COMPLETE MUON COOLING CHANNEL DESIGN AND SIMULATIONS*

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Abstract

While considerable progress has been made in developing promising subsystems for muon beam cooling channels to provide the extraordinary reduction of emittances, there is no end-to-end design that is capable of matching between or within the various subsystems. We present concepts to match emittances between and within muon beam cooling subsystems via the Helical Cooling Channel (HCC), which allows a general analytic approach to guide designs of transitions from one set of cooling channel parameters to another. These principles are demonstrated between segments in an existing cooling channel design, resulting in better performance (elimination of particle losses and colder muons) achieved in a channel approximately half its original length! These techniques will allow for a design of a complete cooling channel in a Muon Collider (MC) applicable to a Higgs Factory and an Energy Frontier machine.

INTRODUCTION

Promising subsystems for muon beam cooling channels exist that provide the extraordinary reduction of emittance required for a MC that serves as a Higgs Factory and/or an Energy Frontier machine. A high-performance front end from target to cooling systems has been designed and simulated [1], along with advances in theory, simulation codes, and hardware development. However, the various proposed subsystems have not been consolidated into an integrated design. We present the principles to match emittances between muon beam cooling subsystems or segments having different characteristics. We will exploit the theoretical framework of the Helical Cooling Channel (HCC) [2], which allows a general analytical approach to guide the transition from one set of cooling channel parameters to another. Longitudinal and transverse emittance matching techniques are extended from prior studies [3] and will be elucidated via matching between a pair of segments in an existing cooling channel design [4] while also demonstrating its success.

BASICS OF HELICAL COOLING CHANNEL THEORY

In a HCC [2], a solenoid field is augmented with a transverse helical dipole field that provides constant dispersion along the channel for emittance exchange that allows longitudinal cooling plus addition of a helical quadrupole field to provide beam stability. The solenoid magnet creates an inward radial force due to the transverse momentum of the particle, while the helical dipole magnet creates an outward radial force due to the longitudinal momentum of the particle:

$$F_{h-dipole} \approx p_z \times b; b \equiv b_\phi; \quad F_{solenoid} \approx -p_\perp \times B; B \equiv B_z, \qquad (1)$$

where B is the field of the solenoid, the axis of which defines the z axis and $b=b_\phi$ is the field of the transverse helical dipole. The equilibrium orbit satisfies:

$$p(a) = \frac{\sqrt{1+\kappa^2}}{k}\left[B - \frac{1+\kappa^2}{\kappa} b_\varphi\right] \qquad (2)$$

where

- p is reference momentum; a is reference radius
- $k = 2\pi/\lambda$; λ is helix period
- $\kappa = p_{transverse}/p_z = 2\pi a/\lambda$ = helix pitch

Conditions for transverse stability about the equilibrium orbit are:

$$0 < G \equiv \left[\left(\frac{B\sqrt{1+\kappa^2}}{pk} - 1\right) + \left(\frac{\left(1+\kappa^2\right)^{3/2}}{pk^2}\left\langle \left.\frac{\partial b_\varphi}{\partial \rho}\right|_a \right\rangle\right)\right]\hat{D}^{-1} < R^2 \equiv \frac{1}{4}\left(1 + \frac{\left(\frac{B\sqrt{1+\kappa^2}}{pk} - 1\right)^2}{1+\kappa^2}\right)^2 \qquad (3)$$

where the dispersion factor $\hat{D}$ is:

$$\hat{D}^{-1} = \frac{a}{p}\frac{dp}{da} = \frac{\kappa^2 + (1-\kappa^2)[(B\sqrt{1+\kappa^2}/pk) - 1]}{1+\kappa^2} - \frac{(1+\kappa^2)^{3/2}}{pk^2}\left.\frac{\partial b_\phi}{\partial \rho}\right|_a \qquad (4)$$

and $\left.\frac{\partial b_\phi}{\partial \rho}\right|_a$ is the gradient of the dipole field.

The motion of particles around the equilibrium orbit is shown schematically in Figure 1.

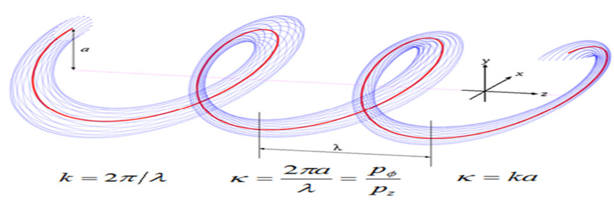

Figure 1: Schematic of beam motion in a HCC using G4beamline [5]. Reference trajectory is shown in red.

A CHALLENGING TRANSITION IN AN EXISTING COOLING CHANNEL DESIGN

The matching techniques will be applied to the most challenging transition between segments in an existing cooling channel design [4], as indicated by the red box shown in Table 1. This is the most challenging in two respects. First, the change in HCC period λ is typically 0.1m between segments, but is 0.3m in the transition from segment 4 to 5. Additionally, there is an RF frequency change from 325 MHz to 650 MHz. Figure 2 shows the ε_{6D} blow-up and associated 27% particle loss in the downstream segment 5 when there is no matching performed other than placing the centroid of the particles

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at the end of segment 4 on reference at the start of segment 5. It is this transition from segment 4 to 5 to which we will apply the longitudinal and transverse emittance matching techniques to demonstrate its success.

Table 1: Parameters in an Existing HCC Design [4]

	Z	b	b'	b_z	λ	ν	ε_T	ε_L	ε_{6D}	ε
unit	m	T	T/m	T	m	GHz	mm rad	mm	mm3	Transmission
1	0	1.3	-0.5	-4.2	1.0	0.325	20.4	42.8	12900	
2	40	1.3	-0.5	-4.2	1.0	0.325	5.97	19.7	415.9	0.92
3	49	1.4	-0.6	-4.8	0.9	0.325	4.01	15.0	10.8	0.86
4	129	1.7	-0.8	-5.2	0.8	0.325	1.02	4.8	2.0	0.73
5	219	2.6	-2.0	-8.5	0.5	0.65	0.58	2.1	3.2	0.66
6	243	3.2	-3.1	-9.8	0.4	0.65	0.42	1.3	0.14	0.64
7	273	4.3	-5.6	-14.1	0.3	0.65	0.32	1.0	0.08	0.62
8	303	4.3	-5.6	-14.1	0.3	1.3	0.32	1.0	0.07	0.60

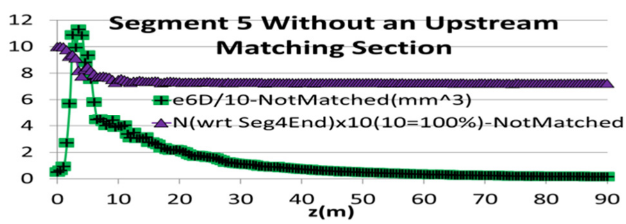

Figure 2: ε_{6D} and survival rate in downstream segment 5 without a matching section.

MATCHING DESIGN OVERVIEW

Matching will proceed in two steps, starting longitudinally and followed by a transverse match. We match longitudinally first for practical reasons in order to introduce the smaller higher frequency RF cavities earlier to allow it to fit into the subsequent smaller-aperture transverse matching section. Segment 4a is introduced as an intermediate virtual channel (zero length) to facilitate the design of the matching section by defining the configuration at end of the longitudinal matching section and start of the transverse matching section. It has the same λ=0.8m as upstream segment 4, but RF frequency of 650 MHz as the downstream segment 5. Figure 3 illustrates the breakdown of the matching channel into the longitudinal that transforms the phase space of segment 4 into segment 4a, followed by the transverse matching section that transforms segment 4a into the targeted downstream segment 5.

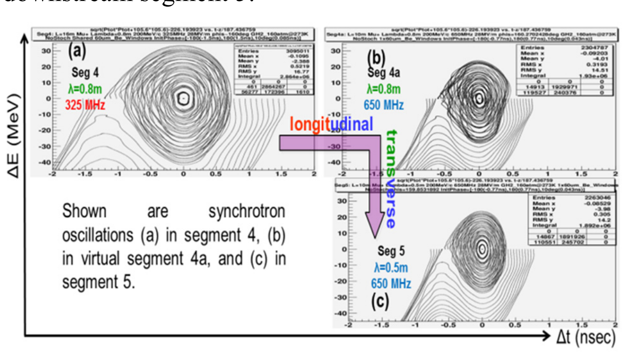

Figure 3: Longitudinal phase space evolution in longitudinal and transverse matching sections. Segment 4 (a) is longitudinally matched into segment 4a (b), which is transversely matched into segment 5 (c).

LONGITUDINAL MATCHING DESIGN

The longitudinal matching concept is adapted from a method used to match rapidly between initial and final voltages V_i and V_f by jumping to an intermediate voltage $V_{\text{int}} = \sqrt{V_i V_f}$ for a quarter synchrotron period. Note that longitudinal dynamics is controlled by the slope of the accelerating voltage about the synchronous particle, which is proportional to both (a) accelerating voltage and (b) RF frequency. However, the segments to be matched here have the same accelerating voltage, so we'll use RF frequency to adjust the slope for matching. In our implementation, we bypass the need for introducing a RF cavity operating with $f_{\text{int}} = \sqrt{f_i f_f}$ by using an appropriate mix of RF cavities operating at the upstream and downstream frequencies. It can be shown that this mix should be 59% of the upstream and 41% of the downstream frequencies. For practical reasons, we used an admixture of 60% upstream (325 MHz) and 40% downstream (650 MHz). Figure 4 shows the layout of the longitudinal matching section where the admixture of the cavities (shown in red) can be seen.

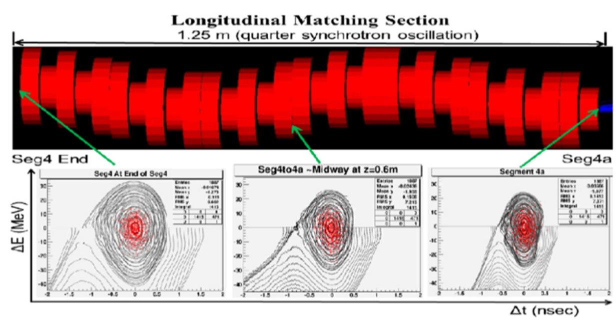

Figure 4: Display of longitudinal matching section and snapshots of longitudinal phase space.

Note that if higher gradients are allowed for higher frequencies, then voltage, frequency, and synchronous phase should be adjusted to preserve the following across the matching section:

$$V_{\text{int}} f_{\text{int}} \cos(\phi_{s,\text{int}}) = \sqrt{V_1 V_2 f_1 f_2 \cos(\phi_{s,1}) \cos(\phi_{s,2})} \quad (5)$$

TRANSVERSE MATCHING DESIGN

The transverse matching design recognizes the strong inherent coupling between the longitudinal and transverse dynamics that exists in any six dimension cooling channel which necessarily incorporates emittance exchange. The transverse matching proceeds by conserving the slip factor η_H throughout the transverse matching channel. The slip factor η_H can be controlled in an HCC and is related to its channel parameters via [2]:

$$\eta_H = \frac{\sqrt{1+\kappa^2}}{\gamma\beta^3}\left(\frac{\kappa^2}{1+\kappa^2}\hat{D} - \frac{1}{\gamma^2}\right) \quad (6)$$

We maintain a constant value for η_H by keeping values of its components unchanged. In particular, the dispersion factor $\hat{D}$, reference energy (γ and β), and helix pitch κ will all remain fixed in the transverse matching section.

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Our application of this principle in matching from segment 4 to 5 will be to linearly evolve the helix period λ from 0.8m to 0.5m over a longitudinal distance of 4m, which corresponds to one full synchrotron oscillation. Evolving λ also requires a corresponding change to the reference radius *a* in order to maintain $\kappa=1=2\pi a/\lambda$. Figure 5 shows the layout of the transverse matching section where the helix period λ evolves linearly from 0.8m to 0.5m over a distance of 4m. The blue tracks are μ^+s traversing the red RF cavities; the magnetic fields are generated analytically. Also shown are the longitudinal dynamics that benefit from the constant slip factor.

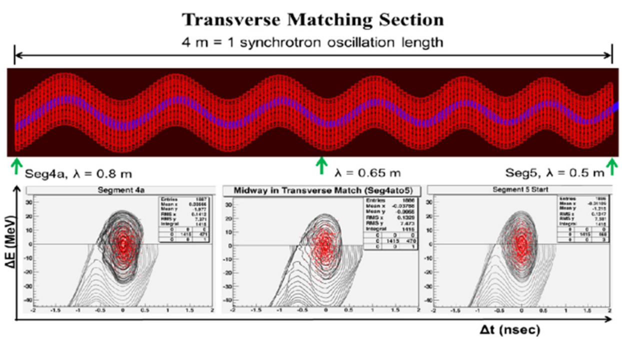

Figure 5: Display of transverse matching section and snapshots of longitudinal phase space.

MATCHING RESULTS

Pulling together the longitudinal and transverse matching, Figure 6 shows no particle loss across the matching section, little or no ε_{6D} growth in the longitudinal portion, and ε_{6D} reduction in the transverse. However, the figure of merit lies in particle survival rate and ε_{6D} in the downstream segment 5 shown in Figure 7, where the ~27% particle loss and ε_{6D} blow-up observed in the unmatched case have been eliminated! A zoomed in view on ε_{6D} in Figure 7 is shown in Figure 8, where it is seen that the unmatched segment 5 takes an additional ~45m to recover to its initial ε_{6D} value from the blow-up in its first 5m; it also takes ~57m to cool to the level that is achieved in the 5.25m long matching channel. Considering only the matched segment 5, a length of 40m is a reasonable choice where there is onset of diminishing returns and hence reducing the initial 90m section 5 to 45.25m (including matching section), while eliminating particle losses and even achieving lower ε_{6D} (matched ε_{6D} at z=40m is less than that of unmatched at z=90m).

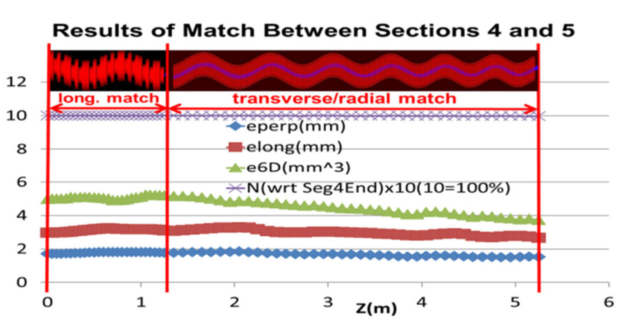

Figure 6: Dynamics & survival rate in matching section.

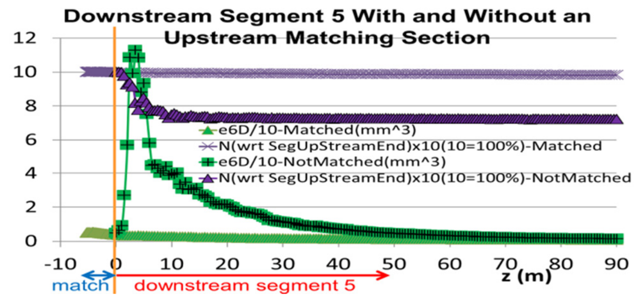

Figure 7: ε_{6D} and survival rate in downstream segment 5 with and without a matching section.

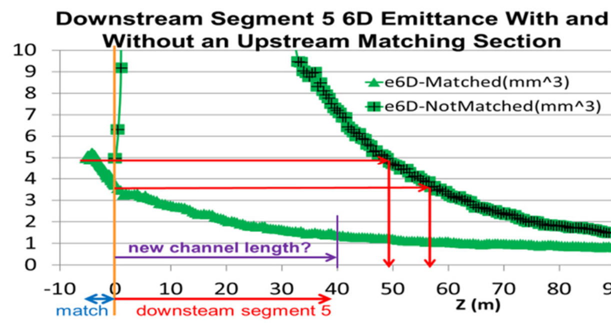

Figure 8: Zoomed in view of ε_{6D} in segment 5 with and without a matching section.

SUMMARY & FUTURE

We presented a matching scheme that is especially beneficial to any 6D cooling channel, where emittance exchange occurs that creates strong coupling between longitudinal and transverse dynamics. The results of its application to a lossy transition in an existing design achieved remarkable success, where a 5.25m long matching section eliminated particle losses (27%) and created a colder beam, while reducing the channel length from 90m to 40m! We will revisit the existing HCC design [4] armed with this new matching technique and other schemes to increase the acceptance and provide the coldest muon beam possible to enable a MC applicable for a Higgs Factory and/or an Energy Frontier machine.

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