

# EVALUATION OF ZERO-FAILURE DATA IN TRANSIENT IONIZING RADIATION BASED ON ORDERING METHOD IN THE SAMPLE SPACE

Bai Xiaoyan, Jin Xiaoming, Li Ruibin, Liu Yan, Ma Qiang, Qi Chao,  
State Key Laboratory of Intense Pulsed Radiation simulation and effect,  
Northwest Institute of Nuclear Technology, Xi'an, China

## Abstract

The conventional method for the evaluation of data in lot acceptance testing (LAT) of transient ionizing radiation is non-parametric method. But the evaluation results are very conservative. After the discovery of data in transient ionizing radiation belonging to one universal data model "case 1 interval censored data", ordering method in the sample space was introduced and applied to evaluate zero-failure data and was compared with non-parametric method both theoretically and via a practical LAT on QG- I. Through the comparisons, it is concluded that ordering method can expand the scope of dose rate corresponding to the same lower confidence limit. It improves data utilization and this improvement could have practical significance in LAT. It can reduce requirements for the radiation source and can also reduce the number of trials.

## INTRODUCTION

Either of the below two radiation sources can be used for dose-rate testing: a flash x-ray machine or an electron linear accelerator. 11/0 test, where 11 parts randomly selected from the lot are irradiated and tested at the specified dose rate level and where if no part fails the test the lot is accepted, is often used in LAT of transient ionizing radiation [1,2]. The conventional method for the evaluation of data in LAT is non-parametric method. But the radiation source which can be used for LAT in China has a limited capacity: smaller target face, poorer stability and repeatability. As a result, the dose rates received by different parts in a sample are often very different: the smallest may be lower than the specified level while the largest may be several times the specification. To meet requirements of LAT under this situation, classic non-parametric method [3] requires the dose rate of every part in the sample no less than the specified level. The requirement is obviously too conservative and undoubtedly increases test cost. The test information of the larger dose rate where parts survive is unutilized. There are a lot of researches [4,5,6] on mechanisms of transient ionizing radiation but few public reports on methods of data evaluation. We aim to find a suitable mathematical method which could make more use of test information.

There are many methods on processing zero-failure data [7]. We found that data in transient ionizing radiation could be classified into one universal data model "case 1 interval censored data" [8,9]. Hence ordering method in the sample space [8] was introduced to evaluate the

transient ionizing radiation data, which was accomplished by Chen Jiading and is widely used in storage reliability of weapons and equipments [8,10,11] but has not been applied to the radiation research. Compared with classic non-parametric method, ordering method makes more use of the test information and can expand the scope of dose rate corresponding to the same lower confidence limit. This improvement will have practical significance in LAT. A practical LAT on QG- I was taken as an example and lower confidence limits of survival probability were computed on the hypothesis of Weibull distribution and the results were compared with classic non-parametric method.

## THEORY OF ORDERING METHOD IN THE SAMPLE SPACE

The radiation source used in transient ionizing radiation works in pulsed conditions. After the test, results are some parts fail and some parts pass; moreover, the dose rates received by them are very different. We found that the data could be classified into universal data model "case 1 interval censored data", which is often encountered in researches of product storage reliability and infectious diseases.

Case 1 interval censored data can be described in a general form as below [8]. Suppose that the lives of  $n$  product  $X = (x_1, \Lambda, x_n)$  follow a distribution function  $F(x, \theta)$ , where  $\theta$  is the unknown parameter. For a series of observed time  $T = (t_1, \Lambda, t_n)$ , where  $t_i > 0$ , set  $y_i = I(x_i > t_i)$  ( $i = 1, \Lambda, n$ ), where  $y_i$  is 0 or 1 based on  $x_i > t_i$  or not. The observed data are known as case 1 interval censored data of  $X$ , which cannot be observed. There is an enormous amount of literatures [8,9,12] about the processing methods of these data, among which ordering method in the sample space can obtain the confidence limit of  $g(\theta)$  in the situations where there is no-restriction on the number in the sample or the number of failure devices. When  $g(\theta) = 1 - F(x, \theta)$ , it is survival probability. The method was initiated by started Buehler in 1950s and was generalized by Chen Jiading in 1990s and has been widely used in missile storage life but not in radiation researches.

There are basically four steps to obtain the lower confidence limit of  $g(\theta)$  with the confidence level  $1 - \alpha$  in the ordering method. The first step is to define an

ordering relation in sample space  $E = \{(y_1, \Lambda, y_n) : y_i = 0 \text{ or } 1\}$ . When either of the following two conditions is satisfied,  $(y_1, L, y_n) \geq (\bar{y}_1, L, \bar{y}_n)$  is determined:

$$\textcircled{1} \sum_{i=1}^n y_i > \sum_{i=1}^n \bar{y}_i ; \textcircled{2} \sum_{i=1}^n y_i = \sum_{i=1}^n \bar{y}_i \text{ and } \sum_{i=1}^n y_i t_i \geq \sum_{i=1}^n \bar{y}_i t_i .$$

The second step is to establish the likelihood function  $P_\theta(y) = \prod_{i=1}^n (F(t_i, \theta))^{1-y_i} \bullet (1 - F(t_i, \theta))^{y_i}$ .

Combing the above two steps, come the function  $G_n(y, \theta) = \sum_{y \geq \bar{y}} P_\theta(\bar{y}) = P_\theta(Y \geq y)$  which can be finally

computed. Then comes the last step: the lower confidence limit of  $g(\theta)$  can be expressed as [8,10]  $g_L(Y) = \inf\{g(\theta) : G_n(Y, \theta) > \alpha\}$ .

Ordering method is a parametric approach and hence the distribution type of  $X$  must be known to compute  $g_L(Y)$ . However, until now the proper statistical distribution describing the variation of data with the dose rate is still under discussion. We adopted a hypothesis of Weibull distribution based on below two reasons : firstly, Weibull distribution is widely used in reliability and life data analysis [6,8]; secondly, there were authors [13] suggesting that the burning data in transient ionizing radiation may follow a Weibull distribution, which is

$$F(t, \eta, \beta) = \begin{cases} 0 & t \leq 0 \\ 1 - \exp\left\{-\left(\frac{t}{\eta}\right)^\beta\right\} & t > 0, \end{cases} \quad (1)$$

where  $\eta$  and  $\beta$  are unknown scale and shape parameter.

In special cases where there are no failures, the lower confidence limit of survival probability has the following analytical expression [8,11] :

$$R_L(t) = \begin{cases} 0 & t > t_{(n)} \\ \alpha^{1/p} & t = t_{(n)}, p = \#(i, t_i = t_{(n)}) \\ \alpha^{1/f(m^*)} & \left(\prod_{i=1}^n t_i\right)^{1/n} < t < t_{(n)} \\ \alpha^{1/n} & 0 < t \leq \left(\prod_{i=1}^n t_i\right)^{1/n} \end{cases}, \quad (2)$$

where  $1 - \alpha$  is the confidence level,  $t_{(n)} = \max(t_1, t_2, \dots, t_n)$ ,  $\#A$  denotes the number of elements in collection  $A$ ,  $f(m) = \sum_{i=1}^n (t_i / t)^m$ ,  $m^*$  is the root of equation(3).

$$\sum_{i=1}^n \left(\frac{t_i}{t}\right)^m \ln\left(\frac{t_i}{t}\right) = 0, \quad m > 0 \quad (3)$$

If the range of shape parameter  $\beta$  has been known, i.e.  $\beta \in [\beta_1, \beta_2]$  ( $0 < \beta_1 < \beta_2 < \infty$ ), then a larger lower confidence limit  $\tilde{R}_L(t)$  can be obtained [14]

$$\tilde{R}_L(t) = \alpha^{\frac{1}{f(m_0)}}, \quad (4)$$

Where  $f(m) = \sum_{i=1}^n (t_i / t)^m$  and

$$m_0 = \begin{cases} \beta_1 & 0 < t \leq \left(\prod_{i=1}^n t_i\right)^{1/n} \\ \beta_2 & t \geq t_{(n)} \\ \max(\beta_1, \min(m^*, \beta_2)) & \left(\prod_{i=1}^n t_i\right)^{1/n} < t < t_{(n)} \end{cases}. \quad (5)$$

The definitions of  $t_{(n)}$  and  $m^*$  are exactly the same as they are in equation 2.

Next we compare the results computed by non-parametric method and ordering method when none of the  $n$  parts in a sample fails. Non-parametric method can obtain that  $R_L(t) \approx \alpha^{1/n}$  if  $t \leq \min(t_1, t_2, \dots, t_n)$ . If there is no knowledge on  $\beta$ , i.e.  $\beta > 0$ , ordering method can obtain  $R_L(t) \approx \alpha^{1/n}$  if  $t \leq t_c$  ( $t_c = \left(\prod_{i=1}^n t_i\right)^{1/n}$ ), which is the geometric mean.

Obviously ordering method significantly expands the scope of  $t$  corresponding to  $R_L(t) \approx \alpha^{1/n}$ . If  $\beta \geq \beta_1$  ( $\beta_1 > 0$ ), ordering method can obtain  $R_L(t) = \alpha^{1/f(\beta_1)}$  if  $t \leq t_c$ . In this condition, apart from the previous advantage, ordering method also improves the lower confidence limits because of  $R_L(t) = \alpha^{1/f(\beta_1)} > \alpha^{1/n}$  which is deduced from the facts that  $f(m)$  is an increasing function of  $m$  when  $t \leq t_c$  and the minimum of  $f(m)$  is  $n$ . Hence it can be clearly seen that ordering method has distinct advantages in all conditions compared to non-parametric method.

### TRANSIENT IONIZING RADIATION TEST

Above is a theoretical comparison, next we apply both methods to a practical LAT on QG- I and compare the results. Below are some details about the test. The specified dose rate level  $\dot{D}_0$  and the failure criteria were

specified before the test; 11 samples were randomly selected from the inspection lot ; oscilloscopes were used to record the transient signal and the recovery period of the output voltage of devices; thermo-luminescence dosimeters were used to determine absorbed doses of the devices and a scintillator phototube was used to measure pulse width. There were no failures and the absorbed dose

rate of every device was  $\dot{D}_i (i=1,2,\dots,11)$ , which was normalized by  $\dot{D}_0$  and then listed in Table 1.

Table1: Normalized Dose Rates Received by Devices

<b>Part Number</b>	1	2	3	4	5	6
<b>Dose Rate</b>	2	2.4	1	1.5	1	2.3
<b>Part Number</b>	7	8	9	10	11	
<b>Dose Rate</b>	1.9	1.8	1.8	1.8	1.4	

### RESULTS AND DISCUSSION

Based on the data in Table 1, the lower confidence limits of survival probabilities at different dose rates were computed via both non-parametric method and ordering method at 0.1 significance level. Let  $d$  represents normalized dose rate, i.e.  $d = \dot{D}/\dot{D}_0$ .

Via no-parametric method, the result was: if  $d \leq 1$ ,  $R_L(d) = 0.8$  was obtained.

Via ordering method, the range of  $\beta$  should be supposed. In many engineering practices people know  $\beta$  is in a certain interval. For example, the life of bearings follows the Weibull distribution and the shape parameter is in the vicinity of 1.1. But until now we lack such knowledge in transient ionizing radiation. In this situation, we computed  $R_L(d)$  based on equation (2) in the case  $\beta \in (0, \infty)$ . We obtained  $R_L(d) = 0.8$  if  $d \leq 1.65$ , where 1.65 was the geometric mean of the 11 data in Table 1.

Compared with non-parametric method, ordering method expands the range of dose rate corresponding to the same confidence limit. It improves data utilization and this improvement can make sense to LAT. To meet the requirements in LAT, no-parametric method needs the dose rate of every part in the sample must be equal to or larger than the specified level but ordering method only needs geometric mean of the dose rates of all parts above the specified level. At present, the radiation source in China has a limited capacity and it is very hard to guarantee the dose rate received by every part in only one trial no less than the specified level unless the part size is very small. Therefore ordering method can reduce requirements for the radiation source and it is possible to reduce the number of trials.

As shown earlier in this article, the lower confidence limit will be larger if there is a restriction on  $\beta$  and therefore the number in a test sample can reduce if LAT requirements are the same.

### CONCLUSION

Data in LAT of transient ionizing radiation can be classified into one universal model “case 1 interval censored data” and hence ordering method in the sample space was introduced and applied to evaluate zero-failure data. Compared with the classic non-parametric method, ordering method improves data utilization and expands the range of dose rate corresponding to the same lower confidence limit. The improvement could have the practical significance in LAT. It can reduce requirements for the radiation source and may reduce the number of trials.

In this article, no restriction was put on  $\beta$ . If we can obtain a general but narrower range of the shape parameter based on the basic mechanism of transient ionizing radiation or the analysis of historical data or the testings of different types of electronic parts, then the lower confidence limit will increase, which would have a greater significance in practical work.

### REFERENCES

- [1] GJB548B-2005~5005.2, *Qualification and Quality Conformance Procedures*.
- [2] MIL-STD-883H~5005.15, *Qualification and Quality Conformance Procedures*.
- [3] J.F. Lawless, *Statistical models and methods for lifetime data*, (Beijing: Chinese statistics press, 1998), 408.
- [4] G.Z. Wang et al., *High Power Laser and Particle Beams*, 21 (2009) 5.
- [5] C.H. Du et al., *High Power Laser and Particle Beams*, 18 (2006) 2.
- [6] R.B. Li et al., *Science China Technological Sciences*, 55 (2012) 11.
- [7] M. Han, *Advances in Mathematics*, 31 (2002) 1.
- [8] J.D. Chen, *Survival analysis and reliability*. (Beijing: Peking University press, 2005), 215.
- [9] J. Huang and J.A. Weller, *Interval censored survival data: A Review of Recent Progress. Lecture Notes in Statistics*. (New York:Springer-Verlag, 1996)
- [10] The Research Group of Life and Reliability, *Chinese Journal of Applied Probability and Statistics*, 6 (1990) 4.
- [11] J.D. Chen et al., *Acta Mathematicae Applicatae Sinica*, 18 (1995) 1.
- [12] M. Banerjee and J.A.Weller, *Scandinavian Journal of Statistics*, 32 (2005) 3.
- [13] D.W. Egelkrout, *IEEE Trans On Nucl .Sci*, 25 (1978) 6.
- [14] Z.P. Wei and J.D. Chen, *Acta Mathematicae Applicatae Sinica*, 29 (2006) 1.