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RFQ SOLVER BASED ON THE METHOD OF MOMENTS

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Abstract

The Method of Moments (MoM) [2] is widely used for radiation and scattering simulations. LINear ACcelerator (LINAC) such as Radio Frequency Quadrupole (RFQ) [3], [8], can also be simulated with the MoM. Present solvers for LINACs are quite intensive. Faster and more accurate solvers would allow better analyses and potential numeric optimisations of LINACs. In this paper, the MoM is used to simulate a section of the Myrrha's RFQ. First, the MoM is recalled. Then, the low-frequency breakdown [1], [5] is explained. Next, a method based on the loop-tree decomposition is proposed to solve the problem of low-frequency breakdown due to the unavoidable very fine mesh used for the simulation of a four-rod RFQ. Finally, some results obtained with the MoM with Rao-Wilton-Glisson (RWG) [2] basis functions for the Myrrha's RFQ are presented.

THE METHOD OF MOMENTS

The MoM is a method used to solve the harmonic Maxwell's equations. Because of the linearity of the equations and thanks to some theorems of vectorial analysis, one can get a diffeo-integral expression of the electric and magnetic fields in function of the current distribution. One of the notable advantage of this method is that it requires only unknowns on the boundaries of the conductors of the system one wants to simulate. The fact that one doesn't need to deal with the fields in free-space itself is because the Green's function, obtained by the resolution of the Helmholtz equation with a Dirac distribution as current source, contains all the physics encoded in the harmonic Maxwell's equations. This gives a significant advantage in comparison to methods that aims to solve the full set of Maxwell's equations without taking advantage of the linearity and distribution theory tools. For instance, finite elements methods need unknowns in the whole volume where the fields must be calculated. The expression of the fields is given by

$$\overline{E} = L[\overline{J}] \tag{1}$$

with $\overline{E}$ the electric field, $\overline{J}$ the current on the conductors boundaries and L a linear operator given by

$$L[\overline{J}] = -(j\omega + \frac{j\nabla\nabla\cdot}{\omega\mu\epsilon})\int_{\partial D} G(\overline{r},\overline{r}')\overline{J}(\overline{r}')dS \tag{2}$$

$$G(\overline{r},\overline{r}') = \frac{e^{-jk|\overline{r}-\overline{r}'|}}{4\pi|\overline{r}-\overline{r}'|} \tag{3}$$

with G the Green's function, ω the angular frequency, μ the free-space permeability, ϵ the free-space permittivity and $j = \sqrt{-1}$.

In order to invert the linear operator L, one needs to discretized the problem

$$\overline{J} \approx \sum_{i=1}^{n} \alpha_i \overline{J}_i \tag{4}$$

with $\overline{J}_i$ the basis functions, α_i the current distribution coefficients that must be determined. In order to find the coefficients, the boundary conditions are enforced with the help of a scalar product and testing functions. The scalar product is defined by

$$< \overline{f}|\overline{g} >= \int_{\partial D} \overline{f}^* \cdot \overline{g} dS \tag{5}$$

with $\overline{f}^*$ the conjugate function of $\overline{f}$. The total electric field can be decomposed as the superposition of the scattering field due to the current and an external source field as follows

$$\overline{E} = \overline{E}_{scatter} + \overline{E}_{source} \tag{6}$$

If one assumes a system made only of perfect conductors and because of the linearity of L and the scalar product one gets

$$< \overline{E}|\overline{T}_k > \approx < L(\sum_{i=1}^{n} \alpha_i \overline{J}_i) + \overline{E}_{source}|\overline{T}_k >$$

$$< \overline{E}|\overline{T}_k > \approx \sum_{i=1}^{n} \alpha_i < L(\overline{J}_i)|\overline{T}_k > + < \overline{E}_{source}|\overline{T}_k >$$

with $\overline{T}_k$ a testing function. Since the parallel component of the electric field on a perfect conductor must vanish one has

$$e_k =< \overline{E}_{source}|\overline{T}_k >= -\sum_{i=1}^{n} \alpha_i < L(\overline{J}_i)|\overline{T}_k >$$

This equation is called the Electric Field Integral Equation (EFIE) [2]. The matrix $Z_{ki} =< L(\overline{J}_i)|\overline{T}_k >$ is called the MoM impedance matrix and e_k the excitation. In order to solve this system of equations, one needs a number of testing functions equal to the number of basis functions. In general, the set of testing functions is identical to the set of basis functions. This testing method is called a Galerking test and is widely used because of the well conditioned system it provides.

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LOW-FREQUENCY BREAKDOWN

In order to simulate a four-rod RFQ with the MoM one needs basis functions of dimension close to a thousandth of the wavelength. This fine mesh is required in order to model the shape of the rods accurately. Fine meshes lead to the well known low-frequency breakdown [1] which means that the system of equations becomes badly conditioned. With a little bit of algebraic manipulations and with the help of few analysis theorems [2] one can rewrite the EFIE as follows

$$Z_{ki} = j\mu c \int_{\partial D}\int_{\partial D} (k\bar{J}_k \cdot \bar{J}_i - \frac{1}{k}[\nabla \cdot \bar{J}_k][\nabla \cdot \bar{J}_i]) G dS dS' \quad (7)$$

with k the wave number, c the speed of light. For a given set of basis functions, if the wave number becomes small, the divergence term in the integral equation becomes dominant. Physically, it means that the (almost electrostatic) contribution of the charges to the fields value becomes dominant in comparison to the (almost magnetostatic) contribution of the current.

In order to explain the bad conditioning of the matrix, one will use a simple example. Consider the mesh fig. 1

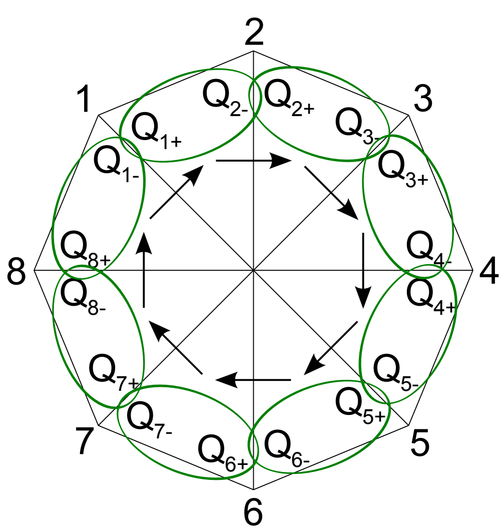

Figure 1: Low-frequency breakdown example.

The mesh is made of 8 isometric triangles. The basis functions used are the so-called Rao-Wilton-Glisson basis functions and are defined as follow

$$\bar{J}(\bar{r}) = \frac{L}{2S}(\bar{v}^- - \bar{r}) \text{ with } \bar{r} \in T^- \quad (8)$$

$$\bar{J}(\bar{r}) = \frac{L}{2S}(\bar{r} - \bar{v}^+) \text{ with } \bar{r} \in T^+ \quad (9)$$

with S the surface of the triangle, the other variables are defined on fig. 2

The divergence of such current distribution has the following expression

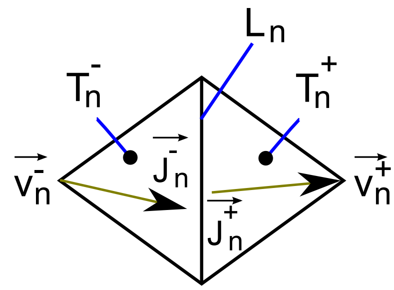

Figure 2: RWG basis function.

$$\nabla \cdot \bar{J} = \frac{L}{S} \text{ with } \bar{r} \in T^- \quad (10)$$

$$\nabla \cdot \bar{J} = -\frac{L}{S} \text{ with } \bar{r} \in T^+ \quad (11)$$

Now, if one considers the following current distribution $\bar{J} = \sum_{i=1}^{8} \bar{J}_i$, then the EFIE becomes

$$Z_{ki} = j\mu c \int_{\partial D}\int_{\partial D} k\bar{J}_k \cdot \sum_{i=1}^{8} \bar{J}_i G dS dS' \quad (12)$$

for any testing function $\bar{J}_k$. For such current distribution the divergence term completely disappears. It means that if the wave number k is small the image of the current $\bar{J}$ through L is very small in comparison to the image of current distributions through L for which the divergence term would not vanish.

LOOP-TREE METHOD

A solution to the low-frequency breakdown is the so-called Loop-Tree method. With the example of the previous section, one pointed out a current distribution that participates in the ill conditioning of the system. The idea is to generate a new set of basis functions separated in two categories. The first category consists of basis functions that are divergence free, which means that the divergence term in the EFIE vanishes and the second consists of basis functions that are non-divergence free. The current distribution can be therefore expressed as follows

$$\bar{J} = \sum_{i=1}^{k} \bar{J}_{df,i} + \sum_{i=1}^{l} \bar{J}_{ndf,i} \quad (13)$$

where df stands for divergence free and ndf stands for non-divergence free. The total number of basis functions must remain the same so $l + k = n$. Since the divergence free basis functions give entries in the MoM impedance matrix proportional to the wave number the idea is simply to rescale them by dividing them by the wave number. For the non-divergence free basis functions, one can see in the EFIE that the divergence term is inversely proportional to the wave number. In order to avoid large value, one

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multiplies the non-divergence free basis function by the wave number.

There is still one remaining question, how does one generate the new set of basis functions. In fact, it is possible to prove that the loop basis functions such as the one that was introduced in the example of the previous section are the only kind of non-divergence free basis functions. Therefore, the generation of the new set of basis functions consists simply of generating such a loop basis functions and keeping a number of RWG basis functions for the non-divergence free basis functions.

RESULTS

In this section, some simulations of the Myrrha's RFQ will be presented. For computational reason, so far only a section of the RFQ has been simulated. The presented geometry of the Myrrha's RFQ is actually a reconstruction based on partial data of the true geometry. Thus, the model in this paper is an approximation of the true Myrrha's RFQ. The mesh is presented in fig. 3

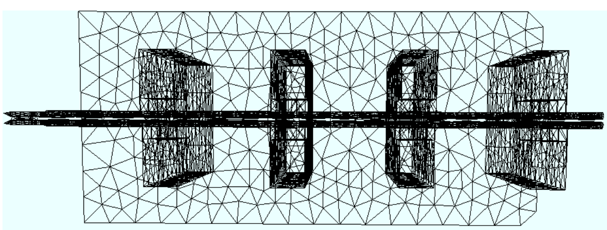

Figure 3: Mesh Myrrha's RFQ - Top view.

The feeding was performed directly on the rods at the beginning of the accelerator (extreme left of fig. 3). The frequency of the simulation is 176MHz. The current distribution obtained is shown fig. 4

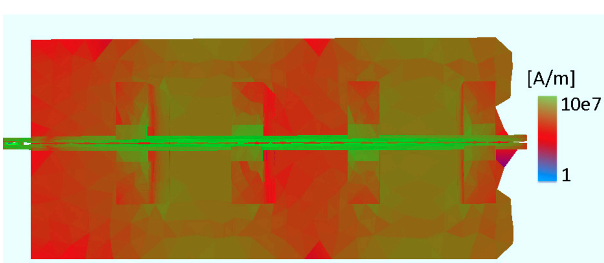

Figure 4: Current distribution Myrrha's RFQ - Top view.

The conditioning number of the impedance matrix was 1.2 10^6. The system can still be inverted with traditional methods but iterative solvers such as GMRES has shown a very slow convergence.

The fields distribution in a slice of the RFQ is presented on fig. 5.

The two pictures on the left represent the fields at time $t_0 = 0$ while the two pictures on the right represent the fields at time $t_1 = \frac{\pi}{\omega}$. Hence, the fields shown on the right

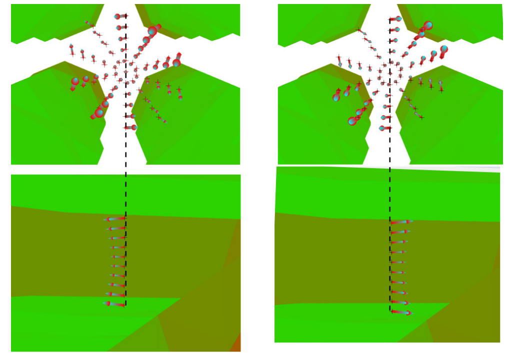

Figure 5: Fields distribution Myrrha's RFQ.

have an opposite value than the fields on the left. The top pictures show the quadrupole behaviour of the fields while the two bottom pictures show the accelerating field.

CONCLUSION

A section of the Myrrha's RFQ has been simulated and comparison with software such as Toutatis is foreseen shortly. The conditioning number must still be improved if one wants to use iterative solvers such as GMRES. The Loop-Tree method is currently under investigation to achieve this objective. Fast methods such as the Macro Basis Functions [7], [4], [6] approach are currently under implementation in order to improve the computation time.

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