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IBS SIMULATIONS WITH COMPUTE UNIFIED DEVICE ARCHITECTURE (CUDA) TECHNOLOGY*

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Abstract

A program code for 6D tracking has been developed taking into account IBS (Intra-Beam Scattering) and Touschek effect and using Monte-Carlo method. The simulation algorithm has been developed on the basis of well-known IBS theory presented in [1]. The resulting program can be executed using GPGPU devices (General-Purpose Graphics Processing Units) supporting CUDA technology (Compute Unified Device Architecture).

INTRODUCTION

IBS and Touschek effect are the important factors limiting the performance of modern particle accelerators. Often they mainly determine the size, lifetime and other characteristics of the beams in both existing and projected machines. It is, therefore, important to be able to assess the impact of these factors at the design stage. Estimation techniques can vary significantly, even in the choice of a mathematical model of the effect. Also within each mathematical model different assumptions and approximations can be made which are valid for some particular sets of parameters.

Initially, the algorithm that allows simulation of IBS was proposed by A. Piwinski in smoothed optics approximation [2]. Later J. Bjorken and S. Mtingwa proposed the technique that allows taking into account optical functions variations along the accelerator ring [3]. There is also an approach based on the methods of stochastic dynamics (using the solution of the Fokker-Planck equation) [4].

There are other ways of studying and describing these effects. However, all of them are based on the calculation of momentum transfer during Coulomb interaction between charged particles. At this stage one usually establishes the basic restrictions on the input parameters, i.e. selects approximations that will be used later. This choice is mainly due to the intended field of application of the model being created.

The purpose of this work was to create a program code for IBS and Touschek effect simulation for the case of ultrarelativistic ($\gamma \gg 1$) flat ($\sigma_y \ll \sigma_x$) bunches of light particles (e^- and e^+). Therefore, a detailed examination of the kinematics of the collision was omitted. Instead, we used a one-dimensional model of the collision.

Special attention was paid to the implementation of the selecled algorithm. The original idea was to perform parallel computations using the capabilities of modern graphics processors. CUDA technology developed by NVIDIA company was chosen as the basis. Therefore, we have to create program code not only for IBS and Touschek effect simulation, but also for tracking particles along the accelerator ring.

MATHEMATICAL MODEL

Following [1], we consider the one-dimensional model of IBS, i.e. we assume that particles receive only an energy gain during the collision. We also assume that the relative motion of particles in a bunch is non-relativistic and occurs only along the radial axis.

In this model the intrabeam scattering is divided into single and multiple scattering. Single scattering events are rare interactions with a large energy transfer. Therefore, each event should be simulated separately. Multiple scattering events are frequent interactions with a small energy transfer. The net energy gain after a large number of events can be considered as gaussian noise. Thus, two values should be introduced: lower relative energy transfer limit for multiple scattering ε_0 and similar value ε_1 for single scattering.

The keystone for this model is the calculation of scattering rate from unit phase space with relative energy transfer greater than ε. We denote this value by $\Omega(\varepsilon, \mathbf{X})$.

The value ε_0, according to [1] and [5], is determined using maximum impact parameter b_{max}

$$\varepsilon_0 = \sqrt{\frac{r_0}{b_{max}}}, \quad \text{where } r_0 \text{ — the classical electron radius.}$$

$$b_{max} = \min\left\{\sigma_y, \left(\frac{\gamma V_b}{N_p}\right)^{1/3}\right\}, \text{ where}$$

$V_b = 8\pi^{3/2}\sigma_x\sigma_y\sigma_s$ — bunch volume, N_p — number of particles in the bunch.

We denote 6D particle coordinates by $\mathbf{X}$, second-order moments matrix by $R_{ij} = \left\langle X_i X_j \right\rangle$ and then introduce $\tilde{\mathbf{X}}$ vector (spatial part of $\mathbf{X}$) and $\hat{\mathbf{R}}$ matrix ($\mathbf{R}$ matrix with 4-th and 6-th rows and columns canceled).

Then $\Omega(\varepsilon, \mathbf{X})$ is written as follows

$$\Omega(\varepsilon, \mathbf{X}) = \rho(\mathbf{X}) N_p \frac{{r_0}^2 c}{2\pi\gamma^3\varepsilon^2} \frac{\mathrm{F}(a,b)}{\sqrt{\det \hat{\mathbf{R}}}} \exp\left(-\sum_{i,j=1}^{3} \hat{S}_{ij}\tilde{X}_i\tilde{X}_j\right), \tag{1}$$

where

$$\hat{\mathbf{S}} = \begin{pmatrix} \xi_{11}/2 & \xi_{13} & \xi_{14} \\ 0 & \xi_{33}/2 & \xi_{34} \\ 0 & 0 & \xi_{44}/2 \end{pmatrix},$$

$$\xi_{ij} = \hat{R}^{-1}_{ij} - \frac{\hat{R}^{-1}_{i2}\hat{R}^{-1}_{j2}}{\hat{R}^{-1}_{22}},$$

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$$\mathrm{F}(a,b) = e^{-b^2}\int_1^\infty \frac{u^2-1-\ln u}{u^3} e^{-a^2u^2}\cosh(2abu)du\,,$$

$$a = \frac{\sqrt{2}\varepsilon}{\gamma\sigma_{x'}}\,,$$

$$b = \sqrt{\frac{\hat{R}_{22}^{-1}}{2}}x' + \frac{\hat{R}_{12}^{-1}}{\sqrt{2\hat{R}_{22}^{-1}}}x + \frac{\hat{R}_{32}^{-1}}{\sqrt{2\hat{R}_{22}^{-1}}}y + \frac{\hat{R}_{42}^{-1}}{\sqrt{2\hat{R}_{22}^{-1}}}z\,.$$

Suppose that scattering occurs only at a certain finite set of points along the accelerator ring (scattering points). If the distance between any two adjacent points (scattering interval) is small then optical functions and second order moments can be considered as constants in this part of the trajectory. Computational power is insufficient for tracking of every particle in the bunch, so we perform tracking of a sample (number of particles in the sample is much less than in the bunch).

Single Scattering

Single scattering events are those with relative energy transfer greater than ε_1. Suppose that one particle experiences, at the most, one such scattering per interval, since the following ones are suppressed. Then single scattering probability in the interval of the length Δs is $\frac{\Delta s}{c}\Omega(\varepsilon_1,\mathbf{X})$. Note that

$$\Psi(\varepsilon,\mathbf{X}) = \begin{cases} 0, & \varepsilon < \varepsilon_1 \\ 1 - \dfrac{\Omega(\varepsilon,\mathbf{X})}{\Omega(\varepsilon_1,\mathbf{X})}, & \varepsilon \geqslant \varepsilon_1 \end{cases}$$

is the distribution function of the relative energy transfer absolute value, i.e. $\Psi(\varepsilon,\mathbf{X}) = \mathrm{P}\left(\left|\frac{\Delta E}{E}\right| < \varepsilon\right)$. Knowing $\Psi(\varepsilon,\mathbf{X})$ we can generate this value using inversion method. To do this, we generate a random variable ξ, uniformly distributed on the interval $[0;1]$. After that such ε_* is required that $\Psi(\varepsilon_*,\mathbf{X}) = \xi$. a_* can be found as the limit of the series

$$a_{n+1} = a_n + \frac{\mathrm{F}(a_n,b) - {a_n}^2\dfrac{\mathrm{F}(a_1,b)}{{a_1}^2}\xi}{\dfrac{2}{a_n}\mathrm{F}(a_n,b) - \mathrm{F}'(a_n,b)}\,,$$

where

$$a_1 = \frac{\sqrt{2}\varepsilon_1}{\gamma\sigma_{x'}}\,, \qquad \mathrm{F}'(a_n,b) = \frac{\partial \mathrm{F}(a_n,b)}{\partial a}\,.$$

Note that in (1) individual particle coordinates appear only in the exponent and the second parameter of $\mathrm{F}(a,b)$. To use piecewise approximation, we calculate an array of $\mathrm{F}(a,b)$ values for different b for each scattering point before the tracking procedure. Intrabeam scattering changes the elements of the $\hat{\mathbf{S}}$ matrix, therefore, after each certain number of turns these arrays must be recalculated. Interaction is pairwise, hence it must be simulated twice for every particle in the sample.

Multiple Scattering

There is a great number of events with relative energy transfer $\varepsilon_0 < \varepsilon < \varepsilon_1$ in each scattering interval. So, the net energy gain for one particle has centered gaussian distribution with the witdth equal to

$${\sigma_\varepsilon}^2 = 2\rho(\mathbf{X})N_p\frac{{r_0}^2\Delta s}{2\pi\gamma^3}\frac{\mathrm{H}(a_0,a_1,b)}{\sqrt{\det\hat{\mathbf{R}}}}\exp\left(-\sum_{i,j=1}^{3}\hat{S}_{ij}\tilde{X}_i\tilde{X}_j\right)\,,$$

where

$$\mathrm{H}(a_0,a_1,b) = \int_{a_0}^{a_1} a^2\frac{\partial}{\partial a}\left(\frac{\mathrm{F}(a,b)}{a^2}\right)da\,.$$

USAGE OF CUDA TECHNOLOGY

Bunch sizes can be considered as constants over one turn (or even several successive turns). Thus, the motion of each individual particle in the sample can be simulated independently over this period. So, simulation algorithm can be paralleled using SIMD principle (Single Instruction Multiple Data). This approach has been successfully implemented in GPGPU devices (General-Purpose Graphics Processing Units) [6].

Program code developed as a part of the present work is called TrackKing. TrackKing simulation code uses CUDA technology (Compute Unified Device Architecture) by NVIDIA company. It is a hardware and software system that allows usage of the extended C programming language, so that a part of the code is executed on the graphics card processor (GPU).

TrackKing program consists of CPU and GPU parts. CPU part takes magnetic structure of the ring in the MAD (Methodical Accelerator Design) format [7] and the beam parameters as the input and gives GPU part source code and approximation data for each scattering point as the output. GPU part performs tracking of the sample particles. Program logic can be represented as the scheme (Fig. 1).

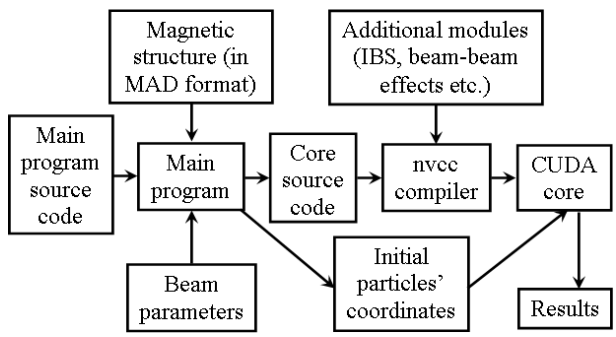

Figure 1: Scheme of TrackKing simulation program.

SIMULATION RESULTS

Electron bunch size growth due to intrabeam scattering at VEPP-4 has been studied for different beam current values. Two independent simulation codes were used: the first one is TrackKing, the second one is program code by

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Table 1: Comparison of the Simulation Results Obtained with TrackKing and CIMP (in square brackets)

N_p	$\varepsilon_x/\varepsilon_{x0}$	$\varepsilon_y/\varepsilon_{y0}$	σ_L/σ_{L0}	σ_E/σ_{E0}
$1 \cdot 10^8$	1.3 [1.25]	1.1 [1.25]	1.2 [1.12]	1.2 [1.12]
$1 \cdot 10^9$	2.1 [2.0]	1.6 [2.0]	1.6 [1.4]	1.6 [1.4]
$1 \cdot 10^{10}$	4.4	2.7	2.5	2.5
$1 \cdot 10^{11}$	9.2 [7.98]	5.2 [7.98]	3.7 [2.8]	3.7 [2.8]
$1 \cdot 10^{12}$	18.7	9.8	5.4	5.4
$1 \cdot 10^{13}$	32.7	16.7	7.0	7.0

S. Sinyatkin using CIMP (Completely Integrated Modified Piwinsky High Energy Approximation) [8] (see Table 1).

In the experiment carried out at VEPP-4 the equilibrium bunch length was measured for different energy values and then the energy spread was calculated. Comparison of the experimental data and TrackKing simulation results is represented in Fig. 2.

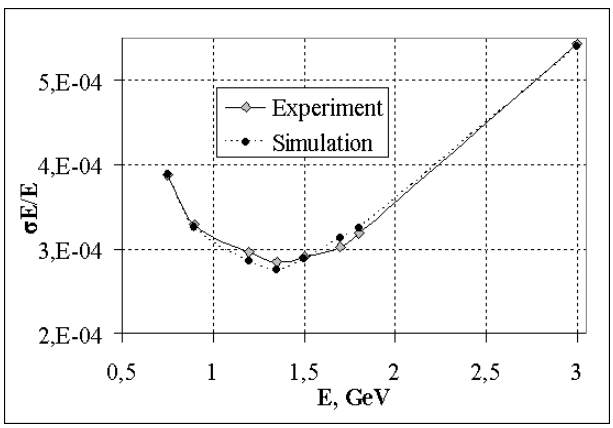

Figure 2: Comparison of the experimental data and simulation results.

CONCLUSION

A program complex called TrackKing has been developed for 6D tracking taking into account IBS and Touschek effect and using Monte-Carlo method. The simulation program can be executed on single PC with CUDA enabled graphics card. Simulation results with good accuracy repeat the experimental data and the results obtained with other simulation programs.

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