SPIN TUNE DECOHERENCE EFFECTS IN ELECTRO- AND MAGNETOSTATIC STRUCTURES

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Abstract

In Electric Dipole Moment search experiments with polarized beams the coherence of spin oscillations of particles has a leading role [1]. The decoherent effects arise due to spin tune dependence on particle energy and particle trajectory in focusing-bending fields. They are described through the n-th order spin tune aberrations. Since the first order is suppressed by RF field, the second order begins to play a crucial role. It depends on the orbit lengthening and on the odd order field components. We consider the spin decoherence effects and methods of their compensation in different channels, electrostatic, magnetostatic linking the decoherence effects with common characteristics such as the momentum compaction factor, the chromaticity and others.

SYNCHROTRONOUS PRINCIPLE

The basic "synchrotronous acceleration principle" of Veksler and McMillan is formulated by simple system equation:

$$\frac{d\varphi}{dt} = -\omega_{rf}\eta\delta$$
$$\frac{d\delta}{dt} = \frac{eV_{rf}\omega_{rf}}{2\pi h\beta^2 E}\sin\varphi \qquad (1)$$

where $\delta = \dfrac{\Delta p}{p_s}$ is momentum deviation from equilibrium synchronous level of momentum $\Delta p = p - p_s$, φ is phase deviation from synchronous phase $\varphi_s = 0$ (no acceleration in storage ring), η is sleep-factor, E is full energy, β is relative velocity, eV_{rf} is energy gain per turn with V_{rf} voltage gap, $\omega_{rf} = 2\pi h f_{rev}$ is angular frequency of RF field, h is a harmonic number, $f_{rev} = 1/T_{rev}$ is revolution frequency.

The first equation of system (1) comes directly from "synchrotronous acceleration principle": particle having shorter revolution time arrives earlier and gets in an earlier phase of the RF field:

$$\frac{\Delta\varphi}{\varphi} = -\frac{\Delta T_{rev}}{T_{rev}} \qquad (2)$$

In the first approach change of orbit length C versus momentum deviation is defined by $\Delta C / C = \alpha_0 \cdot \delta$, and we can write:

$$\frac{\Delta T_{rev}}{T_{rev}} = \frac{\Delta(C/v)}{C/v} = \frac{\Delta C}{C} - \frac{\Delta v}{v} = \left(\alpha_0 - \frac{1}{\gamma^2}\right)\cdot\delta \quad (3)$$

Slip factor $\eta = \alpha_0 - 1/\gamma^2$ is introduced by ratio between revolution time and momentum deviations:

$$\frac{\Delta T_{rev}}{T_{rev}} = \eta \cdot \delta \qquad (4)$$

Obviously, in case of bunched beam the revolution time deviation averaged over one synchrotron oscillation is zero. In first approach it follows from solution of (1). However using the higher expansion in power of momentum compaction factor $\alpha = \alpha_0 + \alpha_1 \cdot \delta$ and velocity $(v_s + \Delta v)^{-1} = v_s[1 - \Delta v / v_s + (\Delta v / v_s)^2 - ...]$ the deviation of time revolution deviation can be represented in form:

$$\frac{\Delta T_{rev}}{T_{rev}} = \frac{\Delta C}{C} - \frac{\Delta v}{v_s} - \frac{\Delta C}{C}\cdot\frac{\Delta v}{v_s} + \left(\frac{\Delta v}{v_s}\right)^2$$
$$= \left(\alpha_0 - \frac{1}{\gamma^2}\right)\cdot\delta + \left(\alpha_1 - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\delta^2 \qquad (5)$$

In addition, we must include to the orbit lengthening value $(\Delta L / L)_\beta$ due to the betatron motion:

$$\frac{\Delta T_{rev}}{T_{rev}} = \left(\alpha_0 - \frac{1}{\gamma^2}\right)\cdot\delta + \left(\alpha_1 - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\delta^2 + \left(\frac{\Delta L}{L}\right)_\beta \quad (6)$$

Then the longitudinal motion equations can be written as:

$$\frac{d\varphi}{dt} = -\omega_{rf}\left[\left(\alpha_0 - \frac{1}{\gamma^2}\right)\cdot\delta + \left(\alpha_1 - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\delta^2 + \left(\frac{\Delta L}{L}\right)_\beta\right]$$
$$\frac{d\delta}{dt} = \frac{eV_{rf}\omega_{rf}}{2\pi h\beta^2 E}\sin\varphi \qquad (7)$$

Let us assume that $\varphi << 1$, that is $\cos\varphi \approx 1$, and write the equation for momentum deviation δ as:

$$\frac{d^2\delta}{dt^2} + \frac{eV_{rf}\omega_{rf}^2}{2\pi h\beta^2 E}\left(\alpha_0 - \frac{1}{\gamma^2}\right)\cdot\delta =$$
$$-\frac{eV_{rf}\omega_{rf}^2}{2\pi h\beta^2 E}\cdot\left[\left(\alpha_1 - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\delta^2 + \left(\frac{\Delta L}{L}\right)_\beta\right] \qquad (8)$$

From (6) we see that average value $\overline{\Delta T_{rev} / T_{rev}} \neq 0$ is not zero, and it is defined by $\alpha_0, \alpha_1, \gamma$ and $(\Delta L / L)_\beta$:

$$\overline{\frac{\Delta T_{rev}}{T_{rev}}} = \left(\alpha_1 - \frac{\alpha_0}{\gamma^2} + \frac{1}{\gamma^4}\right)\cdot\overline{\delta^2} + \overline{\left(\frac{\Delta L}{L}\right)_\beta} \qquad (9)$$

As follows from equation (8) the orbit lengthening must be compensated by an equilibrium momentum level rising to be consistent with the basic "synchronous acceleration principle". Solving (8) using asymptotic

methods [2] we can define an influence of the betatron oscillation, the square term of momentum compaction factor α_1 and the slip factor η onto the equilibrium level energy shift $\Delta\delta_{eq}$:

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \frac{\alpha_0}{\gamma_s^2} + \frac{1}{\gamma_s^4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right] (10)$$

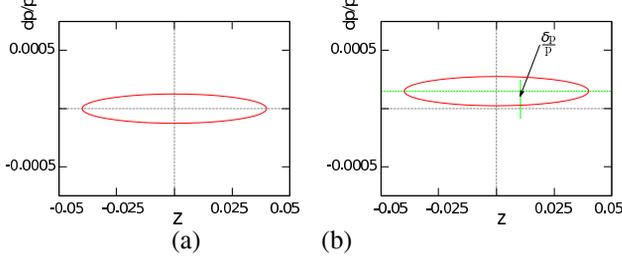Expression (10) means that the equilibrium momentum is different for every particle in bunch.

Figure 1: Phase trajectory in longitudinal plane for initial coordinates x=0, y=0 (a) and x=3 mm, y=0 (b).
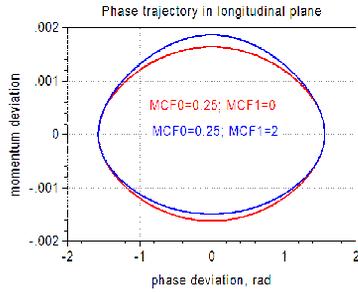

Figure 2: Phase trajectory in longitudinal plane for $\alpha_1 = 0$ and $\alpha_1 \neq 0$ without betatron oscillation.

As example Fig. 1 shows the results of COSY Infinity calculation in electrostatic ring [3] when the equilibrium momentum level rises up due to betatron motion. In the same time due to the non-zero second order momentum compaction factor $\alpha_1 \neq 0$ and $(\Delta v/v_s)^2$ phase trajectories lose symmetry in the longitudinal plane in direction of the momentum, and thus lead to a shift of the equilibrium momentum value. Figure 2 shows the cases of an axial particle with zero and non-zero second order momentum compaction factor. The effect with $\alpha_1 \neq 0$ is similar to sextupole effect in the transverse plane when the phase trajectory begins to take a shape of a triangle.

ORBIT LENGTHENING

Betatron Motion

Now we define parameters α_0, α_1 and $(\Delta L/L)_\beta$ on the basis of simple geometric considerations. Figure 3 shows the orbit lengthening due to momentum spread δ and betatron oscillation $(\Delta L/L)_\beta$. Let us begin from the case when the orbit lengthening arises due to betatron oscillation (Fig. 3a). Assume that the particle has

parameters $\{x_\beta, x'_\beta\}$ at a time. Due to a larger radius $\rho + x_\beta$ the orbit is longer by factor $(\rho + x_\beta)/\rho$ and due to x'_β is longer by factor $1/\cos x_\beta$. Together with the vertical motion the factor of lengthening is $1/\cos\vartheta$, where $\vartheta = \sqrt{x'^2_\beta + y'^2_\beta}$.
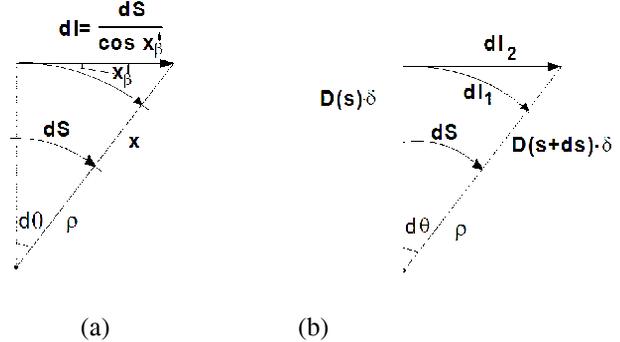

Figure 3: Orbit lengthening due to betatron oscillation (a) and momentum spread (b)

Then the orbit lengthening due to betatron oscillation is:

$$\left(\frac{\Delta L}{L}\right)_\beta = \frac{1}{L}\oint\left(\frac{\rho + x_\beta}{\rho\cos\theta} - 1\right)ds = \frac{1}{L}\oint\left(\frac{x_\beta}{\rho} + \frac{x'^2_\beta + y'^2_\beta}{2}\right)ds$$

Since $\left\langle\frac{x_\beta}{\rho}\right\rangle = 0$, $\langle x'^2_\beta\rangle = \frac{1}{2}\left\langle\frac{\varepsilon_x}{\beta_x}\right\rangle$, $\langle y'^2_\beta\rangle = \frac{1}{2}\left\langle\frac{\varepsilon_y}{\beta_y}\right\rangle$ and $\langle 1/\beta_{x,y}\rangle = v_{x,y}/\overline{R}$ the orbit lengthening due to the betatron motion is:

$$\left(\frac{\Delta L}{L}\right)_\beta = \frac{\pi}{2L}\left[\varepsilon_x v_x + \varepsilon_y v_y\right] (11)$$

Momentum Deviation

Now let us go back to the orbit lengthening due to the momentum deviation. For that we will appeal to figure 3b. All notations of variables are introduced in accordance with this figure. First of all we define linear and angular dispersion:

$$D(s,\delta) = D_0(s) + D_1(s)\cdot\delta$$
$$D'(s,\delta) = D'_0(s) + D'_1(s)\cdot\delta \quad (12)$$

In arbitrary point along $ds = \rho d\theta$:

$$dl_1 = \left(\rho + D_0\cdot\delta + D_1\cdot\delta^2\right)d\theta = \left(1 + \frac{D_0}{\rho}\cdot\delta + \frac{D_1}{\rho}\cdot\delta^2\right)ds ,$$

$$dl_2 = dl_1\sqrt{1 + (D'_0\delta)^2} =$$
$$\left(1 + \frac{D_0}{\rho}\cdot\delta + \frac{D_1}{\rho}\cdot\delta^2\right)\cdot\left(1 + \frac{1}{2}(D'_0\delta)^2\right)ds \quad (13)$$

As a result we have:

$$l_2 = \oint\left[1 + \frac{D_0}{\rho}\cdot\delta + \left(\frac{D_1}{\rho} + \frac{1}{2}D'^2_0\right)\cdot\delta^2\right]ds \quad (14)$$

Thus, the orbit lengthening due to the momentum deviation is:

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(15)

$$\frac{\Delta C}{C} = \frac{l_2 - C}{C} = \alpha_0 \delta + \alpha_1 \delta^2 + ...$$

$$\alpha_0 = \left\langle \frac{D_0}{\rho} \right\rangle; \quad \alpha_1 = \left\langle \frac{D_1}{\rho} \right\rangle + \frac{1}{2} \left\langle D_0'^2 \right\rangle$$

In result we have the equilibrium momentum spread due to the betatron motion and non-zero second order momentum compaction factor:

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2 \alpha_0 - 1} \left[\left(\alpha_1 - \frac{\alpha_0}{\gamma_s^2} + \frac{1}{\gamma_s^4} \right) \cdot \frac{\delta_m^2}{2} + \frac{\pi}{2L} \left(\varepsilon_x \nu_x + \varepsilon_y \nu_y \right) \right]$$

(16)

ORBIT LENGTHENING AND SPIN DECOHERENCE

As we know the spin tune $\nu_s = \gamma G$. If the equilibrium energy $\Delta\gamma_{eq}$ depends on the particle parameters the spin tune spread for N_t turns has incoherent spread:

$$2\pi \langle \Delta \nu_s \rangle = 2\pi G \langle \Delta \gamma_{eq} \rangle N_t \qquad (17)$$

It reduces spin decoherence time. For example, let us consider the case with the spin decoherence time limited by 1000 seconds ($\sim 10^9$ turns) and $\langle \Delta\gamma_{eq} / \gamma \rangle < 1\,\text{rad}/2\pi\gamma GN_t = 7\cdot 10^{-11}$. Then using expression (10) we can define limit for momentum spread:

$$\left\langle \delta_m^2 \right\rangle < \left\langle \frac{\Delta\gamma_{eq}}{\gamma} \right\rangle \cdot \frac{2}{\beta^2} \cdot \frac{\gamma_s^2 \cdot \left(\gamma_s^2 \alpha_0 - 1 \right)}{\gamma_s^4 \alpha_1 - \gamma_s^2 \alpha_0 + 1} \quad (18)$$

In COSY ring $\alpha_0 = 0.2$, $\gamma_s = 1.248$, $\alpha_1 = 2$ taking zero contribution from betatron motion ($\varepsilon_{x,y} \sim 0$) RMS momentum spread should not exceed the value $\langle \delta_m \rangle < 8\cdot 10^{-6}$. Reducing the second order of MCF up to $\alpha_1 = 0.01$ we get $\langle \delta_m \rangle < 2\cdot 10^{-5}$. In order to exclude completely the momentum spread influence on the orbit lengthening the coefficient in (10) has to be zero

$$\alpha_1 - \alpha_0 / \gamma_s^2 + 1/\gamma_s^4 = 0 . \qquad (19)$$

Now let us estimate the restriction for the emittance value:

$$\varepsilon_{x,y}^{rms} < \left\langle \frac{\Delta\gamma_{eq}}{\gamma} \right\rangle \cdot \frac{1}{\beta^2} \cdot \frac{\gamma_s^2 \alpha_0 - 1}{\gamma_s^2} \cdot \frac{L}{\pi\nu_{x,y}} \quad (20)$$

In COSY ring taking $\langle \delta_m \rangle \ll 10^{-5}$ the emittances should be $\varepsilon_{x,y}^{rms} < 1.4\,\text{mm mrad}$. Thus, we can conclude that contribution to the spin tune decoherence is the same for $\varepsilon_{xyrms} \approx 1\,\text{mm mrad}$ and $\delta_{rms} \approx 10^{-5}$.

SEXTUPOLE CORRECTION OF ORBIT LENGTHENING

Let us consider single sextupole effecting on the orbit lengthening. The second order momentum compaction

factor plays the crucial role in the orbit lengthening due to momentum and simultaneously it depends on normalized sextupole strength $S_{sext} = \left(1/2B\rho\right)\cdot \partial^2 B_y / \partial x^2$ as

$$\Delta\alpha_{1,sext} = -\frac{S_{sext} \cdot D_0^3}{L} . \qquad (21)$$

We can correct α_1 up to required value

$$\alpha_1 + \Delta\alpha_{1,sext} = \frac{\alpha_0}{\gamma_s^2} - \frac{1}{\gamma_s^4} \qquad (22)$$

Simultaneously the sextupole affects on the orbit lengthening directly:

$$\left(\frac{\Delta L}{L} \right)_{sext} = \mp \frac{S_{sext} D_0 \beta_{x,y} \varepsilon_{x,y}}{L} . \qquad (23)$$

Based on (11,23) a simple solution follows:

$$S_{sext} = \mp \frac{\pi\nu_{x,y}}{2D_0\beta_{x,y}} \qquad (24)$$

Together with α_1 correction three families of sextupoles are required, and they should be placed in locations with maximum/minimum value of D_0 and $\beta_{x,y}$ functions.

FEATURES OF ELECTROSTATIC RING

In electrostatic deflector due to converting of kinetic energy to potential the velocity spread $\Delta v / v$ reduces by some factor F_v dependent on the optics with almost unchanged trajectory. Then for arbitrary particle we have additional factor $1/F_v$ in expression (6):

$$\frac{\Delta T_{rev}}{T_{rev}} = \left(\alpha_0 - \frac{1}{F_v}\frac{1}{\gamma^2} \right)\cdot \delta$$
$$+ \left(\alpha_1 - \frac{1}{F_v^2}\frac{\alpha_0}{\gamma^2} + \frac{1}{F_v^2}\frac{1}{\gamma^4} \right)\cdot \delta^2 + \left(\frac{\Delta L}{L} \right)_\beta$$

(25)

It means in order to compensate the decoherence effect coming from momentum spread we have to adjust α_1 by factor F_v smaller. Simultaneously, we take note that the slip factor can even change sign when $F_v > 1/\alpha_0\gamma^2$, and behavior of beam will be like at higher critical energy, in particular, the synchronous phase change by π.

ACKNOWLEDGMENT

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