# HIGH-GAIN X-RAY FELS USING A TRANSVERSE GRADIENT UNDULATOR IN A DIFFRACTION-LIMITED STORAGE RING*

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*Abstract*

Storage ring FELs operated in the low-gain regime use optical cavities and are limited by mirror technology to 200 nm radiation wavelength. The development of the so-called diffraction-limited storage ring requires both transverse emittances on the order of 10 pm, on par with the beam quality obtained from linac-based x-ray FELs. However, the relatively large energy spread in such a diffraction-limited storage ring usually leads to a rather long gain length for a high-gain FEL in the x-ray wavelengths. The energy spread can be mitigated by dispersing the electron beam in a transverse plane and by using a transverse gradient undulator. Using PEPX as a representative example, we show from theory and simulations that a high-gain FEL at 1 nm radiation wavelength can be achieved in a bypass line with the undulator length of about 100 m.

## INTRODUCTION

Linac-based x-ray free-electron lasers (FEL), with a peak brightness of approximately ten orders of magnitude over third-generation light sources, have shown remarkable scientific capabilities in different research disciplines. The typical repetition rate of those facilities such as LCLS and SACLA is on the order of 100Hz or less, which is mainly limited by the normal conducting rf structure. At the same time, there is a growing scientic interest in X-ray FEL sources that can provide a continuous train of evenly spaced, low peak power, coherent photon pulses at repetition rates of above 1 kHz. These CW sources would enable dynamic imaging of materials undergoing transitions in millisecond or less time scales, and lead to a better under-standing of electronic and nuclear dynamics in materials.

To provide such a CW mode x-ray FELs, superconducting-based accelerators would be an ideal choice. On the other hand, there is a worldwide interest in developing so-called diffraction-limited storage ring (DLSR) light sources. This would provide spectral brightness two orders of magnitude higher than the present 3rd generation sources and also large coherent flux in the multi-keV photon energy range [1]. A recent study based on a fourth-order achromatic lattice [2] shows a natural emittance on the order of 10 pm-rad which reaches the diffraction limit for 10-keV x-ray photons.

With the achievable low-emittance electron beam in a DLSR, the transverse brightness is sufficient to drive a high gain x-ray FEL. However, the low peak current and large energy spread prevent obtaining a reasonable gain. For example, a typical peak current in a DLSR is a few tens Amperes, and relative energy spread is on the order of $10^{-3}$. To achieve a higher e-beam peak current, a scheme of shortening the electron bunch has been discussed in [3] by increasing RF focusing. Using PEPX as an example, superconducting RF cavities operating at 1428MHz in CW mode was suggested to reduce the bunch length by a factor 10. At the same time, a transverse-gradient undulator was also proposed to compensate the large energy spread in a DLSR. In this paper, we expand the discussions in [3] to explore optimizations of the DLSR FEL performance based on theory and simulations.

## TRANSVERSE-GRADIENT UNDULATOR

The concept of using a "transverse gradient wiggler (undulator)" (TGU) was proposed about thee decades ago as a way to overcome electron energy spread in FEL oscillators [4]. More recently the use of TGUs has been studied for high-gain FELs in laser-plasma accelerators [5]. The idea is illustrated in Fig. 1. By canting the magnetic poles, one can generate a linear $y$ dependence of the horizontal undulator field so that

$$\frac{\Delta K}{K_0} = \alpha y \,. \tag{1}$$

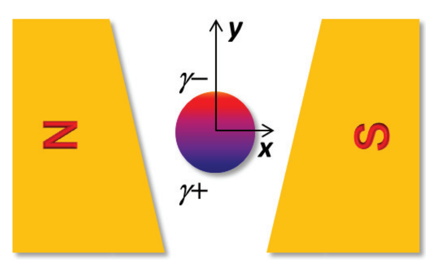


Figure 1: Transverse gradient undulator by canting the magnetic poles. Each pole is canted by an angle $\phi$ with respect to the $yz$ plane. The higher energy electrons are dispersed to the higher field region (negative $y$) compared to the lower energy electrons to match the FEL resonant condition.

We choose a planar undulator having the horizontal magnetic field. The vertical emittance in a storage ring can be

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much smaller than the horizontal emittance, and is advantageous to be used as the dispersive plane. Consider dispersing the electron beam vertically according to its energy such that $y = \eta\Delta\gamma/\gamma_0$. By choosing the dispersion function

$$\eta = \frac{2 + K_0^2}{\alpha K_0^2} \tag{2}$$

and keeping it constant in the TGU, the change in the electron's energy is now exactly compensated by the change in the magnetic field so that every electron satisfies the resonant condition in the undulator.

The operation of a TGU-based high gain FEL in the small-signal regime can be described by the linearized Vlasov-Maxwell equations of the system. In this framework, we derive an equation for the guided FEL eigenmodes, i.e. the solutions of the form $A(\mathbf{x})e^{i\mu z}$ for the complex amplitude of the radiation [6]. Disregarding emittance effects and the natural undulator focusing and assuming a Gaussian beam with an rms energy spread $\sigma_\delta$ and beam sizes $\sigma_x$ and $\sigma_y$ (in the absence of dispersion), we find that the growth rate $\mu$ and the mode profile $A(\mathbf{x})$ satisfy the relation

$$\left(\mu - \frac{\nabla_\perp^2}{2k_r}\right) A(\mathbf{x}) = -8\hat{\rho}^3 k_u^3 A(\mathbf{x}) \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\hat{\sigma}_y^2}\right) \times \int_{-\infty}^{0} d\xi \xi e^{i(\mu - \Delta\nu k_u)\xi - 2\hat{\sigma}_\delta^2 k_u^2 \xi^2} \exp\left(-2ik_u\xi \frac{\sigma_y^2}{\hat{\sigma}_y^2}\frac{y}{\eta}\right) \tag{3}$$

Here, $k_u = 2\pi/\lambda_u$, $\Delta\nu = \nu - 1$ (where $\nu$ is the frequency variable scaled by the resonant frequency $\omega_r = ck_r$) while $\hat{\sigma}_y$, $\hat{\sigma}_\delta$ and $\hat{\rho}$ are the vertical size, effective energy spread and FEL parameter of the dispersed beam [5]. The last three quantities are given by $\hat{\sigma}_y = \sigma_y R$, $\hat{\sigma}_\delta = \sigma_\delta/R$ and $\hat{\rho} = \rho R^{-1/3}$, where $R = (1 + \eta^2\sigma_\delta^2/\sigma_y^2)^{1/2}$ and $\rho$ is the FEL parameter for $\eta = 0$. Using a variational technique, we solve Eq. (3) for the fundamental FEL mode, which yields the power gain length $L_G = (2\,|\mathrm{Im}[\mu]|)^{-1}$ in terms of $\Delta\nu$ and $\eta$.

## PEPX-FEL OPTIMIZATIONS

We use PEPX as an example for FEL optimization studies. As discussed in [3], to drive an FEL in a DLSR, we will periodically switch a stored electron bunch into a bypass that contains the FEL undulator. After lasing process, the bunch with an increased energy spread will be sent back in the storage ring for a sufficient time for damping down. While during this damping period, another bunch can be switched into the bypass FEL beam line. Based on the beam dynamics studies including the intra-beam scattering and multi-bunch instability, we can keep a repetition rate of $\sim 10$ kHz in the bypass FEL beamline for PEPX.

We summarize the main parameters to be used in the FEL studies in Table 1. As shown in this table, we set 1% coupling of the emittance between $x$ and $y$, which makes a flat beam with the vertical size much smaller than the

Table 1: Main parameters for PEP-X FEL.

| Parameter | @200A | @300A | unit |
|---|---|---|---|
| RF voltage [MV] | 125 | 282 | MV |
| RF frequency [MHz] | 1428 | 1428 | MHz |
| e-Beam energy | 4.5 | 4.5 | GeV |
| Bunch Charge | 0.75 | 0.75 | nC |
| norm. emittance, x | 1.23 | 1.45 | mm-mrad |
| norm. emittance, y | 0.0123 | 0.0145 | mm-mrad |
| Energy spread [$10^{-3}$] | 1.5 | 1.55 | |
| rms Bunch length | 0.45 | 0.3 | mm |
| Undulator period | 2 | 2 | cm |
| Undulator $K_0$ | 3.68 | 3.68 | |
| Transverse gradient | $\sim 38$ | $\sim 38$ | $\mathrm{m}^{-1}$ |
| Dispersion | $\sim 3$ | $\sim 3$ | cm |
| FEL wavelength | 1 | 1 | nm |
| FEL peak power | $\sim 100$ | $\sim 200$ | MW |
| FEL pulse energy | $\sim 0.2$ | $\sim 0.25$ | mJ |
| Saturation length | 100 | 80 | m |
| FEL repetition rate | 10 | 10 | kHz |

horizontal size. By introducing a vertical dispersion, we finally make an almost round beam inside the undulator, but the electron energy is correlated with the $y$ dimension. To evaluate FEL performance, we first use the parallel beam theory discussed earlier to scan the operating point. That is, by changing vertical dispersion and also adjusting the transverse gradient based on Eq. (2), we expect to have the best gain at a particular dispersion and transverse gradient configuration. This is because a larger dispersion gives a smaller energy spread in each transverse mode, but it also makes a lower e-beam density. On the other hand, if the dispersion is too small, the large energy spread cannot be effectively compensated by a TGU. Then we perform FEL simulations with modified Genesis code [7] . The dispersed beam is prepared separately and imported into Genesis.

Figure 2 shows the FEL gain vs. dispersion with different detunes based on the theoretical calculations obtained from Eq. (3). According to this result, we performed Genesis simulations, and Fig. 3 shows the FEL power gain curves at 1nm wavelength with 200A beam current. We used the beam parameters listed in 1, and varied the dispersion values (and the transverse gradient was adjusted accordingly for each dispersion). At a dispersion of 3-5 cm, the FEL saturates at about 100 m, with its peak power about 100 MW. This gives about 0.2-mJ pulse energy assuming 1.5-ps rms pulse length. Also please note that the dispersion is not very sensitive, which means if the condition of Eq. (2) is satisfied, we have a pretty wide working range for good FEL performance. When the dispersion is too big or too small, such as 9cm or 2cm, the gain is much reduced. This is consistent with what we discussed earlier, and agrees reasonably well with the theoretical prediction




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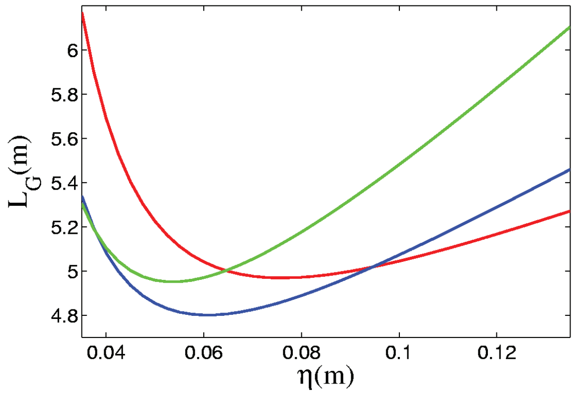


Figure 2: FEL gain length vs dispersion using a parallel beam theory, at 1nm with 200A. Different colors show the detune of the resonant wavelength: red/blue/green represents 1.0/1.00035/1.0005 nm.

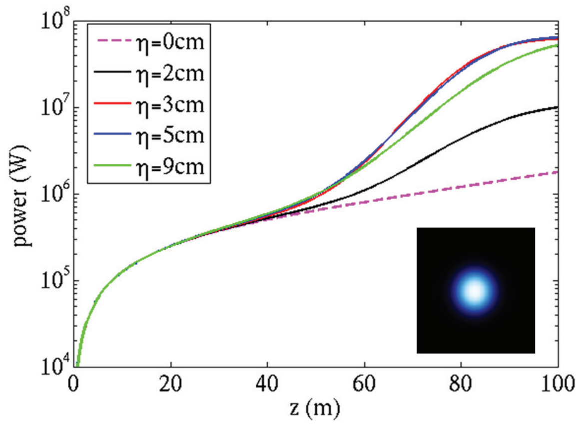


Figure 3: Simulated FEL power gain curve using a TGU with 200A peak current. The FEL wavelength length is 1nm. The inset shows the far field image at the end of undualtor for 3cm dispersion case.

in Fig. 2. We also showed a dashed curve with intentionally setting dispersion to be zero. In this case, we just used a regular undulator without transverse gradient. The final power at the undulator exit is about 2 orders lower.

For simplicity, we do not use any quads in the undulator beamline and hence the vertical dispersion is kept constant. In the horizontal dimension, where emittance is 100 times larger, we take the advantage of undulator natural focusing. The average beta function is about $\beta_x = \gamma\lambda_u/(A_w 2\pi) = 11m$. Using the emittance in Table-1, the rms beam size in x is about 39 $\mu$m. In the vertical dimension, we make a converging beam and the beam waist is at the middle of the undulator. The initial beta function is 125m at the undulator entrance (z=0) and 25m in the middle (z=50m). The beam size changes from 13$\mu$m to 6$\mu$m before adding dispersion. With a dispersion, for example, at 3cm, then the vertical beam size is about 45 $\mu$m, which is mainly dominated by the dispersion. This configuration finally makes an almost round beam in the system, and we get a fully coherent transverse mode the end of the undualtor, as shown one example in the inset of Fig. 3.

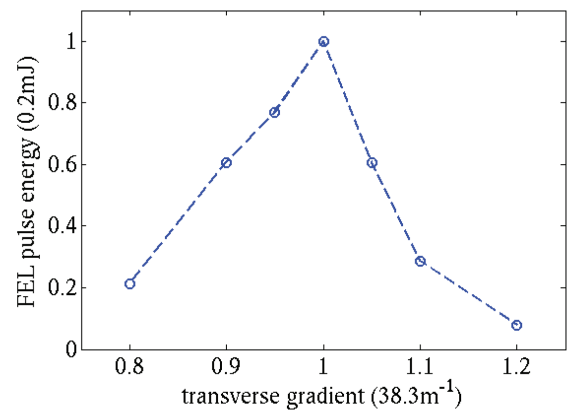


Figure 4: FEL pulse energy vs. transverse gradient settings. Dispersion is 3cm with 200A beam current.

Figure 4 shows the sensitivity of transverse gradient at a fixed dispersion 3cm with a beam current of 200A. From Eq. (2), the matched transverse gradient is 38.3 $m^{-1}$, where we get the maximum pulse energy. When the transverse gradient deviates from this value, we see the FEL pulse energy drops about 20% with a 5% gradient deviation. Also note the curve is asymmetric, and it is less sensitive on the side of a smaller gradient at a fixed dispersion (or a larger dispersion at a fixed gradient). This can be explained by finite transverse emittance effect, see the Eq. (9) in ref. [5].

If we can raise to 300A by increasing the rf voltage, the FEL saturates at about 80m, with a peak power about 200MW. Since the bunch length is reduced, the total pulse energy is only increased by 25% compared with 200A case. We summarize the results in Tabel-1.

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