SPACE CHARGE SIMULATION BASED ON MEASURED OPTICS IN J-PARC MR

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Abstract

Linear optics parameters, beta, alpha, phase, x-y coupling and dispersion are measured by phase space monitor and/or other tools. Nonlinear effects due to the space charge and magnets are dominantly determined by linear optics. For example, the beam distribution is mainly determined by linear optics, and error of beta function at a sextuple magnet is larger than error of magnet strength in $K_2\beta^{3/2}$ generally. This means space charge simulation based on the measured optics takes into account of the major part of errors. We discuss how beam loss degrades and which resonances are induced by the errors in the simulations and algebraic analysis.

INTRODUCTION

Emittance growth and beam loss are caused by chaotic behavior near nonlinear resonances induced by space charge force and nonlinear accelerator components. Estimation of errors of accelerator elements is inevitable to study beam loss in high intensity proton machines, because the errors excite and enhance resonances. Normally people generate errors using random number to realize the errors. Using measured optics is alternative way to evaluate effects of the errors.

One turn map, which characterizes the chaotic behavior, is determined by integration of the nonlinear forces and linear optics at the nonlinear elements.

$$\mathcal{M}(s)=\prod_{i=0}^{N-1} M(s_{i+1},s_i)e^{-:H_I(s_i):} \quad (1)$$

$M(s_{i+1},s_i)$ is the transfer map from s_i to s_{i+1}: that is, it can be transfer map with a weak nonlinearity like an edge field of magnets. For simplicity, M is regarded as linear transfer map in the explanation.

The transformation in exponential expression is

$$e^{-:H_I(s_i):}\boldsymbol{p}=\boldsymbol{p}-\frac{\partial H_I(s_i)}{\partial \boldsymbol{x}} \quad (2)$$

For the case of sextupole magnet, H_I is expressed by

$$H_I(s_i)=\frac{K_2(s_i)}{6}(x^3-3xy^2) \qquad K_2=\frac{eB"}{p_0} \quad (3)$$

For the space charge force,

$$H_I=\Phi(x,y,s), \quad (4)$$

where Φ which is the space charge potential is given by solving Poisson equation.

M is diagonalized blockwisely using eigenvector matrix, V.

$$M(s_2,s_2)=V(s_2)^{-1}U_{21}V(s_1) \quad (5)$$

where

$$V(s)=B(s)R(s)H(s) \quad (6)$$

B,R,H are represented by Twiss parameters (α,β), x-y coupling ($r_{1\text{-}4}$) and dispersion (η,ζ) [1], respectively. Twiss parameters are designed ones and x-y coupling is nothing ($r_{1\text{-}4}$=0, R_0=I: unit matrix) in general. Twiss (B), x-y coupling parameters (R) and tunes (U) are measurable quantities. Dispersion is not taken into account in this paper (H=I). The measured transfer map is related to the design one as follows,

$$\begin{aligned}M(s_{i+1},s_i)&=V^{-1}(s_{i+1})U_{i+1,i}\Delta U_i V(s_i)\\&=V^{-1}(s_{i+1})V_0(s_{i+1})M_0(s_{i+1},s_i)V_0^{-1}(s_i)\Delta U_i V(s_i)\end{aligned} \quad (7)$$

The one turn map is expressed by designed transfer map and corrected nonlinear map K_I,

$$\mathcal{M}(s)=\prod_{i=0}^{N_I-1} M_0(s_{i+1},s_i)e^{-:K_I(s_i):} \quad (8)$$

where M_0 is the designed transfer map (matrix) written by design Twiss parameters,

$$M_0(s_{i+1},s_i)=V_0^{-1}(s_{i+1})U_{0,i+1,i}V_0(s_i)\,. \quad (9)$$

The corrected nonlinear map is expressed by

$$e^{-:K_I:}=V_0^{-1}(s_i)\Delta U_i V(s_i)e^{-:H_I:}V^{-1}(s_i)V_0(s_i)\,, \quad (10)$$

MEASUREMENT OF LINEAR OPTICS

Linear optics parameters were obtained by measurement of 4 dimensional phase space trajectory excited by X or Y mode [1,2]. Second order moments $\langle x_i x_j\rangle$ (i=1,4) of betatron motion in 4 dimensional phase space give linear optics parameters. Figure 1 shows measured optics parameters, Twiss parameters (α,β), x-y coupling ($r_{1\text{-}4}$). Twiss parameters (α,β) deviate from the design values about 5%.

x-y coupling is measured by detecting a small y signal from x mode excitation, vice versa. We had limitation in accuracy of the measurement in the present technique. The rotation error corresponding measured x-y coupling, which is 1-2 mrad, is larger than position survey data. Calibration of monitors may be insufficient for measurement of x-y coupling.

SPACE CHARGE SIMULATION USING MEASURED LINEAR OPTICS

We study effects of measured optics in each nonlinear element of one turn map in Eqs.(8) and (10). Measured beta, alpha, x-y coupling at sextupoles and space charge elements are taken into account in simulation. We can find which contribution is dominant for the beam loss.

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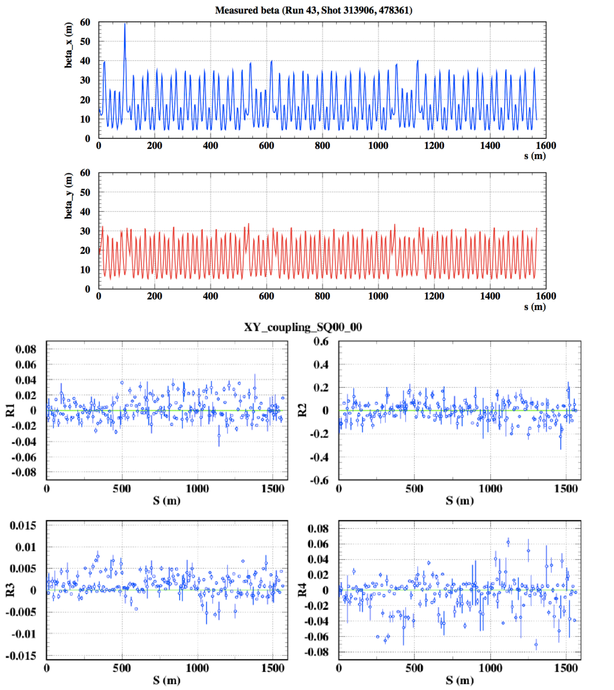

Figure 1: Measured optics parameters, Twiss parameters (α,β), x-y coupling ($r_{1\text{-}4}$).

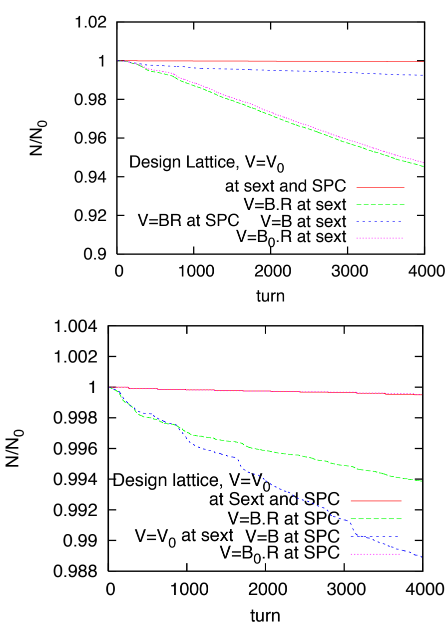

Figure 2: Beam loss given by space charge simulation with the measured optics. Top and bottom plots depict beam loss for optics errors in sextupoles and space charge elements, respectively.

Figure 2 shows beam loss given by space charge simulation with the measured optics. In the top picture, measured optics V=BR is adopted at space charge elements, while V=BR, V=B and V=B_0R are adopted at the sextupoles, in Green, Blue and Magenta lines, respectively. Strong beam loss is seen in Green (V=BR) and Magenta (V=B_0R) lines. This behavior indicates that x-y coupling in sextupoles is dominant for the beam loss.

In the bottom picture, V=B_0 is adopted in sextupoles, while V=BR, V=B and V=B_0R are adopted at the space charge elements in Green, Blue and magenta lines, respectively. The loss rate is far less than the cases with optics erros in sextupoles. Error of beta function dominates in the beam loss compare than x-y coupling for space charge elements.

LATTICE RESONANCE INDUCED BY MEASURED OPTICS

We now study resonance characteristics induced by sextupole magnets for the measured linear optics. One turn map is modeled by tune spread due to space charge potential ($U_0(J_x,J_y)$) and resonance ($G_m(J_x,J_y)$) from sextupoles hereafter as follows,

$$H = \boldsymbol{\mu} \cdot \boldsymbol{J} + \sum_{\boldsymbol{m}=1} G_{\boldsymbol{m}}(\boldsymbol{J}) \exp(-i\boldsymbol{m} \cdot \boldsymbol{\phi}) + U(J_x, J_y) \tag{11}$$

where resonance terms due to space charge and tune spread due to sextupoles are neglected. This model is motivated by the simulation results, in which the beam loss is mainly caused by optics error in sextupoles (Figure 2).

Beam particles are diffused by a resonance and its modulation due to synchrotron motion [3]. Resonance island width is essential parameter to characterize the emittance growth,

$$\Delta J_x = 2m_x \sqrt{\frac{G_{m_x,m_y}}{\Lambda}}, \tag{12}$$

where Λ, which is tune sloop in J space, is represented by second derivatives of the space charge potential,

$$\Lambda = m_x^2 \frac{\partial^2 U}{\partial J_x^2} + m_x m_y \frac{\partial^2 U}{\partial J_x \partial J_y} + m_y^2 \frac{\partial^2 U}{\partial J_y^2}\bigg|_{J=J_R}. \tag{13}$$

Figure 3 shows d^2U/dJ_x^2 of the space charge potential for a round beam. d^2U/dJ_y^2 and d^2U/dJ_xdJ_y have similar behavior. The tune sloop of space charge is far larger than that of lattice nonlinearity. The values of z axes are $d^2U/dJ_x^2=3\times10^6$ for $J/\varepsilon=4$ (2σ), 1×10^6 for $J/\varepsilon=9(3\sigma)$. The space charge force induces resonances far from the operating point, while their widths are small due to the large tune sloop. Lattice tune spread (weak space charge) induces resonances near the operating point, while their widths are large due to the small tune sloop.

The tune shift (spread) of the space charge potential is shown in Figure 4. Black and red dots show tune for peak and its half line density of the J-PARC beam in every 0.5 ε step ($0<J_{xy}<16\ \varepsilon$). Resonance lines overlapping the tune spread area are indicated by (m_x,m_y), where the operating point is (ν_x, ν_y)=(0.4,0.75).

The resonance strengths (G_m) of sextupoles are evaluated by polynomial one turn map by 12-th order. The map is factorized by

$$\begin{aligned} \mathcal{M} &= M_0 \exp(-:H_{nl}:) \\ H_{nl} &= \sum_{\boldsymbol{m}=1} G_{\boldsymbol{m}}(\boldsymbol{J}) \exp(-i\boldsymbol{m} \cdot \boldsymbol{\phi}) \end{aligned} \tag{14}$$

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To evaluate the resonance width, G_m has to be evaluated at amplitude, where the resonance condition is satisfied at (J_{xR},J_{yR}). For simplicity, the resonance strength is evaluated at $J_{xR}+J_{yR}=9\ \varepsilon\ (3\sigma)$. Tune sloop, which also depends on (J_{xR},J_{yR}), is 10^6 as a typical value shown in Figure 3. Table 1 shows the widths of resonances overlapping with the tune spread due to the space charge potential. Right three columns show resonance widths for $V=B_0$ (design lattice), V=B (without x-y coupling) and V=BR (measured). Skew resonances (1,1), (2,±1), (0,3), (1,±3) and (3,±1) appear for the measured optics parameters, V=BR. Enhancement of the resonance strength due to beta function modulation is seen in (3,0) and (1,±2). The resonance widths, which are around 0.1-1 mm.mrad, are not negligible for the beam emittance ε=4mm.mrad. Note that they have ambiguity of factor, because of the choice of (J_{xR},J_{yR}) discussed above.

The linear optics parameters are distorted by the space charge force. The distorted parameters are evaluated by envelope formalism in 4 dimensional phase space [1,2]. It may be better to parameterize the nonlinear components under linear optics containing linearized space charge force in Eq.(14).

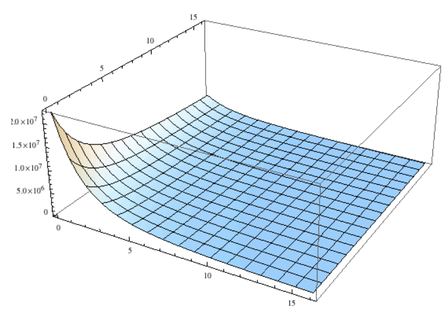

Figure 3: Second derivative of space charge potential, $d\nu_x(J_x,J_y)/dJ_x$ for a round beam, where x-y axes are J_x/ε_x-J_y/ε_y.

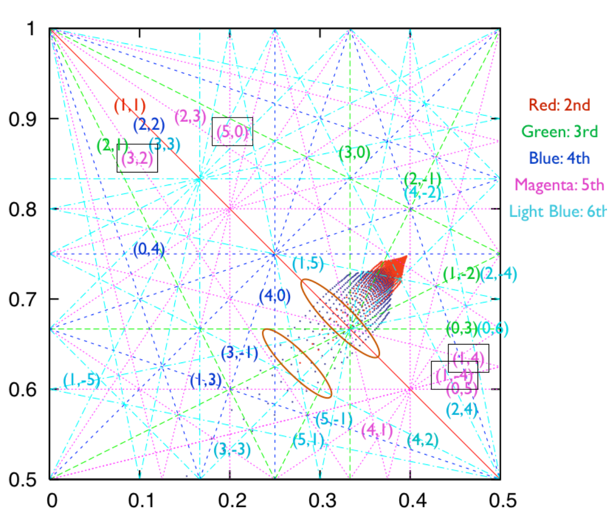

Figure 4: Tune spread and related resonances for J-PARC MR. Horizontal and vertical axes are ν_x and ν_y, respectively. The integer part is 22(x) and 20(y).

SUMMARY AND CONCLUSIONS

Estimation of errors of accelerator elements is inevitable to study beam loss. Normally people generate errors using random number to realize the errors. Using measured optics is alternative way to evaluate the errors. Accuracy of the measurement is still a question and should be cleared. We have studied space charge effects based on the measured optics parameters. X-y coupling in sextupole magnets degraded the beam loss performance, while the beta function modulation little degraded. Optics errors in space charge elements were smaller effects than those in sextupoles.

Resonance widths were evaluated based on the measured optics parameters. Some resonances were induced and enhanced by the optics errors. The widths (ΔJ) were 0.1-1 mm.mrad were not negligible for the emittance ε=4mm.mrad. Modulation of the resonances with large widths due to the synchrotron motion causes emittance growth and beam loss.

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Table 1: Resonance widths determined by linear optics at sextupoles. The unit of ΔJ is mm.mrad.

m_x	m_y	ΔJ (B_0)	ΔJ (B)	ΔJ (BR)
1	0	0.440	0.868	0.863
2	0	0.315	0.426	0.432
1	1	0.000	0.000	0.127
0	2	0.149	0.125	0.104
3	0	0.467	0.719	0.727
2	1	0.000	0.000	0.754
2	-1	0.000	0.000	0.564
1	2	0.432	0.843	0.855
1	-2	0.768	1.044	1.044
0	3	0.000	0.000	0.662
4	0	1.001	1.002	1.002
3	1	0.000	0.000	0.165
3	-1	0.000	0.000	0.053
2	2	0.315	0.154	0.149
2	-2	0.225	0.183	0.179
1	3	0.000	0.000	0.119
1	-3	0.000	0.000	0.058
0	4	0.219	0.241	0.238

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