0}{r} \cdot 2\pi r dr = 2\pi\varepsilon E_0 H_0 r_0^2 \ln\frac{r_0 + \Delta r}{r_0},$$
$$P_d = \int_{r_1}^{r_2} \frac{E_0 r_0}{r} \cdot \frac{H_0 r_0}{r} \cdot 2\pi r dr = 2\pi E_0 H_0 r_0^2 \ln\frac{r_2}{r_0 + \Delta r}, \qquad (4)$$

where E_0 and H_0 are the electric and magnetic fields on the surface of the inner conductor. Because of enhancement of electric field in the gap the fraction of power leaking through the gap is proportional to ε:

$$p_i = P_g/(P_d + P_g) \approx 1.2\varepsilon \cdot \Delta r/r_0 \qquad (5)$$

for the inner gap and $p_o \approx 1.2\varepsilon \cdot \Delta r/r_2$ for the gap between the washer and the outer conductor. Accordingly, the power loss in the dielectric will be decreased by the factor of $1 - p_i$ or $1 - p_o$ given the respective gaps. Fig. 5 illustrates the enhanced power flow through the gap.

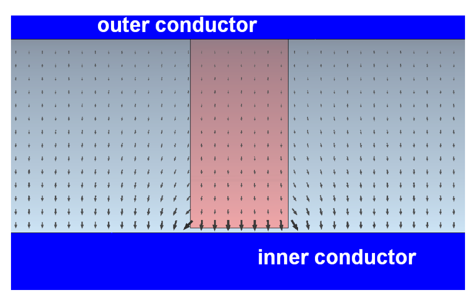

Figure 4: Fringe fields near the gap found with MWS.

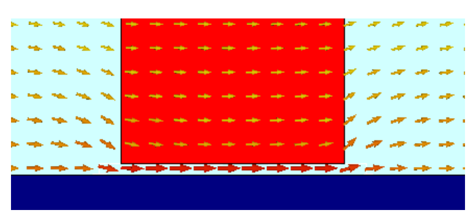

Figure 5: Power flow in the presence of a gap.

COMPARISON WITH SIMULATIONS

For comparison, actual values of real and imaginary parts of permittivity for washers made of silicon carbide (SiC) measured in the frequency range from 1 to 12.4 GHz were used. First of all, the algorithm for calculation of ε and μ from measured parameters was verified. For this purpose, for given values of ε and μ a geometry with a washer in a coaxial line (Fig. 1) was calculated and values of S-parameters were found. Then, these S-parameters were used for calculation of ε and μ with the same algorithm which was used for processing the measured S-parameters in actual measurements. With a number of meshcells about 100000 and time of calculation about 5 minutes, deviations of newly calculated Re ε and Im ε were less than 0.5 % and 5 %, respectively, from the input values in the frequency range from 1 to 12.4 GHz. SiC is not a magnetic material, deviation of calculated values of μ from $1 - i \cdot 0$ was about 0.01 for the real part and 0.001 for the imaginary part of μ. Then, the same calculations were done for a washer with a 0.05 mm gap from the inner and outer parts of the washer.

As can be seen from Fig. 6, the algorithm used for processing the measurements without a gap proved correct, and corrections found with MWS for a model with a gap are in a good agreement with the analytical solution. We used the exact solution for two capacitors in series and the correction for stray fields.

The solution for the imaginary part of ε (Fig. 7) is also in a good agreement for the case without a gap. However, too simplified solution in the presence of the gap gives 25 – 30 % higher value than the MWS results which we believe is correct with better accuracy. The error obviously goes from ignoring the reflected wave, standing wave within the washer, and the phase shift between voltage and current in the lossy dielectric. Nevertheless, even this solution is much better than the result without considering the gaps. Surprisingly, linearization of the ratio $p_i = P_g/(P_d + P_g)$ with P_g and P_d from equations (4) into the simplified form (5) is in a very good agreement with the MWS results. Of course, this is just an empirical fitting but we will use it if it works! However, we should check whether it works if the value of either real or imaginary part of ε is well away from one used in this case.

Actual samples of materials inserted in the measuring line have gaps from both inner and outer side. To calculate a correction in real life we generalize the upper model to a model with three capacitors in series: two empty and one filled with the measured material.

CONCLUSION

Lossy materials, ceramics and ferromagnetics, are used in the RF absorbers. Accuracy of measurements of the dielectric permittivity and magnetic permeability with help of transmission lines can be substantially improved if the defects of the samples are taken into account. It is shown that in the gaps between dielectric samples and the metal surface the enhanced electric field leads to an error in the real part but also the power flow through the gaps leads to a substantial error in the imaginary part of the permittivity. For the samples used in measurements with a coaxial line, it appeared easier to precisely measure the dimensions of the samples than accurately machine them with a needed tolerance. Corrections found analytically for both real and imaginary part of the permittivity are in a good agreement with simulations. Correction for the permeability are not as big as for permittivity and can be also taken into account.

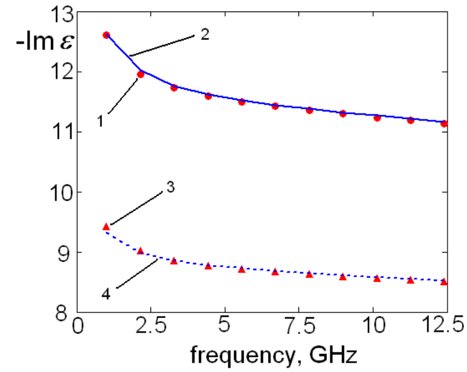

Figure 6: Re ε vs frequency. 1 and 2 – input and output values for the MWS calculations without a gap, 3 – value of Re ε calculated from the S-parameters found with MWS, 4 -analytically calculated values of the equivalent Re ε. Cases 3 and 4 are calculated for a gap of 0.05 mm on the inner side of the washer.

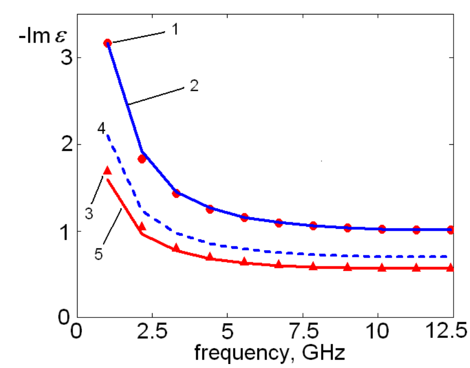

Figure 7: Curves 1–4 same as in Fig. 5 but for the Im ε. 5 – linearized analytical solution, see text.

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