OPTIMIZATION OF ELLIPTICAL SRF CAVITIES FOR $\beta < 1$*

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Abstract

A systematic approach to optimization of SRF cavities done earlier [1] for $\beta = 1$ is extended to $\beta < 1$. Some improvements for earlier developed designs are proposed.

INTRODUCTION

Elliptical superconducting radio-frequency (SRF) cavities are used for accelerating electrons and positrons travelling close to the speed of light ($\beta = v/c \approx 1$). For the acceleration of heavier particles, β can be considerably less than one. If β is not too low ($\beta > 0.4$ according to [2]) it is still effective to utilize an elliptical SRF cavity, which has lower cost of production due to the simplicity of its shape.

Elliptical cavities with $\beta = 0.47$ are used in the driver linac for rare isotope production at the MSU [3]. Two different β are used in the cavities of elliptic shape in the Project-X at FNAL [4]). An effective application of $\beta < 1$ elliptic cavities is the Spallation neutron source at Oak Ridge [5]. Many accelerating systems with $\beta < 1$ are now working around the world. BARC is developing an accelerator for the transmutation of nuclear waste where the elliptic cavity for $\beta = 0.49$ is proposed [6].

We will compare the optimization of a cavity for $\beta < 1$ with the results for $\beta = 1$ [1]. We will analyze the results from JLab and BARC [6] where the presented analysis seems insufficient. We will allow also for the effect of scaling the length of a single-cell cavity on the ratio of the peak magnetic field to the accelerating field (B_{pk}/E_{acc}).

CELL GEOMETRY

Figure 1 represents the cross-section of a half-cell of an elliptical cavity. The profile of the cavity is made up of two elliptical arcs connected by a tangent segment. The cell half-length is given as

$$L = \beta_g c/4f. \quad (1)$$

In this formula we use geometrical beta, β_g, instead of β because the electric field is not zero at the ends of a single-cell cavity and becomes decelerating when the particles are in the pipe. To reduce this harmful effect, the cavity is made shorter, $\beta_g < \beta$. By the same token the end cells of a multi-cell cavity are made shorter. For the inner cells of a multi-cell cavity, typically $\beta_g = \beta$.

For both single- and multi-cell cavities the aperture radius, R_a, is chosen to allow for the propagation of higher-order modes (HOMs) out of the cavity where they can be removed by resistive loads. For the multi-cell cavity, R_a affects cell-to-cell coupling.

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To optimize the cavity's electromagnetic properties, the elliptical half-axes, A, B, a, and b, are used as free parameters. In our case the purpose of optimization of the cell geometry is to allow for a large accelerating field without causing magnetic quenching in the superconductor. The magnetic quench is caused by a high surface magnetic field which can be enhanced by surface defects [8, 9]. Using an arc of optimal shape assures uniformity of the distribution of the magnetic field along the surface, and thus reduces its peak value. This, in turn, leads to lower RF losses [1, 10]. Therefore our goal is to minimize the ratio B_{pk}/E_{acc}.

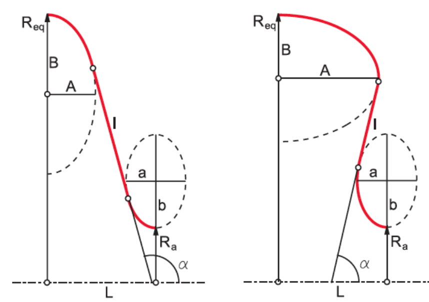

Figure 1: Cross section of an elliptical cavity half-cell. Left: a non-reentrant cell, $\alpha > 90°$; right: a reentrant cell, $\alpha < 90°$.

NUMERICAL SIMULATION

The optimization was done with the code SLANS [11], along with the wrapper code TunedCell [12] written specially for optimization of elliptic cavities. TunedCell adjusts R_{eq} to tune to the given frequency, creates the geometry files for use in SLANS and allows for linear variation of the free parameters. To control these variations, a Matlab formula as a wrapper code for TunedCell was written that optimizes the cavity for minimum B_{pk}/E_{acc}.

OPTIMIZATION OF A MULTI-CELL CAVITY FOR $\beta = 1$

Attempting to alter the geometry of the cavity in order to reduce B_{pk}/E_{acc} tends to cause an increase of E_{pk}/E_{acc}. This places a constraint on our optimization because the maximum electric field at the surface, E_{pk}, should not be too large or it will result in field emission (FE). As in the case of magnetic field, the FE starts from the surface defects or particulates stuck to the surface. The electric field strength required for FE from the niobium is dependent on the treatment applied to the cavity, so this is not a hard limit of the material [13]. Anyway, the value of E_{pk}/E_{acc} doesn't exceed the value of 3 in the most known high-β

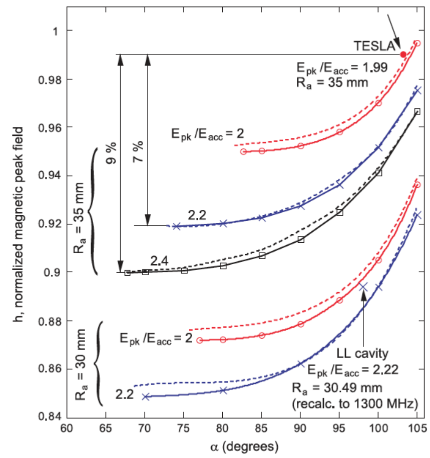

Figure 2: Optimization for $\beta = 1$. Solid lines show min $h = B_{pk}/4.2E_{acc}$ (normalized for TESLA cavity where $B_{pk}/E_{acc} \approx 4.2$ mT/(MV/m)) [1]. Dashed lines show max GR_{sh}/Q.

cavities [14] and the value of 4 in moderately-low-β cavities [7], except of [6] with $\beta = 0.49$ and $E_{pk}/E_{acc} = 4.26$ but this will be discussed separately below.

Decreasing the wall angle of the cell tends to reduce B_{pk}/E_{acc}. Yet the surface treatment on cells with α close to or less than 90° is more difficult because of the liquid-based methods used to treat the surface [13]. The problem of surface treatment was solved for a *single-cell* reentrant cavity with an impressive result: the highest CW accelerating gradient ever realized in a niobium RF resonator [15]. However, this treatment technique should be demonstrated for a multi-cell cavity before the reentrant shape will be accepted by the accelerator community.

Shown in Fig. 2 are results from [1] where $\beta = 1$ for various values of E_{pk}/E_{acc}. Optimizing the cavity for minimum h or minimal losses (maximum GR_{sh}/Q) lead to almost the same shape. This is an important point: having minimal B_{pk}/E_{acc}, i. e. maximal E_{acc} before a quench, you do not need to optimize for minimal losses.

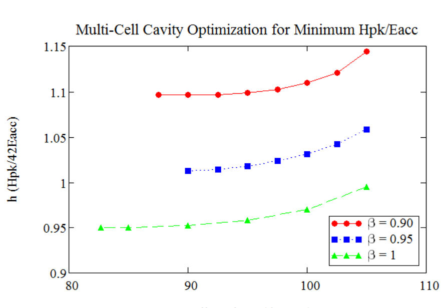

Figure 3: Optimization for min h of an inner cell of a multi-cell cavity with $Ra = 35$ mm and $E_{pk}/E_{acc} = 2$ for $\beta = 1$ from [1] and results for $\beta = 0.90$ and $\beta = 0.95$.

This shape optimization can be not final for high-current accelerators. HOMs are more intense in the case of higher currents. Altering the cavity shape we can allow for the HOMs to propagate out of the cavity, where they can be removed by resistive loads. The best solution would be to improve HOMs propagation changing the shape not much increasing the achieved minimal B_{pk}/E_{acc}.

MINIMIZING B_{PK}/E_{ACC} FOR $\beta < 1$

Cavities with $\beta < 1$ are designed for heavier particles with lower currents that do not excite HOMs. So the shape that results in a minimum h for a $\beta < 1$ cavity may not need to be changed if HOMs are not excited by the beam.

In Fig. 3 the results for minimum h where $\beta = 1$ and $E_{pk}/E_{acc} = 2$ are compared to the results of optimization for minimum h where $\beta = 0.9$ and 0.95 with the same limitation on E_{pk}/E_{acc}. This data shows that h increases with wall angle similarly for $\beta < 1$ as it does when $\beta = 1$. It is also seen that h is increased as β is decreased. This limits the accelerating field of low-beta, elliptical cavities, making the elliptical shape ineffective for low β.

The value of h decreases with decreasing α noticeably for $\beta = 1$, this decrease is less for $\beta = 0.95$, and for $\beta = 0.9$, h increases when α becomes $< 90°$. This data suggests that *the complications that arise in making reentrant cavities may not be worth overcoming for cavities where* $\beta < 1$. To find out what value of β is a limit, the parameters R_a and E_{pk}/E_{acc} should be taken into account.

VERIFICATION OF THE BARC APPROACH [6]

In the paper [6] an attempt was made at minimizing the value of E_{pk}/E_{acc} for a single cell accelerating cavity using the code SUPERFISH. The results from this article for $\beta = 0.49$, $f = 1050$ MHz are shown in the Table 1.

Table 1: Comparison of the Optimizations. Lengths in mm.

	Constraints	Free param.	Result
Optimization from [6]	$f = 1050$ MHz $R_a = 39$ $\beta = \beta_g = 0.49$	$A = 20$ $B = 20$ $a/b = 0.7$ $\alpha = 96.5°$	B_{pk}/E_{acc} = 8.02 mT/(MV/m), E_{pk}/E_{acc} = 4.26 GR/Q $= 1304$ Ohm^2
Our optimization	$L = 34.976$ $R_{eq} = 131.899$ $E_{pk}/Eacc$ = 3.5 min radius of curvature $R_a = 6$ $L = 26.767$ for $\beta_g = 0.375$	$A = 20.811$ $B = 51.3$ $a = 10.51$ $b = 18.41$	B_{pk}/E_{acc} = 8.15 mT/(MV/m), 7.37 after varying β_g. GR/Q $= 1402$ Ohm^2

We tried to recreate this data. Figure 4 shows the results obtained when using the boundary conditions for an *inner cell of a multi-cell cavity* compared with the results from [6] for a single cell. Although this data does not coincide, the qualitative trends are the same, strongly suggesting that [6] utilized the wrong boundary conditions for this

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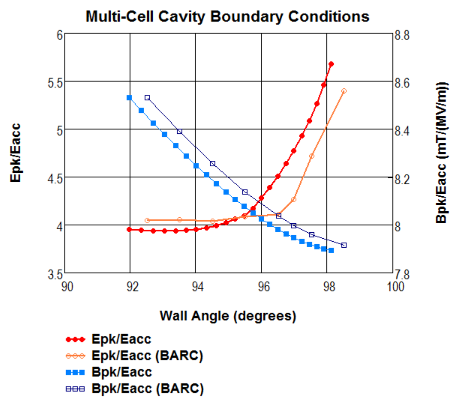

Figure 4: BARC results using SUPERFISH compared with our results from SLANS with multi-cell boundary conditions.

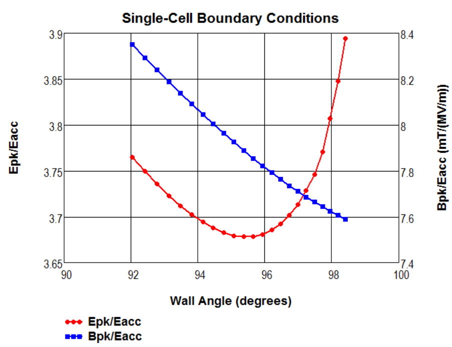

Figure 5: SLANS results for single-cell boundary conditions using BARC constraints (see Table I).

optimization. The discrepancies could be accounted for by different levels of accuracy in the results from SUPERFISH compared to SLANS or from different levels of accuracy in the free parameters. These discrepancies are about 0.1 for both E_{pk}/E_{acc} and B_{pk}/E_{acc}. Using an accurate grid one can find the values of these figures of merit with accuracy of 0.01 as it is regularly referred in different publications, e. g. [7, 14].

If we use the correct boundary conditions the difference between our result and results from [6] becomes large. Figure 5 shows the electromagnetic parameters with respect to wall angle for single-cell boundary conditions, which do not agree with the results from this paper. The data from our simulation has clear minimum for E_{pk}/E_{acc} that is lower than the minimum from [6]. The values of B_{pk}/E_{acc} are also considerably lower than those given by multi-cell (incorrect) boundary conditions.

SINGLE-CELL OPTIMIZATION

Varying Free Parameters Because the results from [6] are based on incorrect boundary conditions we have chosen to complete the optimization for the single-cell cavity. We will minimize the ratio of B_{pk}/E_{acc} in order to reduce the chance for magnetic quenching and losses in the cavity. The values of R_a, β, f and L are the same as those from [6], with the optimization being done by varying our free parameters A, B, a, and b.

For the single-cell cavity the values of E_{pk}/E_{acc} are lower than those found utilizing the multi-cell boundary conditions. Based on the values of E_{pk}/E_{acc} used by the inner cells of the TRASCO-ASH and RIA cavities [7] where $\beta = 0.49$ we chose a maximum of $E_{pk}/E_{acc} = 3.5$.

This optimization quickly led to values of a and b which result in an extremely small radius of curvature for the iris. In order to allow for the fabrication of the cavity from niobium the minimum radius of curvature of the cavity profile should not be too small. We have restricted our radius to twice the thickness of the Niobium sheet from which the cavity is formed: to 6 mm; this minimum radius of curvature is an additional constraint in our optimization.

For the constraints given above the single-cell cavity in Table I was found to have a minimum value of B_{pk}/E_{acc}. Our results for the optimization of the single-cell cavity have a 1.6% higher value of B_{pk}/E_{acc} but we have limited our value of E_{pk}/E_{acc} to be lower than the BARC optimization by 17.8%. However, below we will discuss how B_{pk}/E_{acc} can be lowered further by adding a fifth free parameter: the cell length. Reducing B_{pk}/E_{acc} decreases losses in the cavity, so GR/Q increases, see the Table.

Varying Length Scale Factor, β_g To exclude deceleration in the beam pipe, the cavity optimization from the previous section was extended to a fifth parameter: the cavity length scale factor, β_g.

By shortening the cavity length we were able to reduce the value of B_{pk}/E_{acc} to 7.37 mT/(MV/m) while keeping $E_{pk}/E_{acc} = 3.5$. This is a reduction of 8% in B_{pk}/E_{acc} from the single-cell boundary condition BARC value.

The data, from the SLANS simulation, shows a minimum value of h at approximately $\beta_g = 0.375$.

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