THEORY CALCULATION OF PASER IN GAS MIXTURE ACTIVE MEDIUM*

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Abstract

In the PASER (particle acceleration by stimulated emission of radiation), the energy stored in an active medium transferred directly to the electron beam passing through in discrete amounts by emitting a photon when the bounded electron returns from upper to lower energy state. In this paper, the wake-field generated by a bunch of electrons traversing in an active medium has been discussed. The calculations of the development of amplitude for gas mixture active medium about CO_2 and ArF were made respectively. The results show that the gradient can reach around 1GeV/m. In addition, the electron energy gain occurring as a train of electron micro-bunches traversing in gas mixture was analyzed by a two dimension model. The train of micro-bunches can obviously gain energy from the active medium and the energy exchange is linearly proportional to the interaction length d. The influence of the bunch figure and other quantities on the energy exchange occurring as a train of electron micro-bunches traversing CO_2 gas mixture were investigated. The results illustrate that maximum electron energy gain can be obtained by the train of micro-bunches with optimized parameters.

INTRODUCTION

In 1958 Townes [1] demonstrated that energy stored in atoms may be used for amplification of radiation by a series of multiple collisions of photons with excited atoms, which is well-known as light amplification by stimulated emission of radiation (LASER) (Fig. 1a). Similarly to the LASER, in PASER process(Fig. 1b) [2], the outer electron in the excited atom dropps to the lower energy-state and delivered the energy to the free electron passing near the excited atom, so the electron is accelerated. PASER proof-of-principle experiment in CO_2 gas mixture active medium has successfully been made at the Brookhaven National Laboratory Accelerator Test Facility (BNL-ATF) [3, 4] which shows that a fraction of accelerated electrons have gained more than 200 KeV each in a less than 40 cm long interaction region and the result has good agreement with theory simulation.

In this paper, the calculations about PASER for relativistic electrons in two different cases were made. One is the amplification of a wake field generated by a charged bunch in active medium when the bunch is not modulated. In this case, the wake is amplified to the point that the field-medium interaction reaches saturation where the accelerated bunch should be located. Another is the energy exchange occurring as a train of electron micro-bunches which is modulated in the wiggler or other devices traverses in gas mixture active medium. i.e., the distance between two adjacent micro-bunches of the macro-bunch which consist M micro-bunches is corresponding to the wavelength of the active medium. We use a two-dimensional analytical model to evaluate the influence of the bunch figure and other quantities on the energy exchange. The various parameters about the train of micro-bunch and active medium were optimized.

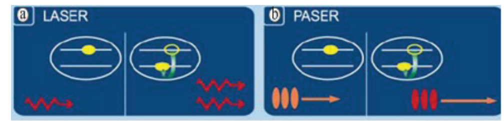

Figure 1: Illustration of the light-electron-atom interaction.

WAKE FIELD GENERATED BY AN ELECTRON BUNCH IN AN ACTIVE MEDIUM

Consider a driving bunch consist of N electrons moves in an active medium, after a series of deduction, the electric field on axis far from the driving bunch can be written as[5]:

$$E_z(r \approx 0, \tau) = \frac{Q}{2\pi\epsilon_0 {R_b}^2}(\frac{2\omega_i}{\omega_{p,res}})^2 e^{\omega_i \tau}\cos(\omega_0 \tau) \quad (1)$$

In which R_b is the bunch radius, Q=Ne, ω_0 is the resonance frequency and ω_i is the corresponding growth rate. $\omega_{p,res}$ is the electronic angular plasma frequency of the atoms that have resonance at ω_0. When considered nonlinear effects [6] the growth rate can be written as [5]:

$$\omega_i(\tau) \approx \overline{\omega_i} e^{-\tau[\overline{E}(\tau)/E_{sat}]^2/2T_2} = \overline{\omega_i} F(\overline{E}, \tau) \quad (2)$$

So the amplitude far away from the driving bunch reads

$$\overline{E} = E_0 F^2(\overline{E}, \tau) e^{\overline{\omega_i}\tau F(\overline{E},\tau)} \quad (3)$$

Wherein $E_0 = (Q/2\pi\epsilon_0 {R_b}^2)(2\overline{\omega_i}/\omega_{p,res})^2$, $E_{sat} \approx \sqrt{3/T_1 T_2}(\hbar/p)$. T_1 is the decay time due to inelastic collisions between atoms, and T_2 is the relaxation time. For typical values [5], $\sqrt{T_1 T_2}$ =0.1ns, $p = (1.6 \times 10^{-19} C) \times (1 \times 10^{-12} m) = 1.6 \times 10^{-31}$Cm, E_{sat}=11.42MV/m. $\lambda = 0.5\mu$m, $\omega_i = 0.005\omega_0$. The calculation result shows that the accelerating gradient can reach more than 1GeV/m [5].

* Work supported by National Natural Science Foundation of China (10675116) and Major State Basic Research Development Programme of China (2011CB808301)
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We make calculations about the development of the amplitude of CO_2 gas mixture and ArF gas mixture respectively (Fig. 2a and Fig. 2b). We find that in the CO_2 gas mixture active medium, the wake amplitude can reach 0.5GeV/m, and in ArF gas mixture active medium the accelerating gradient can reach more than 5GeV/m.

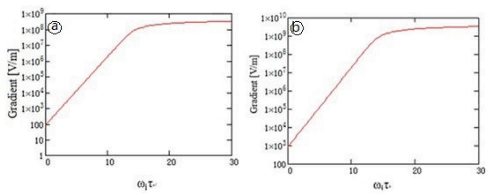

Figure 2: a.The development of the amplitude as a function of $\omega_i\tau$, $\lambda = 10.2\mu$m,T_2 =2ns for CO_2 gas mixture.b.The development of the amplitude as a function of $\omega_i\tau$, $\lambda = 193\mu$m,T_2 =4ns for ArF gas mixture .

KINETIC ENERGY GAINED OF A MACRO-BUNCH CONSISTING M MICRO-BUNCHES IN GAS MIXTURE ACTIVE MEDIUM

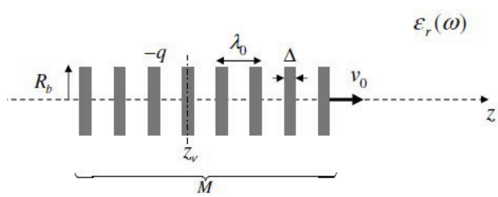

Figure 3: A macro-bunch consisting of a train of M micro-bunches traversing a medium characterized by its dielectric coefficient $\epsilon_r(\omega)$. Each micro-bunch carries a charge -q.

As shown in Fig. 3, consider a macro-bunch consisting M micro-bunches, each one being azimuthally symmetric, having a radius R_b and a length Δ, carrying a charge $-q$ and moving at a velocity v_0, the distance between two adjacent micro-bunches is λ_0. Then the total energy exchange between the macro-bunch and the active medium during the passage d is given by

$$W = \frac{Q^2 d}{4\pi\epsilon_0{\lambda_0}^2\beta^2}\frac{\Omega_p^2}{\Omega_R}F_{\|}(\frac{\Omega_R}{2\beta},\overline{\Delta},M)Re[\Omega_+F_{\perp}(\frac{\Omega_+\overline{R_b}}{\beta})] \tag{4}$$

Wherein $\overline{R_b} = R_b/\lambda_0$,$\overline{\Delta} = \Delta/\lambda_0$,$\Omega = \omega\lambda_0/c$, $\phi = \Omega/2\beta$, are the normalized quantities, $F_{\|}(\phi,\overline{\Delta},M) = sinc^2(\phi\overline{\Delta})sinc^2(\phi M)/sinc^2(\phi)$,$F_{\perp}(u) = (2/u^2)[1 - 2I_1(u)K_1(u)]$ are the longitudinal and transverse form factors, $\Omega = \Omega_\pm = j\alpha\pm\sqrt{2\pi^2+\Omega_p^2-\alpha^2} = j\alpha\pm\Omega_R$ is the poles of the dielectric function, $\alpha = \lambda_0/cT_2$, $\Omega_p = \omega_0\lambda_0/c$, wherein ω_0 is the resonance frequency of the medium, ω_p is the plasma frequency, $\omega_p^2 = e^2n/m\epsilon_0$, with m being the rest mass of the electron and n representing the population density of the resonant atoms. For an excited medium, when the population density is inverted ($n < 0$), the plasma frequency is negative ($\omega_p^2 < 0$). Obviously, the total energy exchange is proportional to Ω_p^2. Because of the population in the medium is inverted, Ω_p^2 is negative, so the total energy exchange W is negative, therefore, the total kinetic energy gain of the macro-bunch ($\Delta E_k = -W$) is positive, which indicates that energy is transferred from the active medium to the macro-bunch. So the relative kinetic energy change of the macro-bunch $\frac{\Delta E_k}{N_{el}mc^2(\gamma-1)}$reads

$$\overline{\Delta E_k} = \frac{4N_{el}d(\pi{r_e}^2)}{\beta^2(\gamma-1)\Omega_R}\frac{w_{act}}{\hbar\omega_0}\times F_{\|}(\frac{\Omega_R}{2\beta},\overline{\Delta},M)Re[\Omega_+F_{\perp}(\frac{\Omega_+\overline{R_b}}{\beta})] \tag{5}$$

Where $w_{act} = -n\hbar\omega_0$ is the density of the energy stored in the medium. Based on the model expressed above, we make some calculations of CO_2 gas mixture ,here, the resonant wavelength is 10.2μm, and the spontaneous decay coefficient T_2 =2ns. Δ = $0.1\lambda_0$ [7]. From equation (4), we know that the energy exchange is linearly proportional to the interaction length d, in the following calculations, the interaction length is set to be 0.5m, ,and the total number of macro-bunch is assumed to be constant $N_{el} = 10^{10}$. Firstly, we consider the relative kinetic energy change of the macro-bunch versus the energy density stored in the CO_2 gas mixture at the resonance frequency. The kinetic energy of each single electron is 45MeV $E_k = mc^2(\gamma-1)$. It illustrates that $\overline{\Delta E_k}$ oscillates as a function of w_{act} (Fig. 4), and the energy densities for which the relative change vanishes correspond to the zero points of the longitudinal form factor. It is shown from Fig. 4a that the optimized w_{act} for the largest relative change in the kinetic of the macro-bunch is R_b independent, but increases with the decreasing of M which shows in Fig. 4b, the reason for this is that when the number of the electrons in the macro-bunch is constant, with the increasing of the electron beam radius the charge density decreases, so the optimal relative kinetic energy change drops, similarly, when the number of the electrons in the macro-bunch is constant, with the decreasing of the micro-bunch numbers, the total amount of charge in each micro-bunch increases, so the optimal relative kinetic energy change increases.

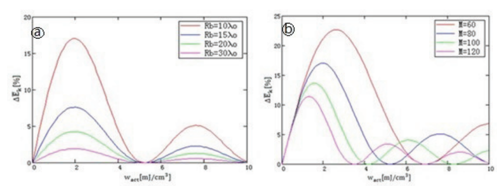

Figure 4: a.The relative kinetic energy change of the macro-bunch versus the energy density stored in the medium with R_b as a parameter and M=80.b.The relative change in the kinetic energy of the macro-bunch versus the energy density stored in the medium with M as a parameter and $R_b = 10\lambda_0$.

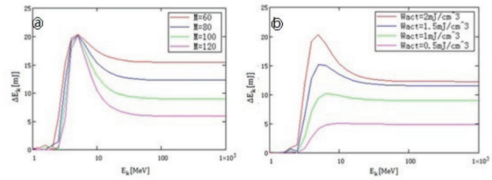

Figure 5: a. The macro-bunch kinetic energy increase versus the average kinetic energy of the electrons with the number of the micro-bunches as a parameter and w_{act} =2000J/m^3.b.The macro-bunch kinetic energy increase versus the average kinetic energy of the electrons with the energy density stored in the medium as a parameter and M=80.

Secondly, the kinetic energy increase versus the initial kinetic energy of the electrons in the relativistic regime was examined (Fig. 5). The macro-bunch radius is set to be $R_b = 10\lambda_0$, and the energy density $w_{act} =$ 2000J/m^3 which is the optimize values obtained from Fig. 4a. It illustrates that the kinetic energy increasing of the electron beam is γ independent for relativistic electrons, as any viable acceleration structure must exhibit. The kinetic energy gain peaks is M independent and is affected by the energy density stored in the medium w_{act} as a function of γ.

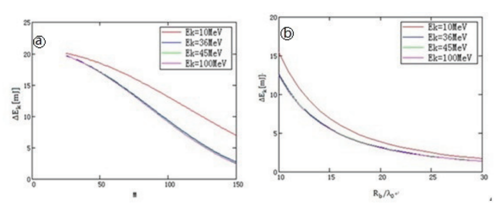

Figure 6: a.The macro-bunch kinetic energy increase versus the number of micro-bunches with E_k as a parameter, $\Delta = 0.1\lambda_0$ and $R_b = 10\lambda_0$.b.The kinetic energy increase of the macro-bunch versus the length of the micro-bunch radius with E_k as a parameter, $\Delta = 0.1\lambda_0$ and M=80.

Thirdly, the macro-bunch kinetic energy increase versus the bunch configurations was calculated. The energy density stored in the medium at the resonance frequency is set to be w_{act} =2000J/m^3. In Fig. 6a, the energy gain drops as the number of micro-bunches increases. As we have already indicated, the acceleration is due to a stimulated process, and therefore, the reason for this is that the energy stored in the medium is maintained constant, with the number of micro-bunches increases, the amount of charge in each micro-bunch decreases. Fig. 6b illustrates that the energy gain decreases as the length of the micro-bunch radius increase.

Finally, the macro-bunch kinetic energy increase versus the bunch configurations while the amount of the charge in the micro-bunch is constant was discussed. Assuming the total amount of the electrons in each micro-bunch is 5×10^7, other quantities are the same as above. From Fig. 6b, it can be seen that for different M, the peak value of the relative kinetic energy change is the same, so it is M independent. Furthermore, the optimized M decreases with the increasing of the w_{act}. Fig. 7a shows that the energy gain is not affected by the pulse duration for relativistic energies.

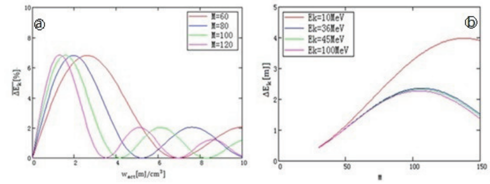

Figure 7: a.The relative kinetic energy change of the macro-bunch versus the energy density stored in the medium at the resonance frequency with M as a parameter and $E_k = 45$MeV.b.The macro-bunch kinetic energy increase versus the number of micro-bunches with E_k as a parameter, and w_{act} =2000J/m^3.

CONCLUSIONS AND DISCUSSION

The calculations above show that the accelerating gradient generated by a small bunch of electrons in gas mixture active medium may reach 1GeV/m or more. When the total number of the electrons in the train of micro-bunches is constant, the optimized energy density stored in the active medium for the largest relative change in the kinetic of the macro-bunch is R_b independent, but increase with the decreasing of the number of the micro-bunch M. For relativistic electrons, the kinetic energy gain peaks which affected by the energy density stored in the medium as a function of γ is M independent. In practice, the length of the micro-bunch is limited primarily by the modulation process and by the space-charge effects within the micro-bunch. When the amount of the charge in the micro-bunch is constant, the peak value of the relative kinetic energy change is not affected by M, each micro-bunch interacts with the medium independent of the others, but the optimized number of micro-bunch decreases with the increasing of the stored energy density.

ACKNOWLEDGMENT

Thanks Prof. Levi Schachter for his useful conversations and assistance which made great help for me.

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ISBN 978-3-95450-122-9