MODELLING OF THE EMMA ns-FFAG RING USING GPT

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Abstract

EMMA (Electron Machine with Many Applications) is a prototype non-scaling Fixed-Field Alternating Gradient (ns-FFAG) accelerator whose construction at Daresbury Laboratory, UK, was completed in the autumn of 2010. The energy recovery linac ALICE [1] serves as an injector for the EMMA ring, within an effective energy range of 10 to 20 MeV. The ring is composed of 42 cells, each containing one focusing and one defocusing quadrupole.

Acceleration over many turns of the EMMA machine has recently been confirmed. In some cases the bunch will traverse upwards of 100 turns, at which point the effects of space-charge may be significant. It is therefore necessary to model the electron beam transport in the ring using a code capable of both calculating the effect of and compensating for space-charge. Therefore the General Particle Tracer (GPT) code [2] code has been used.

A range of injection beam parameters have been modelled for comparison with experimental results and those of other codes. The simulated effects of space-charge on the tune shift of the machine are also compared with those expected from theory.

INTRODUCTION

Commissioning of the EMMA ring (the world's first ns-FFAG) concluded in 2012. The energy recovery linac ALICE acts as the injector for EMMA, providing single bunches of electrons at an energy between 10 to 20 MeV with a maximum bunch charge of 32 pC. A schematic of the EMMA ring is shown in Fig. 1, highlighting some of the important diagnostic components of the beamline.

The beam injected from ALICE into EMMA is at quite a low energy where there may be significant space-charge emittance growth, depending on the input conditions. Therefore rigorous analysis of the effects of space-charge is necessary. Consequently the particle tracking software used must incorporate the ability to model the effects of space-charge; thus GPT was chosen. This paper is an account of the GPT modelling of the EMMA ring using and including the effect of space-charge, for a range of injection parameters.

THE EMMA RING IN GPT

Modelling of the EMMA ring was conducted using GPT due to its ability to model space-charge effects. One revolution of the ring was thus modelled in GPT, transporting a beam of 15 MeV (as well as a nominal bunch charge of 0 pC) with matched twiss parameters from the exit of the EMMA injection line. In addition the bunch was initiated with a uniform energy and finite length for ease of computation. The 42 magnetic cells of the ring were modelled in GPT using the repeated input of a single field-map of one magnetic cell (a schematic of such a magnetic cell can be seen in Fig. 2). This magnetic field was then held constant, allowing for optimisation of the initial beam parameters to achieve a closed orbit solution. Once this process was complete it was repeated for the full range of EMMA injection energies. The average x-position for a single bunch at differing energies can be seen in Fig. 3, demonstrating closed orbits in each case. This plot also serves to demonstrate the increased path length of closed orbits with a decrease in energy, thus defining the parabolic time of flight indicative of a non-scaling FFAG.

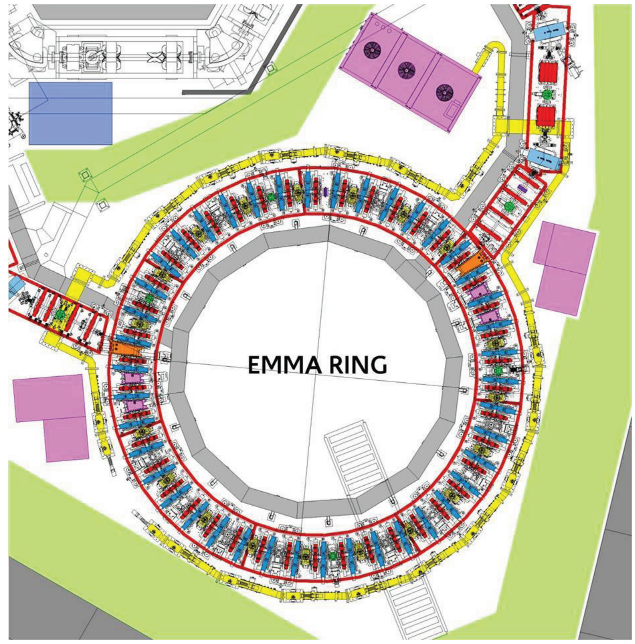

Figure 1: The EMMA ring, with a circumference of 16.57 m.

TUNE CALCULATIONS

The tune of a machine is described as the number of betatron oscillations undergone across a discrete beam length. As previously described, the EMMA lattice is a repeating

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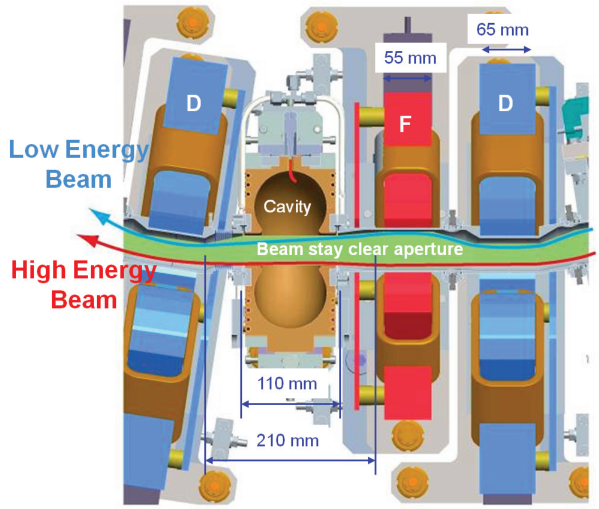

Figure 2: A schematic of a few magnetic cells, which comprise the EMMA ring.

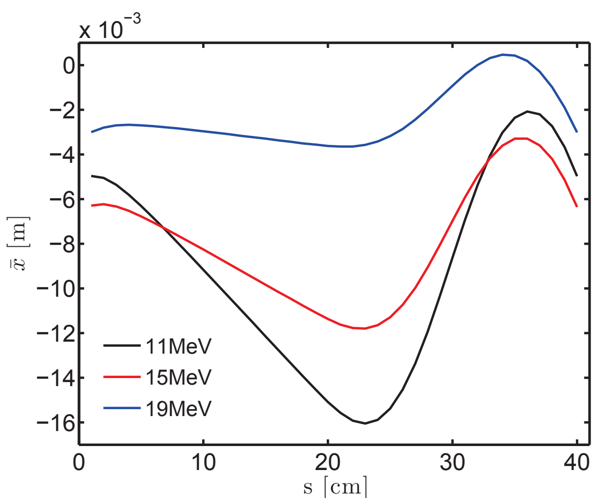

Figure 3: Mean horizontal position for three closed orbits of differing energy in one EMMA cell. The transverse coordinates at the entrance of the cell are equal to those at the exit after a rotation of $2\pi/42$, not represented in this figure.

structure of 42 identical magnetic cells and is thus intrinsically periodic; the beam encounters the same magnet structure once every full revolution, therefore experiencing periodically repeating forces. This tune calculation is achieved by sampling the average x- and y-position of the bunch at the same position in each cell for a large number of revolutions. In this case 1000 turns of the ring are modelled therefore sampling 4200 times. A fast fourier transform (FFT) is performed on these values, resulting in the transverse tune $\nu_{x,y}$. This process is repeated across the full EMMA energy regime, with the results shown in Fig. 4.

Tune calculations have been produced using a number of different codes, all modelling EMMA with the baseline parameters: `MAD` (with displaced quadrupoles) [3], `BERG` (an internal code created specifically by the EMMA collaboration), and `Zgoubi` [4]. The tune calculation results for these different codes, along with `GPT`, can be seen in Fig. 4. All the models demonstrate good agreement, including the `GPT` simulation: the horizontal tune sits slightly higher than the other codes but shows the same shape; the vertical tune lies in the middle of the other codes but is slightly skewed at lower energies.

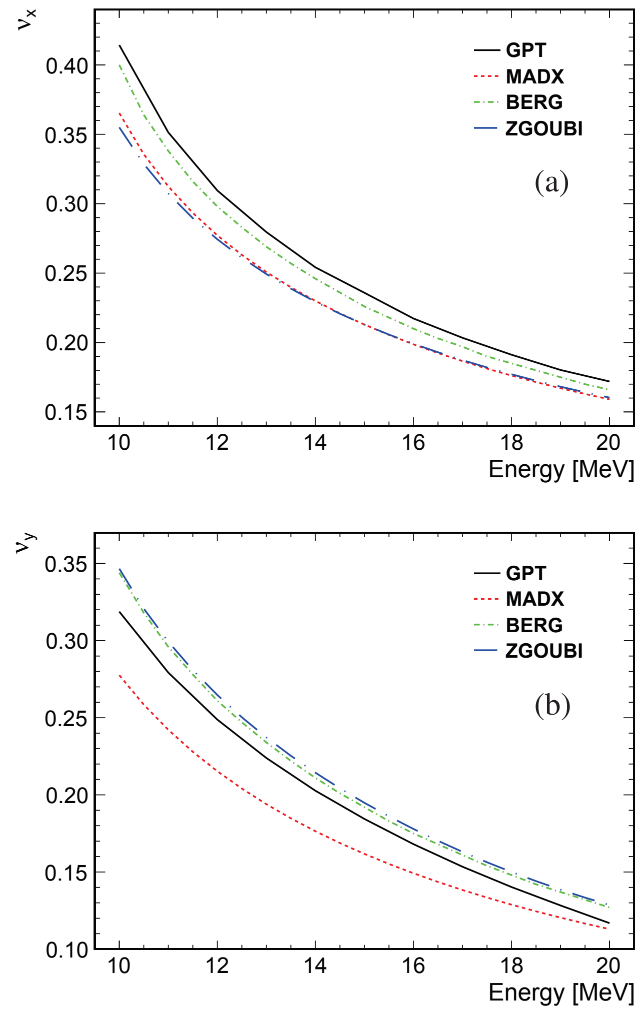

Figure 4: Comparison of the a) horizontal, and b) vertical tune produced by different codes modelling the EMMA ring with baseline parameters.

SPACE-CHARGE EFFECTS

The Coulomb forces between the charged particles of a high-intensity beam in an accelerator create a self-field which acts on the particles inside the beam like a distributed lens, defocusing in both transverse planes. A beam moving at a certain velocity is accompanied by a magnetic field which partially cancels the electrostatic defocusing effect, with complete cancellation at the speed of light. The effect of this direct space-charge alters the number of betatron oscillations per machine turn, ν, by $\Delta\nu$.

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For a Gaussian beam separated into discrete bunches, where space-charge effects are more profound due to an increase in bunch density, the tune is given by

$$\Delta\nu_{x,y} = -\frac{rN}{4\pi\epsilon_{Nx,y}\beta\gamma^2}\frac{C}{\sqrt{2\pi}\sigma_{\rm rms}} , \qquad (1)$$

where β is the Twiss parameter related to spatial beam size and the remaining parameters are defined in Table 1, with their corresponding values for a typical 15 MeV beam in the EMMA ring [5].

Table 1: Bunch parameters for a 15 MeV beam, used to calculate an expected tune shift arising from space-charge at the EMMA bunch-charge regime.

Lorentz factor γ	29.3
Number of particles N	5×10^8 (80 pC)
Normalised emittance $\epsilon_{Nx,y}$	10 π mm mrad
Bunch length $\sigma_{\rm rms}$	1.0 mm
Circumference C	16 m
Electron radius r	2.82×10^{-15} m

This analytical form can then be used to calculate tune shifts independent of the tune value at that energy. The shift for an 80 pC bunch at 15 MeV represents an approximate 3% shift from the nominal value calculated by GPT at this energy for zero bunch-charge. A 3% shift is more than that which is typically tolerable in a high intensity proton synchrotron or storage ring and, although the beam stays in EMMA for only 10 to 20 turns, resonance crossings excited by space-charge are a concern. Figure 5 shows the relation between the GPT results at 0 pC and an arbitrary space-charge regime of 80 pC, across the full EMMA injection energy region.

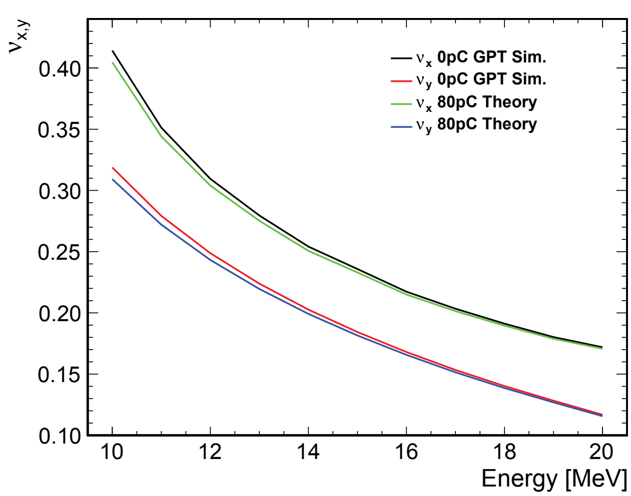

Figure 5: Theoretical tune shift from an 80 pC bunch-charge, compared with the tune calculated from GPT simulations with 0 pC.

The beam energy was then fixed to 15 MeV and the bunch-charge scaled up from zero until a tune-shift was observed. Figure 6 shows the expected linear decrease in tune shift with an increase in bunch-charge. The first data point has no associated uncertainty as it is the GPT output for 0 pC, with all theoretical tune shifts stated relative to this value. These tune values were calculated from a simulation of 100 turns of the machine.

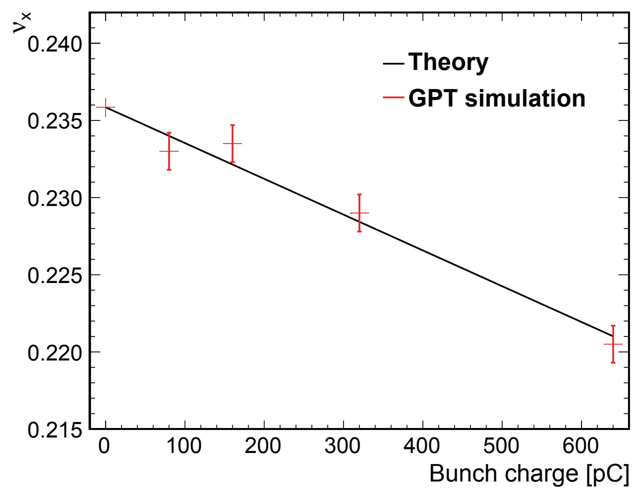

Figure 6: A demonstration of tune shift caused by space-charge over a range of bunch-charges (within and beyond the EMMA regime), with a fixed beam energy of 15 MeV. The theoretical data is taken with respect to the 0 pC simulation as performed by GPT, with no uncertainty on this data point.

CONCLUSIONS

Modelling of the EMMA ring in GPT has been completed. The data generated by GPT's space-charge procedure agrees with the theoretical predictions to within the resolution limit of the FFT routine, thus confirming the validity of the GPT spacecharge3Dmesh routine.

REFERENCES

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