# STUDY ON THE SINGLE BUNCH TRANSVERSE EMITTANCE GROWTH IN BAPS\*

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### Abstract

In order to explore how small the transverse emittance we can get on BAPS, this paper studies the single bunch transverse emittance growth due to short range wakefield according to J. Gao' theory. The mechanism of wakefield induced single bunch emittance is explained first and then the transverse emittance at the design beam current is estimated. Also, the tolerances for the beam current, the transverse loss factor and the vacuum chamber misalignment (or the closed orbit distortion) are presented.

### **INTRODUCTION**

Beijing Advanced photon Source (BAPS) is a proposed next synchrotron radiation facility which has much smaller transverse emittance after SSRF in China, with 5 GeV energy and 1.5 km circumference [1]. The main parameters for BAPS are listed in table 1. As a third type of light source with ultra-low transverse emittance aimed for very high brightness, emittance growth will be definitely a big issue which should be studied carefully.

Parameters	Value
Energy (GeV)	5
Circumference (m)	1549
Bunch number	1000
Average beta x/y (m)	7/9.5
Damping time x/y (ms)	33.6/33.6 (bare lattice)
	10.5/10.5 (60m wiggler)
Nature bunch length (mm)	1.2
Nature emittance x/y	63/1.3 (bare lattice)
(pm-rad)	15/0.7 (60m wiggler)
Design trajectory offset (um)	100/100
Design bunch charge	3.2×10 <sup>9</sup> (100mA)

Table 1: Main Parameters for BAPS

In an electron storage ring, it is observed that with increasing bunch current not only a bunch suffers from bunch lengthening, increase in energy spread, but also transverse emittance growth. The usual explanation to the transverse emittance grow up is based on the intrabeam scattering theory [2-4]. However we found that the intrabeam scattering phenomenon may not always dominate the physical process for the emittance growth in an ultra-low emittance ring especially in the vertical plane [5]. In this paper we will draw attention to another

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important physical cause for the transverse emittance grow up in addition to the intrabeam scattering, i.e. the short range transverse wakefield of the machine.

### **REVIEW OF THEORY**

It is not difficult to imagine that if the closed orbit is distorted and (or) the vacuum chambers are misaligned from the ideal geometric center, the particles in a bunch will suffer from transverse deflections due to the single bunch short range wakefield which result in its emittance growth similar to what happens in a linac when the accelerating structures' axes do not coincide with the trajectory of the passing bunch.

The differential equation of transverse motions of a particle inside the bunch can be regarded as Langevin equation which governs the Brownian motion of a molecule because the transverse wakefield kicks due to vacuum chamber misalignment error and close orbit distortion are random. By solving the Langevin equation with an analogy between the movement of the transverse motion of an electron and that of a molecule, one gets the short range transverse wakefield induced bunch emittance increases which are expressed as [6]

$$R_{\varepsilon,x} = \frac{\varepsilon_{0,x} + \varepsilon_{w,x}}{\varepsilon_{0,x}} = 1 + \frac{\sigma_x^2 \tau_x < \beta_x >}{4T_0 \varepsilon_{0,x} R_{\varepsilon,x}^3} \left(\frac{e^2 N_e k_{\perp,x} (\sigma_{z_0})}{m_0 c^2 \gamma R_z^{0.7}}\right)^2 \tag{1}$$

$$R_{\varepsilon,y} = \frac{\varepsilon_{0,y} + \varepsilon_{w,y}}{\varepsilon_{0,y}} = 1 + \frac{\sigma_y^2 \tau_y < \beta_y >}{4T_0 \varepsilon_{0,y} R_{\varepsilon,y}^3} \left( \frac{e^2 N_e k_{\perp,y} (\sigma_{\varepsilon_0})}{m_0 c^2 \gamma R_{\varepsilon}^{0.7}} \right)^2$$
(2)

where  $\varepsilon_{0,x/y}$  is the design emittance at zero current,  $\varepsilon_{w,x/y}$  is the emittance growth due to transverse short range wakefield which is a function of beam current,  $\langle \beta_{x,y} \rangle$  is the average beta function of the machine,  $\sigma_{z0}$  is the bunch length of zero current,  $k_{\perp,x/y}(\sigma_{z0})$  is the transverse loss factor for the design bunch length  $\sigma_{z0}$  at zero current,  $R_z = \sigma_z/\sigma_{z0}$  is the factor for bunch lengthening,  $N_e$  is the bunch population,  $T_0$  is the circulating period,  $\tau_{x,y}$  is the transverse synchrotron radiation damping time and  $\sigma_{X,Y}$  is the RMS error for beam trajectory which is the combined effects of vacuum chamber misalignment and closed orbit distortion ( $\sigma_{X,Y}^2 = \sigma_{x,y,chamber}^2 + \sigma_{x,y,orbit}^2$ ). It is obvious that to avoid excessive emittance grow ups, both the closed orbit distortions and the vacuum chamber misalignment errors should be under careful control with the same rigour.

## TRANSVERSE LOSS FACTOR ESTIMATION FOR BAPS

The transverse loss factor can be scaled from the exist machine by the formula as follow

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$$k_{\perp,BAPS}(\sigma_{z0, \text{ BAPS}}) = k_{\perp 0}(\sigma_{z0}) \frac{L/L_0}{\left(\frac{\sigma_{z0, \text{ BAPS}}}{\sigma_{z0}}\right)^{0.7}}$$
(3)

where  $k_{\perp 0}(\sigma_{z0})$ ,  $L_0$ , and  $\sigma_{z0}$  are the transverse loss factor, circumference and nature bunch length at zero current for the existing machine respectively,  $k_{\perp,\text{BAPS}}(\sigma_{z0,\text{BAPS}})$ , L, and  $\sigma_{z0,\text{BAPS}}$  are the transverse loss factor, circumference and nature bunch length of BAPS.

By making an average of scaling results from ATF and Super-Aco (see table 2), one gets the approximate value for BAPS's transverse loss factor which is 45783 V/pC/m.

Table 2: BAPS Transverse Loss Factor Estimation from Known Machine

Machine	Circumferenc (m)	Nature bunch length σ <sub>z0</sub> (mm)	Transverse loss factor $K_{\perp 0}(\sigma_{z0}) \ (V/pC/m)$	BAPS Transverse loss factor K <sub>⊥</sub> (σ <sub>z</sub> ) (V/pC/m)
ATF	138	6.8	1020	38493
Super-Aco	72	24	303	53074

# ANALYSIS OF BAPS EMITTANCE GROWTH DUE TO WAKEFIELD

In this section, we will use eq. (1) and eq. (2) to estimate the transverse emittance growth for BAPS with different bunch population, transverse loss factor and trajectory error. Here we assume  $R_z=1$  to simplify the situation because the bunch lengthening effect in BAPS is not serious at the design beam current [7].

Bare lattice







Figure 2: Transverse emittance growth vs transverse loss factor ( $N_e$ =3.2×10<sup>9</sup>,  $\sigma_{X,Y}$ =100um).

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Figure 3: Transverse emittance growth vs trajectory offset  $(N_e=3.2\times10^9, k_{\perp,\text{BAPS}}=45783 \text{ V/pC/m}).$ 

From Fig. 1, one finds that the transverse emittance will increase by 2.7 times for horizontal plane and 7 times for vertical plane at design current. According to the estimation of transverse loss factor (45783 V/pC/m), the transverse emittance will increase by 3 times and 6.5 times for horizontal and vertical direction respectively from Fig. 2. Also, Fig. 3 shows that the transverse emittance will increase by 2.8 times and 7 times for horizontal and vertical direction for design 100 um trajectory error.

Lattice with Damping Wigglers



Figure 4: Transverse emittance growth vs bunch population ( $k_{\perp,BAPS}$ =45783 V/pC/m,  $\sigma_{X,Y}$ =100um, with wigglers).

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Figure 5: Transverse emittance growth vs transverse loss factor ( $N_e$ =3.2×10<sup>9</sup>,  $\sigma_{X,Y}$ =100um,with wigglers).



Figure 6: Transverse emittance growth vs trajectory offset  $(N_e=3.2\times10^9, k_{\perp BAPS}=45783 \text{ V/pC/m,with wigglers}).$ 

From Fig. 4, the transverse emittance will increase by 3 times for horizontal plane and 6.5 times for vertical plane at design current. For 45783 V/pC/m transverse loss factor, the transverse emittance will increase by 2.8 times and 6.5 times for horizontal and vertical direction respectively from Fig. 5. Also, Fig. 6 shows that the transverse emittance will increase by 3 times and 6.5 times for horizontal and vertical direction with the design trajectory error.

Table 3: Machine Tolerances for 100% Emittance Growth

Goal emittance (pm-rad)	Current (mA)	k⊥ (V/pC/m)	Trajectory offset (μm)
63/1.3 (bare lattice)	47/6	18000/4000	50/6
15/0.7 (with wigglers)	41/9	18000/4000	50/10

# Machine Tolerance According to Emittance Growth

One can consider three methods to control the emittance growth due to short range transverse wakefield by decreasing the bunch population, the machine impedance and the vacuum chamber misalignment (or the closed orbit distortion). Assuming the emittance growth is 100%, one gets the machine tolerances for these two kinds of lattice as in table 3.

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From all the figures and table 3, we can see that the emittance growth in vertical plane is much larger than horizontal because the design vertical emittance is much smaller than the design value for horizontal plane. And also, it is almost impossible to realize so small design value for the transverse emittance typically for the vertical plane.

### **CONCLUSION**

In this paper, the single bunch transverse emittance growth due to short range wakefield was estimated based on J. Gao' theory. The machine tolerances for beam current, transverse loss factor and trajectory error including the vacuum chamber misalignment and the closed orbit distortion were introduced. It is obvious that the emittance growth in vertical plane is much more serious than the horizontal plane. It seems that the design vertical emittance for both bare lattice and the one with damping wigglers are not available and even the design horizontal emittance for these two lattices can be a big challenge considering the real technology.

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