

dispersion functions are designed to be small in the IRs. Therefore, the QF8 and QF9 magnets with moderate dispersion functions are selected for matching DX prime. The total number of these magnets are 12 [7].

In addition to DX prime at the snakes, there are constraints on other parameters in the matching to minimize the disturbance to the other optical parameters. These parameters are the DX and beta stars at IP6 and IP8, two interaction points for experiments; the global tunes and beta-beat at one of the quadrupole of each arc. The DX prime difference was reduced from 0.05 to 2×10^{-5} by this scheme. The beta-beat is $\pm 30\%$ peak-to-peak in the horizontal plane, $\pm 8\%$ in the vertical plane. No dispersion peak was introduced along the ring.

The introduced beta-beat are shown in Figs. 2 and 3.

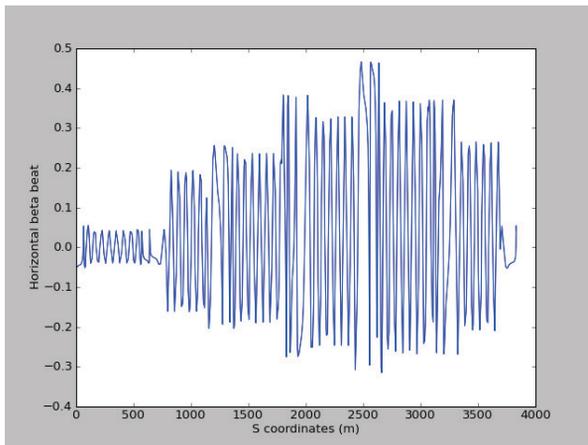


Figure 2: Introduced horizontal beta-beat in QF8+QF9 solution.

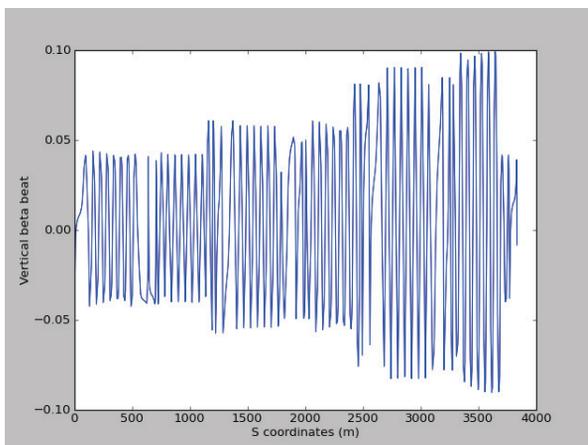


Figure 3: Introduced vertical beta-beat in QF8+QF9 solution.

The horizontal dispersion functions of the ring are shown in Fig. 4.

The initial and final strengths of all the 12 magnets and the relative changes are shown in Table 1.

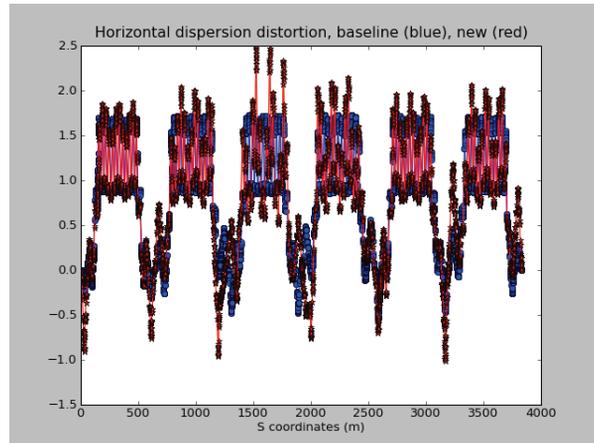


Figure 4: Baseline and new horizontal dispersion functions in QF8+QF9 solution.

Table 1: Initial and Final Strength of QF8 and QF9 Magnets

Magnets	Final	Initial	Relative (%)
qfa6	0.0797930	0.0851501	-6.29
qfa8	0.0875657	0.0851501	2.84
qfa10	0.0862944	0.0807370	6.88
qfa12	0.0826227	0.0807370	2.33
qfa2	0.0843956	0.0823113	2.53
qfa4	0.0823227	0.0807370	1.96
qfb6	0.0840926	0.0845783	-0.57
qfb8	0.0844324	0.0845783	-0.17
qfb10	0.0805808	0.0795547	1.28
qfb12	0.0734157	0.0795547	-7.71
qfb2	0.0839466	0.0812185	3.35
qfb4	0.0758067	0.0795547	-4.71

The allowed relative change of these magnets is -3–0%. There are 5 required magnet strengths exceeding the limits, 7 magnets that need opposite polarity. This scheme would also require certain amount of power supply work.

CONTROLLING REQUIRED STRENGTH CHANGE

In the simulation in the above section, we attempted to put the power supplies' limits in the matching as constraints in order to find a practical solution. No solution could be found by the MAD-X matching module.

A response matrix and SVD (single value decomposition) [8] technique was then employed to match DX prime while being able to control the changes of quadrupole strengths. In this scheme, we first need to check the linearity of the response of parameters of interest to variables (for our case, DPX and tunes to quadrupole strength). The calculation done using MAD-X confirmed the linearity of the response. At the same time, the response matrix for parameters of interest to variables was established and stored.

Then the response matrix was inverted using the SVD algorithm and the required changes of quadrupole strength and the resulting parameters of interest were calculated. As a confirmation, parameters of interest were simulated by MAD-X as well by putting in the changes of strength calculated in the previous step.

With a regular SVD, 2 of the final quadrupole strengths exceeded their limits. Cutting eigenvalues and Tikhonov regulation [9] have been applied to limit the required quadrupole strengths which would end with less optimal matching result. In addition, one other trick is to set the two magnets to be at the maximum strengths allowed. The last one resulted in the best overall performance. DX prime difference of the two snakes was reduced by a factor of 3 with all magnets within limits.

The corresponding beta-beat are shown in Figs. 5 and 6.

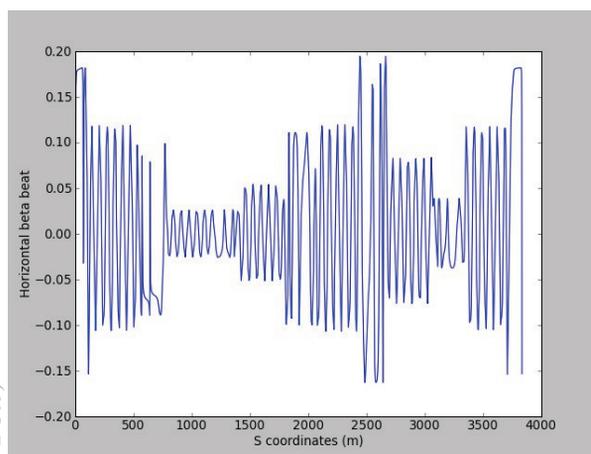


Figure 5: Introduced horizontal beta-beat with controlling required strength change.

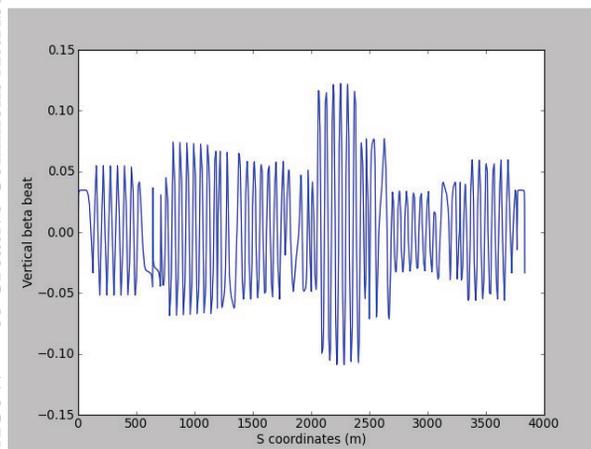


Figure 6: Introduced vertical beta-beat with controlling required strength change.

The horizontal dispersion is shown in Fig. 7.

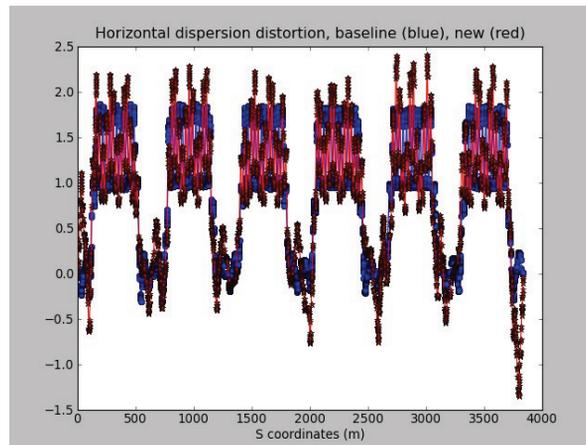


Figure 7: Baseline and new horizontal dispersion functions with controlling required strength change.

SUMMARY

To match DX prime at the two snakes for minimal spin tune spread, the QF8 and QF9 magnets are chosen because of their high dispersion functions. Matching with MAD-X produced good results in terms of DX prime matching, global beta-beat and dispersion. However, the results are not practical because of power supplies' limits. The employment of response matrix and SVD technique is able to keep all magnets within their limits and reduce the DX prime difference by a factor of 3.

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