RF FIELD-ATTENUATION FORMULAE FOR THE MULTILAYER COATING MODEL

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Abstract

Formulae that describe the RF electromagnetic field attenuation for the multilayer coating model with a single superconductor layer and a single insulator layer deposited on a bulk superconductor are derived from a rigorous calculation with the Maxwell equations and the London equation.

INTRODUCTION

An idea to enhance the rf breakdown field of superconducting cavities by multilayered nanoscale coating is proposed by A. Gurevich in 2006 [1]. The model consists of alternating layers of superconductor layers ($\mathcal{S}$) and insulator layers ($\mathcal{I}$) deposited on bulk Nb. The $\mathcal{S}$ layers are assumed to withstand higher field than the bulk Nb and shield the bulk Nb from the applied rf surface field B_0, by which B_0 is decreased down to $B_i < B_0$ on the surface of the bulk Nb. Then the cavity with the multilayered structure is thought to withstand a higher field than the Nb cavity, if B_0 is smaller than the vortex penetration-field [2] of the top $\mathcal{S}$ layer, and B_i is smaller than $200\,\mathrm{mT}$, which is thought to be the maximum field for the bulk Nb. In order to evaluate the shielded magnetic field B_i, the magnetic field-attenuation formulae for the multilayered structure are necessary.

When a magnetic field is applied to a superconductor, the Meissner screening current is induced, which restricts the penetration of the field to a surface layer. This effect is often explained by applying the London equation $d^2B/dx^2 = B/\lambda^2$ to a semi-infinite bulk superconductor in the region $x \geq 0$ with boundary conditions $B(0) = B_0$ and $B(\infty) = 0$, where λ is the London penetration depth and B_0 is the applied surface field parallel to the superconductor surface. The solution is written as $B(x) = B_0 e^{-x/\lambda}$, which means the penetration of the field is restricted to depth λ. This solution, however, is just a solution of the London equation for a special case: a solution for semi-infinite bulk superconductor with boundary conditions given above. For different configurations such as a multilayer coating model, the London equation should be solved with appropriate boundary conditions, and solutions are generally different from $B(x) = B_0 e^{-x/\lambda}$. In this paper the RF electromagnetic field-attenuation formulae for the multilayer coating model are derived from a rigorous calculation with the Maxwell equations and the London equation.

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MAGNETIC FIELD ATTENUATION IN THE MULTILAYER COATING MODEL

Model

For simplicity, let us consider a model with a single $\mathcal{S}$ layer and a single $\mathcal{I}$ layer deposited on a bulk superconductor. The region $x < 0$ is vacuum, the region I ($0 \leq x \leq d_{\mathcal{S}}$) is $\mathcal{S}$ layer with the London penetration depth λ_1, the region II ($d_{\mathcal{S}} < x < d_{\mathcal{S}} + d_{\mathcal{I}}$) is $\mathcal{I}$ layer with permittivity $\epsilon_r \epsilon_0$, in which ϵ_r is a relative permittivity and $d_{\mathcal{I}}$ is assumed to be zero or larger than a few nm to suppress the Josephson coupling [4], and the region III ($x \geq d_{\mathcal{S}} + d_{\mathcal{I}}$) is a bulk superconductor with the London penetration depth λ_2, where all layers are parallel to the y-z plane and then perpendicular to the x-axis [1]. The applied electric and magnetic field are assumed to be parallel to the layers. In order to derive the RF electromagnetic field-attenuation formulae for this model, the Maxwell and London equations should be solved in the region I and III, and the Maxwell equations in the region II. Boundary conditions are given as continuity conditions of the electric and magnetic field at $x = d_{\mathcal{S}}$ and $x = d_{\mathcal{S}} + d_{\mathcal{I}}$.

Electromagnetic Field in the Model

In the region I, the Maxwell and London equations should be solved. The equations [3] are given by

$$\triangle \mathbf{E} = k^2 \mathbf{E}\,, \qquad \triangle \mathbf{B} = k^2 \mathbf{B}\,, \tag{1}$$

where

$$k^2 = \frac{1}{\lambda^2}\Big(1 - i\mu_0 \sigma_{\mathrm{n}} \omega \lambda^2 - \frac{\omega^2}{c^2}\lambda^2\Big)\,, \tag{2}$$

c is the speed of light, ω is the angular frequency, and σ_{n} is a conductivity for the normal conducting carriers. Assuming the typical value of $\lambda \simeq 10^{-7}\,\mathrm{m}$ with $\omega \simeq 10^9\,\mathrm{Hz}$ for rf applications, the third term of the right-hand side of Eq. (2) becomes $\simeq 10^{-12}$. Furthermore, assuming Nb, NbN, MgB_2 or Nb_3Sn as material of the superconductor, normal conductivities σ_{n} are less than $10^7\,\mathrm{S/m}$, and then the second term becomes less than 10^{-3}. Thus contributions from the second and the third terms to the electromagnetic field distribution are negligible. Then Eq. (1) are reduced to

$$\triangle \mathbf{E} = \frac{1}{\lambda^2}\mathbf{E}\,, \qquad \triangle \mathbf{B} = \frac{1}{\lambda^2}\mathbf{B}\,. \tag{3}$$

[1] The layers of our model are not necessarily thin hence the discussion below can be applied to layers with arbitrary thickness.

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The solutions that are consistent with Maxwell equations are given by

$$E_{\rm I} = P_{\rm I} e^{\frac{x}{\lambda_1}} + Q_{\rm I} e^{-\frac{x}{\lambda_1}}, \tag{4}$$

$$B_{\rm I} = \frac{1}{ick\lambda_1}(P_{\rm I} e^{\frac{x}{\lambda_1}} - Q_{\rm I} e^{-\frac{x}{\lambda_1}}), \tag{5}$$

where $E_{\rm I}$ and $B_{\rm I}$ are electric and magnetic field in the region I, respectively, $P_{\rm I}$ and $Q_{\rm I}$ are integration constants, and $k = \omega/c$. In the region II, the Maxwell equations should be solved, where contributions from dielectric losses to the electromagnetic field distribution are neglected. Then the solutions are given by

$$E_{\rm II} = P_{\rm II} e^{i\sqrt{\epsilon_r}k(x-d_{\mathcal{S}})} + Q_{\rm II} e^{-i\sqrt{\epsilon_r}k(x-d_{\mathcal{S}})}, \tag{6}$$

$$B_{\rm II} = \frac{\sqrt{\epsilon_r}}{c}(P_{\rm II} e^{i\sqrt{\epsilon_r}k(x-d_{\mathcal{S}})} - Q_{\rm II} e^{-i\sqrt{\epsilon_r}k(x-d_{\mathcal{S}})}), \tag{7}$$

where $E_{\rm II}$ and $B_{\rm II}$ are electric and magnetic field in the region II, respectively, and $P_{\rm II}$ and $Q_{\rm II}$ are integration constants. In the region III, Eq. (3) should be solved as same as the region I, and the solutions that are consistent with the Maxwell equations are given by

$$E_{\rm III} = Q_{\rm III} e^{-\frac{x-d_{\mathcal{S}}-d_{\mathcal{I}}}{\lambda_2}}, \tag{8}$$

$$B_{\rm III} = -\frac{1}{ick\lambda_2} Q_{\rm III} e^{-\frac{x-d_{\mathcal{S}}-d_{\mathcal{I}}}{\lambda_2}}, \tag{9}$$

where $E_{\rm III}$ and $B_{\rm III}$ are electric and magnetic field in the region III, respectively, and $Q_{\rm III}$ is an integration constant. The constants $P_{\rm I}$, $Q_{\rm I}$, $P_{\rm II}$, $Q_{\rm II}$ and $Q_{\rm III}$ are determined from the boundary conditions: continuity conditions of the electric and magnetic field at $x = d_{\mathcal{S}}$ and $x = d_{\mathcal{S}} + d_{\mathcal{I}}$. Then, we obtain

$$P_{\rm II} = \frac{Q_{\rm III}}{2}\Bigl(1 - \frac{1}{i\sqrt{\epsilon_r}k\lambda_2}\Bigr) e^{-i\sqrt{\epsilon_r}kd_{\mathcal{I}}}, \tag{10}$$

$$Q_{\rm II} = \frac{Q_{\rm III}}{2}\Bigl(1 + \frac{1}{i\sqrt{\epsilon_r}k\lambda_2}\Bigr) e^{+i\sqrt{\epsilon_r}kd_{\mathcal{I}}}, \tag{11}$$

from the continuity conditions at $x = d_{\mathcal{S}} + d_{\mathcal{I}}$, and

$$P_{\rm I} = \frac{1+i\sqrt{\epsilon_r}k\lambda_1}{2} P_{\rm II} e^{-\frac{d_{\mathcal{S}}}{\lambda_1}} + \frac{1-i\sqrt{\epsilon_r}k\lambda_1}{2} Q_{\rm II} e^{-\frac{d_{\mathcal{S}}}{\lambda_1}}, \tag{12}$$

$$Q_{\rm I} = \frac{1-i\sqrt{\epsilon_r}k\lambda_1}{2} P_{\rm II} e^{+\frac{d_{\mathcal{S}}}{\lambda_1}} + \frac{1+i\sqrt{\epsilon_r}k\lambda_1}{2} Q_{\rm II} e^{+\frac{d_{\mathcal{S}}}{\lambda_1}}, \tag{13}$$

from the continuity conditions at $x = d_{\mathcal{S}}$. Substituting Eq.(10) and (11) into Eq.(12) and (13), we obtain

$$P_{\rm I} = \frac{Q_{\rm III}}{2} e^{-\frac{d_{\mathcal{S}}}{\lambda_1}} \Bigl[\Bigl(1 - \frac{\lambda_1}{\lambda_2}\Bigr)\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} + \Bigl(\sqrt{\epsilon_r}k\lambda_1 + \frac{1}{\sqrt{\epsilon_r}k\lambda_2}\Bigr)\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}\Bigr], \tag{14}$$

$$Q_{\rm I} = \frac{Q_{\rm III}}{2} e^{+\frac{d_{\mathcal{S}}}{\lambda_1}} \Bigl[\Bigl(1 + \frac{\lambda_1}{\lambda_2}\Bigr)\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} + \Bigl(-\sqrt{\epsilon_r}k\lambda_1 + \frac{1}{\sqrt{\epsilon_r}k\lambda_2}\Bigr)\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}\Bigr]. \tag{15}$$

Now the integration constants $P_{\rm I}$, $Q_{\rm I}$, $P_{\rm II}$ and $Q_{\rm II}$ are expressed in terms of $Q_{\rm III}$. The constant $Q_{\rm III}$ can be expressed by the surface magnetic field B_0, which is given by

$$\begin{aligned} B_0 &= B_{\rm I}|_{x=0} = \frac{1}{ick\lambda_1}(P_{\rm I} - Q_{\rm I}) \\ &= \Bigl[\Bigl(\frac{\lambda_1}{\lambda_2}\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} - \sqrt{\epsilon_r}k\lambda_1\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}\Bigr)\cosh\frac{d_{\mathcal{S}}}{\lambda_1} \\ &+ \Bigl(\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} + \frac{\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}}{\sqrt{\epsilon_r}k\lambda_2}\Bigr)\sinh\frac{d_{\mathcal{S}}}{\lambda_1}\Bigr] \times \frac{-Q_{\rm III}}{ick\lambda_1}. \end{aligned} \tag{16}$$

Then substituting Eq.(14), (15), (10), (11) and (16) into Eq.(5), (7) and (9), the magnetic fields in the region I, II and III are given by

$$\begin{aligned} B_{\rm I} = &\Bigl[\Bigl(\frac{\lambda_1}{\lambda_2}\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} - \sqrt{\epsilon_r}k\lambda_1\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}\Bigr)\cosh\frac{d_{\mathcal{S}}-x}{\lambda_1} \\ &+ \Bigl(\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} + \frac{\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}}{\sqrt{\epsilon_r}k\lambda_2}\Bigr)\sinh\frac{d_{\mathcal{S}}-x}{\lambda_1}\Bigr] \times \frac{B_0}{D}, \end{aligned} \tag{17}$$

$$\begin{aligned} B_{\rm II} = &\Bigl[\frac{\lambda_1}{\lambda_2}\cos\sqrt{\epsilon_r}k(d_{\mathcal{S}} + d_{\mathcal{I}} - x) \\ &- \sqrt{\epsilon_r}k\lambda_1\sin\sqrt{\epsilon_r}k(d_{\mathcal{S}} + d_{\mathcal{I}} - x)\Bigr] \times \frac{B_0}{D}, \end{aligned} \tag{18}$$

$$B_{\rm III} = \frac{B_0}{D}\frac{\lambda_1}{\lambda_2} e^{-\frac{x-d_{\mathcal{S}}-d_{\mathcal{I}}}{\lambda_2}}. \tag{19}$$

where the denominator D is given by

$$\begin{aligned} D = &\Bigl(\frac{\lambda_1}{\lambda_2}\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} - \sqrt{\epsilon_r}k\lambda_1\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}\Bigr)\cosh\frac{d_{\mathcal{S}}}{\lambda_1} \\ &+ \Bigl(\cos\sqrt{\epsilon_r}kd_{\mathcal{I}} + \frac{\sin\sqrt{\epsilon_r}kd_{\mathcal{I}}}{\sqrt{\epsilon_r}k\lambda_2}\Bigr)\sinh\frac{d_{\mathcal{S}}}{\lambda_1}. \end{aligned} \tag{20}$$

It should be noted that these equations are reduced to the well known expression for the semi-infinite superconductor given by $B = B_0 e^{-x/\lambda_1}$ when the $\mathcal{S}$ layer and the bulk superconductor are the same material ($\lambda_1 = \lambda_2$) and the $\mathcal{I}$ layer vanishes ($d_{\mathcal{I}} \to 0$).

Assuming an insulator thickness $d_{\mathcal{I}} \ll 10^{-2}\,$m, the above equations can be simplified. Completed results and discussions are seen in Ref. [5].

SUMMARY

The formulae that describe the RF field attenuation in the multilayer coating model with a single superconductor layer and a single insulator layer deposited on a bulk superconductor were derived from a rigorous calculation with the Maxwell equations and the London equations. Completed results and discussions are seen in Ref. [5].

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