SPACE-CHARGE STUDIES FOR IONIZATION COOLING LATTICES *

D. Stratakis# and R.B. Palmer, Brookhaven National Laboratory, Upton, NY 11973, USA
D. P. Grote, Lawrence Livermore National Laboratory, Livermore, CA 94550, USA

Abstract
Muon Colliders use ionization cooling to reduce the emittance of the muon beam prior to acceleration. The baseline design for a Muon Collider requires the rms longitudinal emittance to be less than 2 mm while the transverse emittance is a few hundreds of microns. At the last stages of the 6D cooling channel the rms bunch length is just 2cm while it is packed with 4×10^{12} muons at low 200 MeV/c momentum. Such high beam intensity could give raise to space-charge effects. Currently, codes that could study both space-charge and particle-matter-interaction are very limited. Here, with the aid of the particle-in-cell code Warp, a model is developed to examine space-charge for muon ionization cooling lattices and some recent results are presented.

INTRODUCTION

Muon colliders allow the high energy study of point-like collisions of leptons without some of the difficulties associated with high energy electrons, such as the synchrotron radiation requiring their acceleration to be essentially linear and, for this reason, long. Muons can be accelerated in smaller rings and offer other advantages, but they are produced only diffusely and they decay rapidly, making the detailed design of such machines difficult.

A complete scheme for cooling a muon beam sufficiently for use in a muon collider has been previously described [1]. This scheme uses separate 6D ionization cooling channels for the two signs of the particle charge. In each, a channel first reduces the emittance of a train of muon bunches until they can be injected into a bunch-merging system. The single muon bunches, one of each sign, are then sent through a second tapered 6D cooling channel where the transverse emittance is reduced as much as possible and the longitudinal emittance is cooled to a value below that needed for the collider. The beam can then be recombined and sent through a final cooling channel using high-field solenoids that cools the transverse emittance to the required values for the collider while allowing the longitudinal emittance to grow. The baseline cooling requirement for cooling is shown in Fig. 1. This paper will focus on the design of the 6D cooling channel after the bunch merging.

So far, ionization cooling simulations have not considered space-charge effects in any detail. However, numerical estimates predict that at the last 6D cooling stage the rms bunch length is just 2 cm and contains 4×10^{12} muons. Thus, the peak current becomes ~4 kA at a ~200 MeV/c momentum. Due to this large current, space-charge forces can be present, and these may transform the initial Gaussian bunch to parabolic. Thus, there is cause for concern that the existing last 6D cooling lattice design will not work as well as estimated by existing studies. This paper will present results from recent studies on simulating space-charge effects in muon cooling lattices.

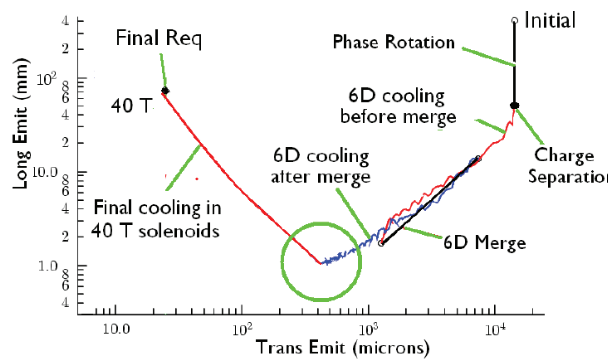

Figure 1: Longitudinal vs. transverse normalized rms emittance baseline requirements for a Muon Collider

SIMULATION MODEL

In all the 6D cooling lattices, the focusing fields include weak dipoles (generated by tilting the solenoids) to generate dispersion at the absorbers. The dipole fields cause the lattices to curve, forming a gentle upward or downward helix [2, 3]. The absorbers are wedge shaped and made of liquid hydrogen (for most cases) or Lithium Hydride (in a few of the last cases). The wedges, in the presence of dispersion reduce the energy of higher energy muons more than that of the lower energy ones, thus reducing the energy spread at the expense of increasing the transverse beam sizes filled absorbers. This thus generates emittance exchange between longitudinal and transverse emittances, allowing cooling in all six dimensions.

Figure 2 shows a cross section of a typical cell of the lattice, with alternating focus coils (magenta), wedge absorbers (cyan), and rf cavities (yellow). Note that the coils are tilted to generate the bending fields as described earlier.

A simplified model of the channel was used for the initial design [4]. This step was useful in order to give guidance on the parameters and performance expected in every stage. The cooling was simulated in a series of straight lattices with a layout similar to the lattice cell in Fig. 2 but without the tilt. This arrangement only produces transverse cooling. Longitudinal cooling requires emittance exchange between the transverse and longitudinal phase space. This was modeled by applying a matrix to the beam coming out of the straight channel.

*Work supported by DE-AC02-98CH10886
#diktys@bnl.gov

ISBN 978-3-95450-122-9

The amount of exchange was controlled using a parameter δ, which reduced the momentum spread and increased the angular divergence of the beam. The matrix acting on $(x, x', y, y', \sigma_z, \sigma_p/p)$ used had the following form:

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1+\delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\delta \end{matrix}$$

Transition from one stage to the next was done with artificial "hard" ends at absorber centers where the axial magnetic fields are always zero. However, the transverse magnetic fields off-axis are non-zero and somewhat different from one stage to the next. So the hard transition is not strictly smooth.

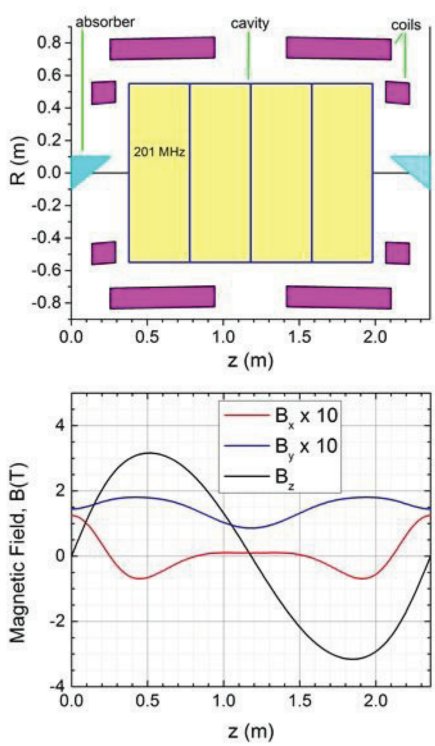

Figure 2: Layout of a single cell (top) and axial profile of the magnetic field profile (bottom) in a realistic channel with tilted solenoids for Stage 2.

The beta function was reduced by scaling down all dimensions and raising the on-axis solenoid field, as shown in Fig. 2. The desired beta function varies from 42 to 3 cm in the post-merge channel, while the on-axis field increases from 2.8 to 17.0 T. The beta function determined from these straight lattices was very similar to those found in the more realistic simulations described later since the required dipole field strength is much smaller than the solenoid field. The RF frequency in the stages increased progressively from 201 to 805 MHz. A gradient of 15.5 MV/m was used for all cavities while the rf phase was set between 34 to 43 degrees. This procedure produced a reference 17-stage post-merge 6D cooling channel design. Detailed lattice parameters are listed in Table 1.

PARTICLE TRACKING

Simulations of the channel performance where done using the ICOOL code [5] using 2,846 particles. We generated 2D cylindrical field maps for each of the stages by superimposing the fields from all the solenoids in the cell and its neighbour cells. The rf cavities were modelled using cylindrical pillboxes running in the TM010 mode. Space-charge effects were studied by using the particle-in-cell code WARP [6]. Warp was originally developed to simulate space-charge-dominated beam dynamics in induction accelerators for heavy-ion fusion (HIF). The code now has an international user base and is being applied to projects both within and far removed from the HIF community. For this work, the tasks to be accomplished were: (1) Incorporate, into the Warp code, a basic model for the cooling particle-matter interaction; and (2) Carry out an initial simulation of a muon cooling channel using Warp's 2-D axisymmetric (r-z) mode, including space-charge and the newly implemented cooling model.

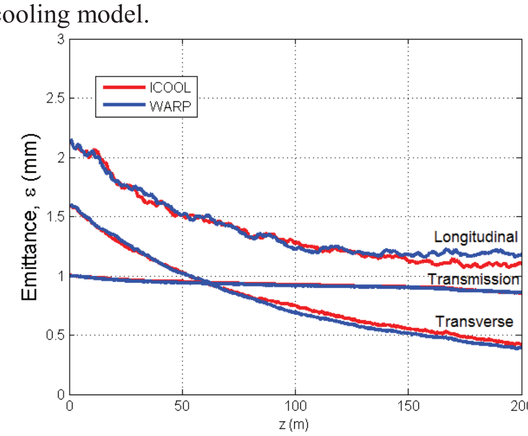

Figure 3: A comparison between ICOOL and WARP for the case without space-charge (Q=1). Result is restricted for stages 10 to 16 only.

The transverse and longitudinal emittance and transmission as a function of distance along stages 10 to 16 is shown in Fig. 3. Muon decays are not considered while the bunch charge was set to 1. The simulation produced a transverse emittance of 0.3 mm and a longitudinal emittance equal to 1.3 mm which is closed to the desired values for a Muon Collider. The quoted values are normalized rms.

Figure 4 displays the same result but with space-charge included. All runs were done with Warp. Clearly, as the bunch charge is increased the cooling becomes less efficient and there is a substantial particle loss. This becomes more severe when Q=4-5x10^{12} which is the intensity regime for a Muon Collider. Interestingly, the transverse emittance is not affected by the bunch charge.

Table 1: Lattice Parameters of the Tapered Cooling Channel

Stage	Cell length [m]	RF Freq. [MHz]	RF Grad. [MV/m]	RF #	RF Length [cm]	Absorber Material	Absorber Lenth [cm]	δ
1	2.75	201	15.48	4	37.6	LH	3.25	1.03
2	2.357	234.7	15.48	4	40.0	LH	2.59	1.026
3	2.021	273.8	15.48	4	34.54	LH	2.03	1.022
4	1.732	319.4	15.48	4	29.6	LH	3.90	1.019
5	1.485	372.6	15.48	4	25.38	LH	2.62	1.016
6	1.273	402	15.48	4	21.75	LH	2.62	1.013
7	1.091	507.1	15.48	4	18.65	LH	1.91	1.010318
8	0.9355	591.5	15.48	4	15.98	LH	1.93	1.008845
9	0.802	690.0	15.48	4	13.70	LH	2.46	1.007582
10	0.6875	805	15.48	4	11.75	LH	2.46	1.0065
11	0.6875	805	15.48	4	11.75	LH	2.79	1.0065
12	0.6875	805	15.48	4	11.75	LH	2.95	1.0065
13	0.6875	805	15.48	4	11.75	LH	3.28	1.0065
14	0.6875	805	15.48	4	11.75	LH	2.87	1.0065
15	0.6875	805	15.48	4	11.75	LiH	2.89	1.006
16	0.6875	805	15.48	4	11.75	LiH	2.97	1.006
17	0.6875	805	15.48	4	11.75	LiH	2.81	1.004

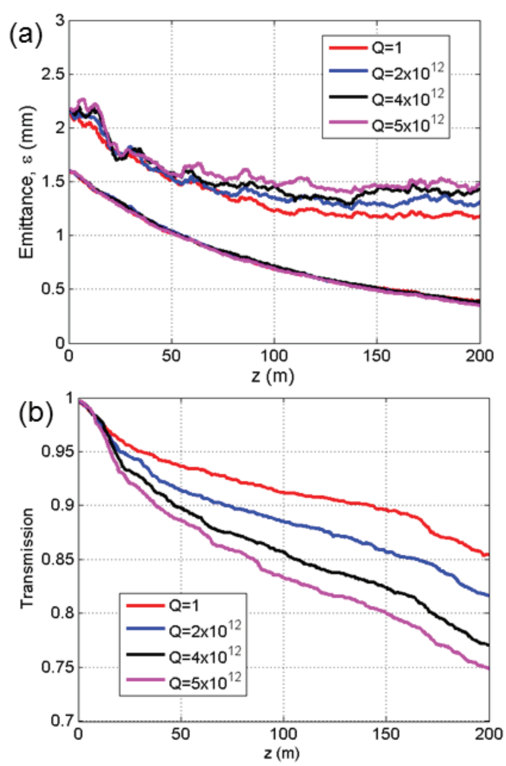

Figure 4: Emittances and transmission as a function of distance along the channel for different bunch charges. Tracking was done with the Warp code.

SUMMARY

Reducing the initial large emittance muon beams is an essential step for a Muon Collider. Initial designs were proposing cooling to a longitudinal rms normalized emittance equal to 1 mm. As this study demonstrated, if the longitudinal emittance approaches ~ 1.5 mm, space-charge is opposing additional cooling. We also showed that space-charge is not affecting the transverse emittance. It is therefore practical to design channels that do not cool less than 2 mm. The next step would be to carry out a full 3D simulation with wedges as absorbers.

ACKNOWLEDGMENT

Thanks to I. Haber, S. Berg and R. Ryne for useful discussions.

REFERENCES

[1] R. B. Palmer, Proc. of PAC 2007, Albuquerque.
[2] R. Palmer et al, *Phys. Rev. ST Accel. Beams* **8**, 081001 (2005).
[3] P. Snopok and G. Hanson, IJMPA 24, p. 987 (2009).
[4] R. B. Palmer, Proc. of PAC 2011, New York. M0P002
[5] R. C. Fernow, Proc. of 1999 PAC, New York, p. 3020
[6] http://hifweb.lbl.gov/Warp/

ISBN 978-3-95450-122-9