PLASMA EFFECT IN THE LONGITUDINAL SPACE CHARGE INDUCED MICROBUNCHING INSTABILITY*

Dazhang Huang†, Qiang Gu, SINAP, Shanghai, 201800, China, P. R.
King-Yuen Ng‡, Fermilab, Batavia, IL 60510, USA

Abstract

Microbunching instability (μBI) usually exists in the LINAC of a free electron laser facility. If it is not well-controlled, the beam quality will be seriously damaged and the machine will not operate properly. In many cases, the longitudinal space charge (LSC) is a dominant factor that generates the instability; therefore its contribution must be studied in detail. The current model of the LSC impedance [1] derived from the fundamental electromagnetic theory [2] has been widely used to explain the physics of the LSC-induced μBI. [3] However, in the case of highly bright electron beams, the plasma effect may also play a role. In this article, the basic model of the LSC impedance including the plasma effect is constructed by solving Vlasov and Poisson equations in the 6-D phase space, and the investigations are carried out to study the modification to the instability gain. The solution indicates that the μBI gain depends not only on the spatial information of the beam, but also on the velocity (momentum) and time information. The form of the solution is also consistent with our expectation and is explainable.

INTRODUCTION

The possibility of oscillation in a plasma due to local separation of charges and the consequent restoring forces was discussed by J. D. Jackson long time ago. [4] The theory is based on a neutral plasma, which has both positively (ion) and negatively (electron) charged components. For a charged particle beam in an accelerator, although it is not neutral in terms of charges, there is still density fluctuation due to the graininess of the individual particles — in our case, the individual electrons. Such graininess is usually smoothed out in the fluid model and ignored in most computations. In a highly intensive beam, however, it may introduce the "plasma-like" oscillation (for convenience, "plasma oscillation" is used hereafter), and must be investigated in details in order to reveal its magnitude and to discover its physics. Similar discussions in the 2-D phase space for this effects on the free electron laser have been addressed by Kim, et al. [5]

In this article, we start our discussions in the 6-D phase space by employing Vlasov and Poisson (Gauss) equations, which describe the evolution of the distribution function of the electron bunch and the electric field induced by the charge distribution. We then use a method similar to Jackson's [4] to linearize the Vlasov equation and obtain the solution of the initial-value problem. The solution includes the contributions from both the perturbed and unperturbed parts of the initial distribution. As the result, the contribution from the velocity distribution is also included. From the solution, we see that the plasma oscillation introduces a "relative dielectric factor (permittivity)" ϵ_r, and also an extra factor coming from the initial velocity distribution. At last, the form of the LSC impedance obtained with Klimontovich particle distribution is given and discussions are made.

SOLUTION OF INITIAL-VALUE PROBLEM

We carry out the investigations with the equations describing the evolution of beam distribution under the influence of the electromagnetic force. The discussion is in laboratory frame hereafter. In cylindrical coordinate system, the linearized Vlasov-Poisson equation and the Poisson (Gauss) equation can be written:

$$\frac{\partial f_1}{\partial t} + v_z \frac{\partial f_1}{\partial z} + v_\perp \frac{\partial f_1}{\partial r} - \frac{eE_z}{\gamma m}\frac{\partial f_0}{\partial v_z} + \frac{F_\perp}{\gamma m}\frac{\partial f_0}{\partial v_\perp} = 0, \quad (1)$$

$$\frac{\partial E_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rE_r) = -\frac{e}{\epsilon_0}\int f_1 dv_z, \quad (2)$$

where $-e$ is charge of an electron, $f(t,\vec{r},\vec{v}) = f_0(t,\vec{r},\vec{v}) + f_1(t,\vec{r},\vec{v})$, with f_0 being the unperturbed background of the beam and f_1 the density perturbation due to plasma oscillation. We assume that f_0, f_1 and $\vec{E}$ have no azimuthal dependence, which is reasonable. The Gauss's law or Poisson equation, Eq. (2), will be solved in the particles' rest frame and then Lorentz transformed to the lab frame in the next section.

Since the transverse velocity $v_\perp$ is small, we can assume $v_z \approx v$ and $F_\perp \ll F_z$. Then Eq. (1) simplifies to

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial z} - \frac{eE_z}{\gamma m}\frac{\partial f_0}{\partial v} = 0. \quad (3)$$

Let us focus on Eqs. (3) and (2). Following Jackson, [4] we perform Fourier transform in z and Laplace transform in t on Eq. (3), and integrate by parts to obtain

$$\int dz \left[e^{-ikz+i\omega t} f_1(v,z,t) \right]_{t=0}^{t=\infty} + \int_{-\infty}^{\infty} dz \int_0^{\infty} e^{-ikz+i\omega t} dt \times \left[(-i\omega + ikv) f_1 - \frac{e}{\gamma m}\frac{\partial f_0}{\partial v} E \right] = 0. \quad (4)$$

* Work supported by National Science Foundation of China (NSFC), Project No. 11275253, and US DOE, contract DE-FG02-92ER40747.
† huangdazhang@sinap.ac.cn
‡ ng@final.gov

For ω in the upper half plane, the upper limit of the first term on the right hand side of Eq. (4) vanishes as $t \to \infty$. We have then the solution in (v, ω, k) space,

$$f_1(v,\omega,k) = \frac{1}{i(kv-\omega)}\left[\Phi(v,k) + \frac{e}{m}\frac{\partial f_0}{\partial v}E(\omega,k)\right], \tag{5}$$

where

$$\Phi(v,k) = \int_{-\infty}^{\infty} dz e^{-ikz} f_1(v,z,t=0). \tag{6}$$

Both Eq. (5) and Eq. (6) form the solution depending on the initial-value of the density perturbation. If we perform inverse Fourier transform on ω, we will obtain the density perturbation $f_1(v,t,k)$ at later time, which represents the time revolution of the density fluctuation. In the regular LSC theory, the density fluctuation is neglected. However, it will be taken into account under certain conditions in the following discussions.

ELECTRIC FIELD INDUCED BY LSC

Based on the classical electromagnetic theory, the solution of Eq. (2) can be written as

$$E_z(\vec{x}) = \frac{e\lambda}{4\pi\epsilon_0}\int G(\vec{x},\vec{x}')\rho(\vec{x}')d^3\vec{x}', \tag{7}$$

with the Green function

$$G(\vec{x},\vec{x}') = \frac{(z-z')\gamma}{[(x-x')^2+(y-y')^2+(z-z')^2\gamma^2]^{3/2}}. \tag{8}$$

Here the same notations of Venturini's [1] have been used, with beam density $\lambda\rho(x,y,z)$, uniform linear density λ, $\rho(x,y,z) = \rho_\perp(x,y)\rho_z(z)$, and normalization $\int \rho_\perp(x,y)dxdy = 1$.

In cylindrical coordinates, the Green function can be expanded as [2]

$$G(\vec{x},\vec{x}') = -\frac{i}{\pi\gamma^2}\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}\int_{-\infty}^{\infty} kdk e^{ik(z-z')} \times I_m\left(\frac{kr_<}{\gamma}\right)K_m\left(\frac{kr_>}{\gamma}\right), \tag{9}$$

where $r_<$ and $r_>$ denotes the smaller and larger between r and r', respectively, and I_m and K_m are the modified Bessel function of the first and second kind. Thus the longitudinal electric field E_z in the k-space becomes

$$E_z(k) = i\frac{e}{4\pi\epsilon}\frac{\lambda}{\pi\gamma^2}\sum_{m=-\infty}^{\infty}\int dV'\rho(r',\phi',z')e^{im(\phi-\phi')} \times ke^{-ikz'}I_m^< K_m^>. \tag{10}$$

In our study, the unperturbed (linear) density distribution serves as the smooth background, therefore it does not contribute to the longitudinal electric field at all. For this reason, only the perturbed (non-linear) part (Eq. (5)) plays a role. Assuming that the beam is of radius r_b and has a uniform transverse distribution $f_\perp = 1/\pi r_b^2$ when $r \le r_b$, we write the beam distribution as $\rho(r,\phi,z) = f_\perp f_1(v,z,t=0)$. For an observation point located on axis (only $m=0$ term contributes), [1] finally we obtain

$$E(\omega,k) = \frac{e\lambda}{2\pi\epsilon_0\epsilon_r\pi k r_b^2}\int_W dv \times \int_{-\infty}^{\infty} dz \frac{e^{-ikz}f_1(v,z,t=0)}{kv-\omega}\Big[1-\xi K_1(\xi)\Big], \tag{11}$$

where $\xi = kr_b/\gamma$, and

$$\epsilon_r = 1 - \frac{e^2\lambda}{2\pi\epsilon_0\gamma m k^2}\int_W \frac{\partial f_0}{\partial v}\frac{dv}{v-\omega/k} \tag{12}$$

is the relative dielectric factor (permittivity). The path of integration W is from $v = -\infty$ to ∞ passing below the pole $v = \omega/k$. This path comes from the analytic continuity from the upper ω-half-plane to the whole ω-plane. Introducing the frequency of plasma oscillation in lab frame, $\omega_p = \sqrt{e^2\lambda/2\pi\epsilon_0\gamma m}$, Eq. (12) can be written as

$$\epsilon_r = 1 - \frac{\omega_p^2}{k^2}\int_W \frac{\partial f_0}{\partial v}\frac{dv}{v-\omega/k}. \tag{13}$$

Equation (13) is also called the dispersion relation, it is a function of the wavenumber k of the density fluctuation. Equation (11) is the expression of the longitudinal electric field induced by the LSC under the influence of density fluctuation (plasma oscillation). Apparently, it includes the contribution due to the velocity distribution of the beam.

In most of the cases, where the momentum and location of the electron are decoupled, the perturbation can also be written as $f_1(v,z,t=0) = f_{v1}(v,t=0)f_{z1}(z,t=0)$. Equation (11) becomes

$$E(\omega,k) = \frac{e\lambda\big[1-\xi K_1(\xi)\big]}{\epsilon_0\epsilon_r\pi k r_b^2}\int_W dv\frac{f_{v1}(v,t=0)}{kv-\omega} \times \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikz}f_{z1}(z,t=0)dz. \tag{14}$$

Equation (14) decouples the contributions from the beam density distribution and velocity/momentum distribution.

INFLUENCE ON μBI

In this section, we start our preliminary discussions on the effects of the modified LSC impedance in microbunching instability. As already known, the gain of μBI due to the LSC impedance (linear in beam current) reads [6]

$$G = Ck|R_{56}|\frac{I_0}{\gamma_0 I_A}\frac{|Z(k)|}{Z_0}\exp\left(-\frac{C^2k^2R_{56}^2\sigma_\gamma^2}{2\gamma_0^2}\right), \tag{15}$$

where C is the compression factor of a bunch compressor (chicane), R_{56} is the transport matrix element of the whole bunch compressor, and σ_r is the uncorrelated energy spread. In Eq. (15), one can see that the gain is proportional to the absolute value of LSC impedance per unit length $Z(k)$.

ISBN 978-3-95450-122-9

The impedance per unit length of the longitudinal space charge is defined as

$$Z(k) = -E_z(k)/I(k), \tag{16}$$

where $I(k)$ is the Fourier transform of the beam current, i.e., $I(k) = ec\lambda\rho_z(k)$. Note that in our discussion $\rho_z(k) = (2\pi)^{-1}\int e^{-ikz} f_1(z, t = 0)dz$. Based on Eq. (14), we can derive the LSC impedance in the existence of density fluctuation:

$$Z(\omega, k) = -\frac{Z_0}{\pi\gamma\epsilon_r r_b}\frac{1-\xi K_1(\xi)}{\xi}\int_W \frac{f_1(v, t=0)dv}{kv-\omega}. \tag{17}$$

Comparing with the regular expression of the LSC impedance without density fluctuation, [1, 3] we can see that the difference comes from the relative dielectric factor ϵ_r and the initial velocity perturbation $f_1(v, t = 0)$. Moreover, it is also a function of the oscillation frequency ω.

To illustrate the problem, plugging in Eq. (11) with the Klimontovich particle distribution at $t = 0$ as the perturbation, which is

$$\begin{aligned}\rho(v, z, t = 0) &= \frac{1}{\pi r_b^2} f_1(v, z, t = 0)\\ &= \frac{1}{\pi r_b^2}\sum_{j=1}^{N_e}\delta(v - v_j^0)\delta(z - z_j^0)\delta(t)\end{aligned} \tag{18}$$

where N_e is the total number of the perturbed electrons.

There are poles, $\epsilon_r(k, \omega) = 0$ and $\omega = kv_j^0$, enclosed by the path of integration over ω. Considering the low energy limit, where $\omega_p \gg kv_j^0$ and $\epsilon_r \approx 1 - \omega_p^2/\omega^2$, and carry out the integral by employing the residual principle, we have:

$$\begin{aligned}E(k, t) &= \frac{e\lambda}{2\pi\epsilon_0\pi k r_b^2}\left[1 - \xi K_1(\xi)\right]\times\\ &\int_W d\omega \frac{e^{-i\omega t}}{\epsilon_r(k, \omega)}\sum_{j=1}^{N_e}\frac{e^{-ikz_j^0}}{kv_j^0 - \omega}\\ &\approx \frac{ei}{\epsilon_0\pi k r_b^2}\left[1 - \xi K_1(\xi)\right]\times\\ &\left[\sum_{j=1}^{N_e}\frac{e^{-ikz_j^0}\omega_p}{2}\left(\frac{e^{-i\omega_p t}}{\omega_p - kv_j^0} + \frac{e^{i\omega_p t}}{\omega_p + kv_j^0}\right)\right.\\ &\left.+\sum_{j=1}^{N_e}\frac{e^{-ik(z_j^0+v_j^0 t)}}{\epsilon_r(k, kv_j^0)}\right]\\ &\approx \frac{ie}{\epsilon_0\pi r_b^2 k}\left[1 - \xi K_1(\xi)\right]\cos(\omega_p t)\sum_{j=1}^{N_e} e^{-ikz_j^0}\end{aligned} \tag{19}$$

In the last step of Eq. (19), $\omega_p \gg kv_j^0$ is applied. Thus the corresponded impedance becomes:

$$\begin{aligned}Z(k, t) &= \frac{E(k, t)}{\rho_k(v, t = 0)ec\pi r_b^2}\\ &= \frac{iZ_0}{\pi\gamma r_b}\frac{1 - \xi K_1(\xi)}{\xi}\cos(\omega_p t)\end{aligned} \tag{20}$$

where $\rho_k(v, t = 0)$ is the Fourier transform of the initial perturbation (18) over z. because $\omega_p \gg kv_j^0$, here we just ignore the initial velocity v_j^0, and $Z_0 = 377\ \Omega$ is the free space impedance.

In the high energy limit, where $\omega_p \to 0$, therefore $\epsilon_r \to 1$, the electric field converges to

$$E(k) = \frac{ie}{\epsilon_0\epsilon_r\pi r_b^2 k}\left[1 - \xi K_1(\xi)\right]\sum_{j=1}^{N_e} e^{-ik(z_j^0+v_j^0 t)} \tag{21}$$

and the impedance reads:

$$Z(k, t) == \frac{iZ_0}{\epsilon_r\pi\gamma r_b}\frac{1 - \xi K_1(\xi)}{\xi} \tag{22}$$

From the above, one can see that in the high energy limit, the impedance of the longitudinal space charge differs from the regular expression [1] only by a small amount which is the high order part of the relative permittivity ϵ_r, and we may certainly ignore it in practice.

In summary, to estimate the effect of the plasma oscillation in the μBI study, the scale of the wavelength (or the frequency) of the plasma oscillation plays an important role. As we have already known, if the wavelength is much larger than the scale of the region that concerns us, the plasma effect can be neglected and our work gives the detail analytically.

CONCLUSIONS

The electric field introduced by the longitudinal space charge (LSC) with density fluctuation is studied in detail by solving the Vlasov and Poisson equations. Its influence on the LSC-induced microbunching instability (μBI) is carried out. The study shows that the gain curve of μBI depends not only on the spatial information of the beam, but also on the beam's momentum/velocity information as well. The investigation with Klimontovich particle distribution shows the result consistent with the property of the plasma oscillation.

REFERENCES

[1] Marco Venturini, Phys. Rev. ST Accel. Beams 11, 034401 (2008).

[2] J. D. Jackson, Classical Electrodynamics (Wiley, 1999).

[3] Z. Huang, et. al., Phys, Rev. ST Accel. Beams 7, 074401 (2004).

[4] J. D. Jackson, J. Nucl. Energy, Part C: Plasma Physics, 1960, Vol 1, 171-189.

[5] K.-J. Kim and R. R. Lindberg, FEL11-proceedings, Aug. 2011, Shanghai, China.

[6] E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, NIMA **483** (2002) 516-520.