EXPERIMENTAL STUDIES OF SOLITON WAVE TRAINS IN INTENSE ELECTRON BEAMS*

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Abstract

Longitudinal perturbations in intense beams can lead to instabilities or degradation of beam quality, ultimately affecting the performance of accelerators, especially near the source where space charge is important. In this experimental study, conducted on the University of Maryland Electron Ring (UMER), large-amplitude perturbations are purposefully generated and their propagation observed over a long transport length. It is found that narrow, large-amplitude perturbations on a long-pulse beam develop into Korteweg-deVries (KdV) type soliton wave trains. Each peak in the wave train has a constant width and amplitude over a long propagation distance, with the amplitude inversely proportional to the square of the width. Furthermore, two such pulses are seen to interact with each other and emerge from the collision unchanged. The experimental data is compared with the KdV model and particle-in-cell simulations with good agreement.

INTRODUCTION

High brightness electron beams have wide applications in accelerator-driven light sources, X-ray, free-electron lasers (FELs), spallation neutron sources and intense proton drivers. Any beam degradation at low energy can be difficult to reduce and therefore also degrade high energy performance. For example a perturbation in current density can be frozen into the beam longitudinal profile as the beam is accelerated and all particles travel at the same velocity. T his eventually leads to consequences such as microbunching instability [1] and coherent synchrotron radiation. These perturbations could be generated by factors such as beam current modulation from the thermionic emission, photoemission or particle velocity modulations, or from the mismatch of the longitudinal focusing channels [2]. In space charge dominated beams, the perturbations could develop into space charge waves [3-4], which is not well understood, When the perturbation becomes nonlinear, solitons, defined as localized persistent waves that behave like particles, preserving their properties (shape, velocity, etc.) over long distances and through interactions and collisions with other solitons, were predicted [5-6]. Recently in the University of Maryland Electron Ring (UMER), we did comprehensive studies on solitons in electron beams with both experiments and simulations [7-8]. UMER is an electron storage ring, a scaled model to investigate the transverse and longitudinal dynamics of space charge dominated beams, with a circumference of 11.52m. The electron beam has 10keV energy, 0.3-3um emittance, and 197ns circulation time. The beam bunch has up to 10^{11} particles, and the duration varies from 25 to 140ns. It could operate in the current range of 0.6 to 100mA.

THEORETICAL MODEL

Small initial perturbations are known to split into two space charge waves, a slow wave and a fast wave, going in opposite directions in the beam frame [3, 9]. Larger perturbations are theoretically predicted to evolve into solitons, if the space charge in the beam is sufficiently strong [7-8]. This occurs because of the nonlinear steepening when the particles on the crest travel faster than the ones on the trough. As the length of the steepening wave front becomes comparable to the pipe diameter, dispersion is non-negligible and it will balance the steepening. In the cold fluid model, if the electric field is written as a series in terms of the derivatives of the line charge density, the first term in that series is proportional to $d\lambda^3/dz^3$, which eventually turns into the dispersion term in the Korteweg-deVries (KdV) equation. The beam evolution can be shown to approximately evolve according to the KdV equation known to describe soliton evolution:

$$\frac{\partial u}{\partial t}+\alpha\cdot u\frac{\partial u}{\partial z}+\beta\cdot\frac{\partial^3 u}{\partial z^3}=0\,,\quad \alpha,\ \beta \text{ constants,} \qquad (1)$$

where $u(z,t)$ is the density or velocity perturbation amplitude, as a function of longitudinal distance z and propagation time t. The second term represents the nonlinear effect that steepens and narrows the perturbation until it is comparable to the pipe diameter, resulting in several sub-pulses. The third term is the dispersion that tends to widen the pulse. The soliton arises from the cancellation of these two terms. An analytical solution to the KdV equation shown above is the single-soliton solution:

$$u(z,t)=\frac{3c}{\alpha}sech^2[\frac{1}{2}\sqrt{\frac{c}{\beta}}(z-ct)]\,, \qquad (2)$$

where c is wave speed. The evolution of a known initial perturbation profile $u(z,t=0)$ can be found by integrating the KdV equation over a time period τ to obtain $u(z,t=\tau)$. A numerical example is shown in [10] that illustrates a soliton train formation from a single initial pulse. We expect a similar perturbation evolution in experiments.

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EXPERIMENTAL SETUP

There are mainly two methods to introduce controllable large perturbations in the UMER beam. The first one is to apply photoemission on the long (~100ns), rectangular thermionic bunch by shooting a narrow laser pulse (~5ns) on the porous tungsten (W) cathode coated with barium oxide and calcium aluminate, so that the beam density is modified. A brief schematic is shown in Fig. 1. Refer to [8, 11] for more details. The other technique is to introduce a short electric pulse at the induction cell [2] inside the ring when the beam passes through, by which a beam velocity perturbation is generated. In this paper, we focus on the experiments with the first method.

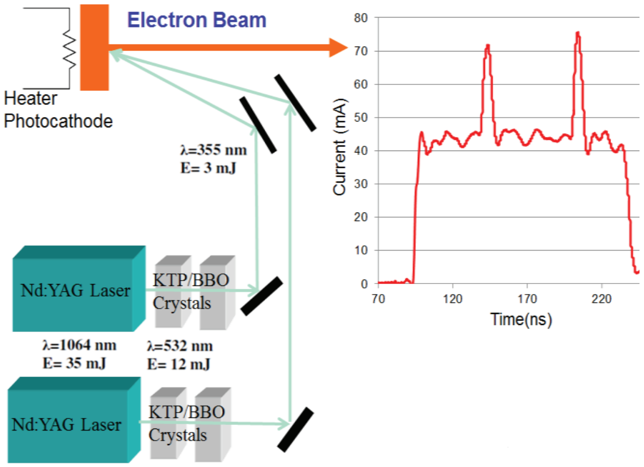

Figure 1: Schematic of the perturbation experimental setup (left) and the perturbed beam current profile (right).

With the above experimental setup, we are able to generate one or multiple perturbations. The perturbation level could be adjusted by the laser power, and its width could be modified by setting the triggering delay between the two perturbations. The initial condition of the perturbed beam is measured at the Bergoz, a fast current transformer 64cm downstream from the electron gun aperture. The multi-turn beam measurement is taken by a wall current monitor (WCM) inside the ring, which is 7.67m from the Bergoz.

EXPERIMENTAL RESULTS

Fig. 1 shows the turn-by-turn plot of the beam current, for a typical experiment with a large-amplitude initial perturbation. The peak beam current is 22 mA, with 5.5 mA above that (i.e., a 25% perturbation). The perturbation is introduced at the trailing edge of the beam so that there is the maximum distance to monitor the fast wave propagation on the beam, while the slow wave rolls off the edge. The beam currents are represented by positive values. For better comparison, the beam current is shifted upward by 12mA on the plot after every turn. As can be seen, the fast wave steepens and develops into a wave train after a few turns. The sub-pulses of the train maintain their shapes thereafter. The amplitude and width of each sub-pulse remain constants within measurement error [Fig. 3]. Also, the sub-pulse width is measured to be about 1 ns, which is 6 cm long while the pipe diameter is comparably 5.08 cm, so that the dispersion effect is sufficient to balance the wave steepening.

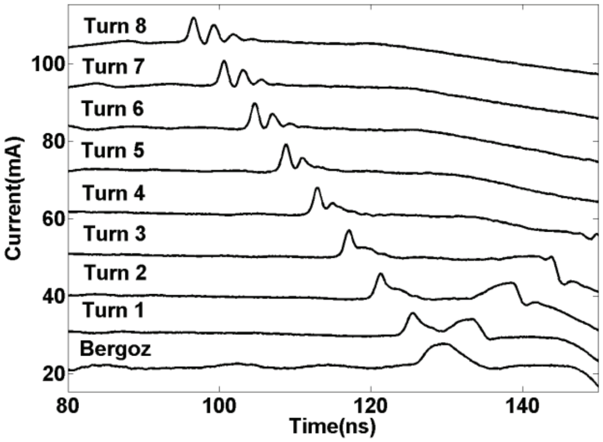

Figure 2: Turn-by-turn plot of 22 mA beam with one 25% density perturbation pulse.

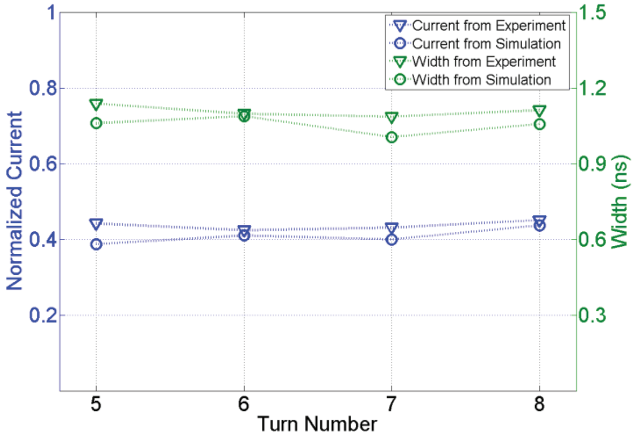

Figure 3: Width and amplitude of the 1st sub-pulse at different turns in the ring.

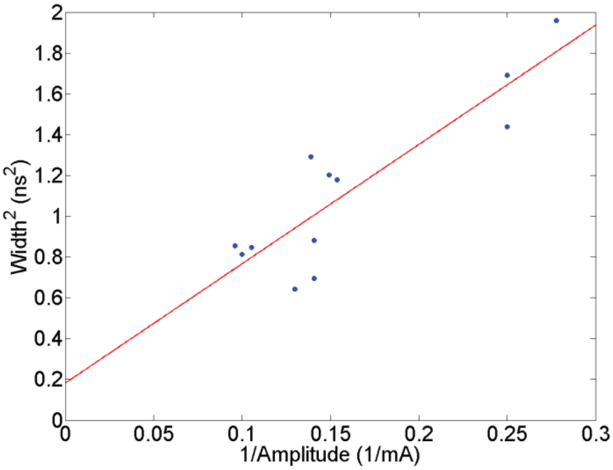

Figure 4: Plot of soliton pulse width^2 vs 1/Amplitude, along with its linear fit.

To support the assertion that solitons are in fact what we observe, the KdV analytic model in Eqn. (1) is used. According to the KdV soliton solution in Eqn. (2), the width of the soliton (w) is inversely proportional to the square root of its amplitude (A) [Fig. 4], in other words, w^2A= constant.

In addition, we performed additional experiments to confirm that the observed waves have another soliton characteristic: "They can interact with other solitons, and emerge from the collision unchanged, except for a phase shift" [12]. Using the setup in Fig. 1, there are two space charge waves propagating towards each other. As Fig. 5 illustrates, the two wave trains emerge completely unmodified by the collision. Comparison with an experiment with only one perturbation reveals little difference in the shape and amplitude of the sub-pulses after the collision.

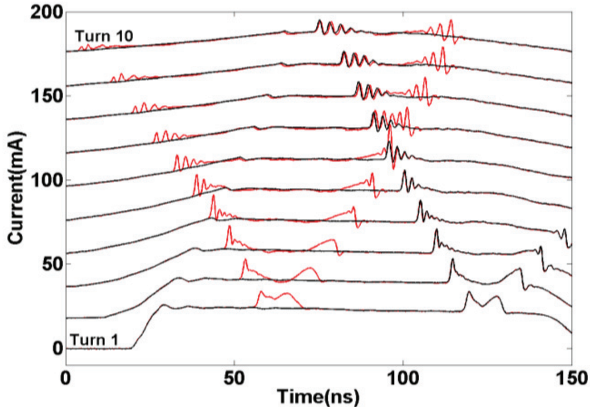

Figure 5: Comparison of two-perturbation experiment and one-perturbation experiment (30mA, 50%). The fast wave of the right perturbation interact with the slow wave of the left perturbation (red), is compared with the fast wave propagation of the one perturbation experiment (black).

SIMULATION RESULTS

Fig. 6 shows the evolution of the beam perturbation using the Warp particle-in-cell (PIC) code [13] in an RZ geometry, and its comparison with experiment. A good agreement is achieved. Refer to [7] for more details.

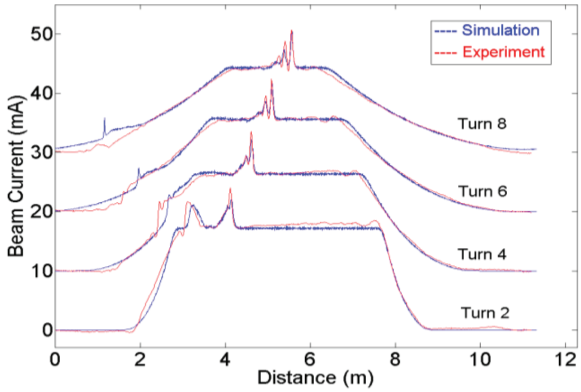

Figure 6: Beam current profile comparison between experiment (red) and simulation (blue) at different turns.

CONCLUSION

An experimental observation of soliton wave trains on an intense electron beam is presented, with proof from the theory and simulation results. These findings presented here are scalable to larger accelerators, provided the relative strengths of space charge to external forces are the same. We expect such a soliton-train modulated electron beam to be potentially used as a tunable, coherent radiation source.

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