

IMPROVEMENT OF THE Q FACTOR MEASUREMENT IN RF CAVITIES*

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Abstract

The Q values of Higher Order Modes (HOMs) in RF cavities are usually calculated from the resonance bandwidth measured at -3 dB level by a network analyzer. The resonant curve distortion is caused by the resonance split due to the ellipticity caused by manufacturing tolerances, and RF ports. Therefore, the measured Q values are usually lower than the simulated or theoretical Q values. In some cases, only one mode's Q can be measured with the -3 dB method. There may be two reasons for this. One is that one mode is excited strongly, while the neighboring split-mode is close to 90° polarized and thus excited weakly; the other reason is that the resonant curve of one mode was distorted by the other mode too much to measure the -3 dB level. In this paper, we resolve this issue by looking into the RF measurement setup, including cavity, input coupler and pick-up coupler, from the equivalent circuit and wave point of view. Using HOM data for a copper prototype of the BNL3 cavity, we compare the results from measurements and simulations.

INTRODUCTION

In room temperature higher-order-mode (HOM) measurements, the Q values of the HOMs are typically measured at -3 dB level using a network analyzer, which excites and picks up RF signal through two ports located at opposite sides of a cavity. These ports could be a fundamental power coupler (FPC) port on one side of cavity and a pick-up port on the other side of the cavity, for example. Dipole, quadrupole and other axially asymmetric modes are degenerate. That is, there are two dipole modes with the same field pattern and resonant frequency, polarization of which are undetermined for axially symmetric geometries. Due to manufacturing tolerances the cavity cells can become slightly elliptical. This would break mode degeneracy and split two identical dipole modes into two polarized modes with very close, but different frequencies. The FPC port, pick-up port, or any other object attached to the cavity will have similar effect on HOMs. The two polarized dipole modes should have similar Q values, unless one of the modes couples stronger to the cavity port(s). The network analyzer would excite both modes and as a result display an interference signal. Additionally, one mode usually has a stronger coupling than the other due to the location of the RF ports. This distorts shapes of the two modes. As a consequence of that, the Q values at -3 dB level are no longer accurate. In this paper, at first we explain the Q

distortion, and then how to improve the Q measurement results. HOM measurements were performed using a copper prototype of the five cell BNL3 cavity [1]. We compare the results of measurements, simulations with Omega3P [2] and improved results.

MEASUREMENT AND SIMULATION RESULTS

The measurement setup for the BNL3 copper prototype cavity is shown in Fig. 1. There is an FPC port at one side and a pickup port at the other side of the cavity. In the warm RF measurement of the copper prototype, either FPC port or pickup port can be used for RF input port or pickup port, and both of them should be weakly coupled. Figure 2 shows the first dipole passband's spectrum taken with a network analyzer. It shows that every dipole mode splits into two modes and their resonant curves are distorted by each other.

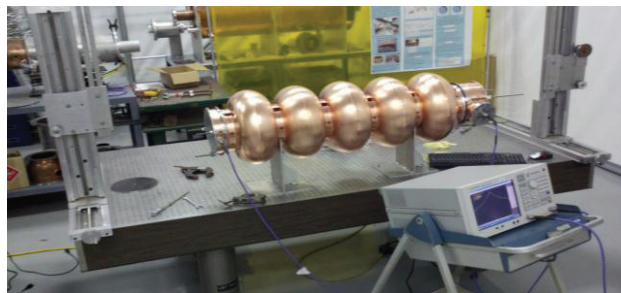


Figure 1: Network analyser measurement setup.

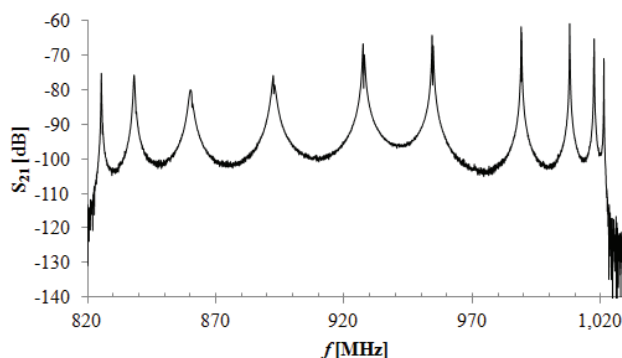


Figure 2: First dipole mode passband.

A 3D cavity model with the FPC port and pick-up port was built for the HOM studies with Omega3P. The comparison of simulation and measurement results of the first dipole passband is presented in Figure 3. First of all, it shows that the simulated Q values are higher than the measured Q values. Secondly, most of the modes come as a pair of split modes because of polarization of the dipole

* This work is supported by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. DOE, and Award No. DE-SC0002496 to Stony Brook University with the U.S. DOE.

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modes. These two phenomena will be explained in the following sections.

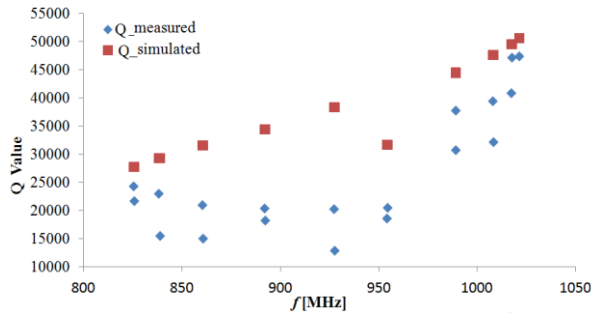


Figure 3: Comparison of the results from measurements and simulations of the BNL3 cavity.

EXPLANATION AND MODEL OF THE Q-DEDUCTION

The Mode Splitting in RF Cavity

Due to manufacturing tolerances and errors, the shape of the circular cross section of the cavity is usually distorted, causing the cross section of the cavity to be slightly elliptical. In addition, RF ports (FPC, pick-up and HOM couplers) also produce polarization of the cavity. The important consequence of the imperfection and polarization is splitting of the originally degenerative mode into two polarized modes. A simple model to explain the mode splitting is to look into the modes in elliptical waveguide, where odd mode and even mode always exist due to the polarized cross section, as it is shown in Fig. 4. Figure 4 (left) shows the split modes in an elliptical waveguide. The total field is obtained by adding up the two modes as in Fig. 4 (right). As far as the cavity is concerned, a dipole mode will split into two modes with close frequencies and similar Q values. Because of this, the network analyzer sees the overlap of the two modes. A typical example of this phenomena, measured in BNL3 copper prototype, is shown in Fig. 5. Both modes are distorted by each other, and the actual bandwidth of the resonant curve is broadened, so that the measured Q values are usually lower than what they should be. Usually, the coupling factors of the two modes to RF ports are different from each other, so that the nominal field amplitudes of the two modes are different. This will cause the resonant curve of one of the modes to be distorted more than that of the other mode.

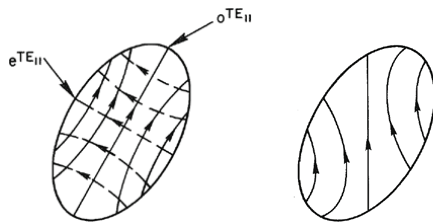


Figure 4: Mode splitting in an elliptical waveguide.

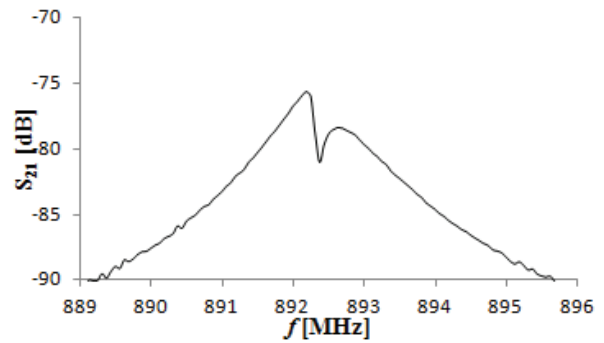


Figure 5: Typical S_{21} curve of the split modes.

The Model and Formula

The theoretical solutions for modes in an elliptical waveguide have been studied and published in references [3]. The field in the elliptical waveguide was solved by using elliptical coordinates, and Maxwell's equations can be separated into Mathieu equations. The discussion of RF theory for splitting is beyond the scope of this paper. However, we will derive the S_{21} formula for two split modes, which will improve the measurement results.

Using either a lumped element model of a cavity with two couplers (RLC circuit and two transformers) or a differential equation describing the setup, one can derive a formula for S_{21} [4, 5]:

$$S_{21} = \sqrt{\frac{4\beta_e \cdot \beta_l}{[1 + \Delta^2]}} (1 - j\Delta),$$

where
$$\Delta = Q_L \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right),$$

β_e and β_l are the constant coupling factors for the input and pick-up RF ports. This is derived assuming that only one mode exists in the cavity. However, in the realistic measurement, the S_{21} curve will be distorted due to the reasons explained above. The excitation signal is common to both split modes, and what is measured by network analyzer is the sum of two phasors as it is shown in Fig. 6.

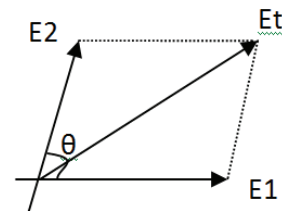


Figure 6: Sum of phasors of two modes.

If the angle between two phasors is $\theta = \Phi_1 - \Phi_2$, then the new S_{21} with contribution from two modes is

$$S_{21} = \sqrt{\frac{D_1^2}{1 + \Delta_1^2} + \frac{D_2^2}{1 + \Delta_2^2} - \frac{2D_1D_2 \cos(\pi - \theta)}{\sqrt{1 + \Delta_1^2} \sqrt{1 + \Delta_2^2}}},$$

where D_1 and D_2 are constants that depend on coupling coefficients of the two modes, respectively. Figure 7

shows examples of S_{21} curves for two closely spaced modes for different angles from 0 to π . The nominal Q values are 20,000, and they change from 18,335 to 29,464 if measured at -3 dB level, depending on the angle between two phasors. Figure 8 shows S_{21} curves for two modes with a fixed angle but different coupling factors (D_1 and D_2). These two figures show the important effect of the phasor sum and coupling factors of two polarized modes.

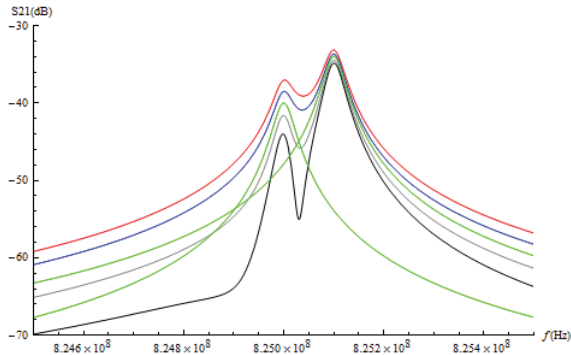


Figure 7: S_{21} of two split modes for different angles: 0 (red), $\pi/2$ (blue), $3.5 \pi/5$ (gray), $4.5 \pi/5$ (black) and two pure modes (green).

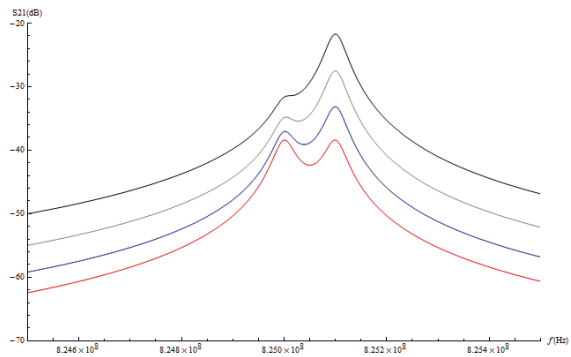


Figure 8: S_{21} of two split modes for different coupling factor of mode 2: 0.01 (red, the same as for mode 1), 0.02 (blue), 0.04 (gray), 0.08 (black).

IMPROVEMENT OF Q -VALUE MEASUREMENT

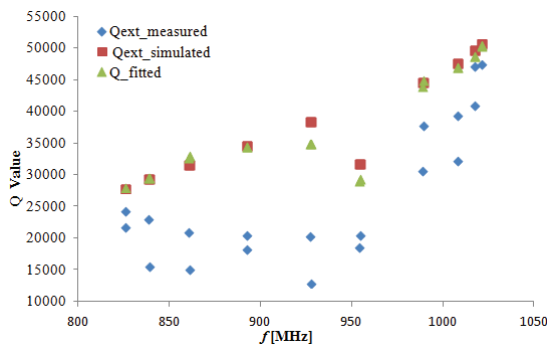


Figure 9: Comparison results from measurements, simulations and fitting for the BNL3 cavity.

As it is described in the above section, the resonant curves are distorted due to mode splitting, so to improve the measurements of Q values, we should use the formula to fit the measured S_{21} curves. We fitted all the dipole modes with “Findfit” function in Wolfram Mathematica. There are seven parameters: two Q values, two frequencies, two coupling factors, and the phasor angle. From the measurement results, we can get good first approximation for the frequencies. The amplitude of the S_{21} is determined by the coupling factor, thus the range for fitting is small. So, most of the efforts are to find the Q values and the phasor angle. The results are shown in Fig. 9. One can see that the fitted Q values agree with the simulated results very well. One typical result of curve fitting is shown in Fig. 10.

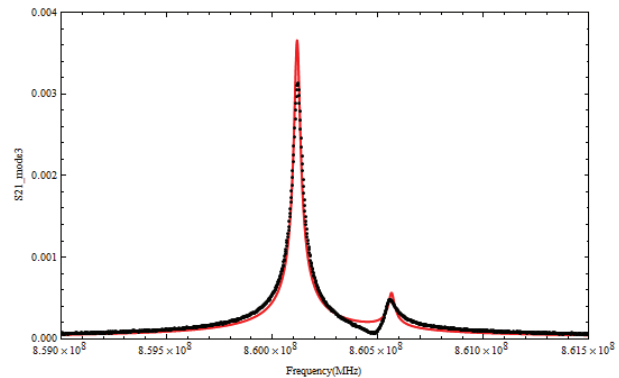


Figure 10: One example of resonant curve fitting.

SUMMARY

The measured Q values of the BNL3 copper prototype cavity are noticeably smaller than the simulation values. This paper discussed the conventional -3 dB method of Q values measurement in the room temperature cavity and the issues of using this method caused by the field polarization of a slightly elliptical cavity. The reasons of cavity polarization, which causes the S_{21} curve distortion, were described. The new S_{21} formula for the split modes was derived and used to fit the measured data. The fitted S_{21} and Q values match the simulated results very well.

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