UNDULATOR PHOTON BEAMS
WITH ORBITAL ANGULAR MOMENTUM

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Abstract

Photons carrying an orbital angular momentum (OAM) are present in the off-axis radiation of higher harmonics of helical undulators. Usually, the purity and visibility of OAM carrying photons is blurred by the electron beam emittance. However, high brightness OAM beams are expected in Ultimate Storage Rings (USRs) and FELs, and they may trigger a new class of experiments utilizing the variability of the topological charge, a 3rd degree of freedom besides the wavelength and the polarization. In 2013 a 99eV OAM beam from a helical undulator has been observed for the first time at the storage ring BESSY II. We discuss the sensitivity of the conducted interference experiment on the emittance and the settings of the two undulators which were involved. Furthermore, we propose another interference experiment which will be very robust against the undulators field setting.

INTRODUCTION

More than 20 years ago singular photon beams, i.e. OAM carrying photons, have been predicted in higher order Laguerre Gaussian modes [1]. These beams are characterized by a cork-screw like wavefront and an annular intensity distribution. Since then, they have been generated in many laboratories utilizing computer-controlled spatial light modulators for the transformation of Hermite Gaussian modes into Laguerre Gaussian modes. In 1997 Sasaki et al proposed the generation of OAM beams with relativistic charged particles in helical undulators [2,3]. Various observation methods have been discussed, among them the use of diffracting elements like wires or slits. Singular photon beams have successfully been generated at the APS via the implementation of a cork screw like phase plate into the radiation of a planar undulator [4]. The first direct production of 99 eV OAM photons with a helical undulator in the storage ring BESSY II was observed in 2013 [5]. The experiment was based on the interference of two undulator beams generated with the APPLE II double undulator UE56-2 [6]. One undulator was tuned to helical mode, generating OAM photons in the 2nd harmonic off-axis radiation. The other undulator produced linear light in the 1st harmonic at the same wavelength and served as reference. The interfering beams were mapped with a pinhole in front of the 1st mirror and the intensity was detected with a diode behind the monochromator. Longitudinally, the two photon beams overlapped only behind the monochromator. The observed interference pattern was in excellent agreement with simulation with WAVE [7] which modelled the realistic 3D geometry. This was a clear proof of the existence of OAM photons. For details we refer to [5]. All simulations in this paper are done with WAVE (code available from HZB).

THEORY

On resonance the complex field amplitude A of a helical undulator is described by the following equation:

$$A \sim \frac{(A_x - iA_y)}{\sqrt{2}} = \sqrt{2}\, e^{\pm i(n-1)\varphi} \left\{ \left(\gamma\theta - \frac{nK}{X} \right) J_n(X) - K J_n'(X) \right\}$$

with n = undulator harmonic number; K = undulator deflection parameter; φ = azimuthal angle; θ = polar angle; $X = 2n\xi\gamma\theta K$; $\xi = 1/(1+\gamma^2\theta^2+K^2)$; γ = Lorentz factor of the relativistic electrons; J_n = n^{th} order Bessel function of the 1st kind; $J_n{}'$ = 1st derivative of J_n with respect to *X*. For n > *1* each photon carries an OAM of $\pm(n-1)\hbar$. The sign depends on the helicity of the 1st undulator harmonic. Fig. 1 mirrors the typical phase distribution in a plane perpendicular to the propagation direction of $\Phi = (n-1)\varphi$ (topological charge of *n*=*1*).

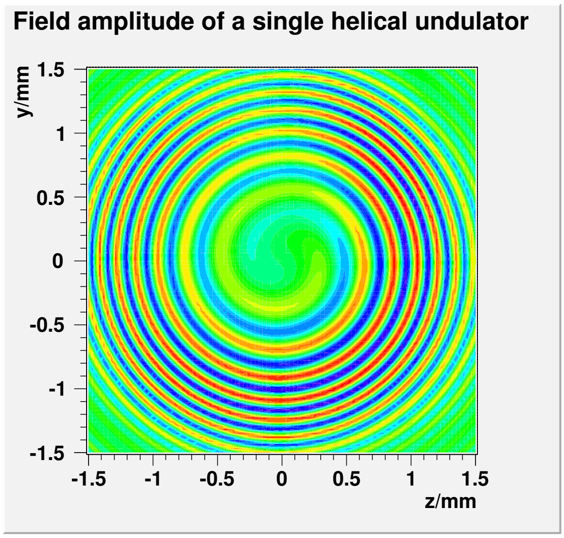

Figure 1: 2nd harmonic horizontal field amplitude (real part) of a helical undulator. The spiral shape mirrors the cork screw like phase distribution of the beam. The spiral has a one-fold symmetry (topological charge of *n*=*1*).

The real parts of the horizontal field amplitudes of the undulator beams have the forms:

$$A(r,\varphi) = \frac{a(r)}{L+d} cos\left(\frac{\pi d}{\gamma^2\lambda} + \frac{\pi}{(L+d)\lambda} r^2 \pm (n-1)\varphi + \frac{2\pi L}{\lambda} - \omega t \right)$$

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$$B(r,\varphi) = \frac{b(r)}{L} cos\left(\frac{\pi}{L\lambda}r^2 + \frac{2\pi L}{\lambda} - \omega t\right)$$

The intensity pattern is derived as:

$$I(r,\varphi) = \frac{\omega}{2\pi}\int_0^{\frac{\omega}{2\pi}} (A+B)^2\,dt =$$

$$\frac{a^2}{2(L+d)^2} + \frac{b^2}{2L^2} + \frac{ab}{L(L+d)} cos\left(\frac{\pi d}{\gamma^2\lambda} - \frac{\pi d}{L^2\lambda}r^2 \pm (n-1)\varphi\right)$$

The argument of the cos-function is zero for:

$$\varphi = \pm\left(-\frac{\pi d}{\gamma^2\lambda} + \frac{\pi d}{L^2\lambda}r^2\right)/(n-1)$$

This simple analytic expression describes nicely the shape of the spiral structure of the intensity distribution.

SENSITIVITY ON BEAM PARAMETERS AND GAP-SETTINGS

Detailed simulations demonstrate the impact of the electron beam parameters on the visibility of the interference pattern (Fig. 2). At 1.72 GeV the energy spread broadens the pattern whereas the emittance complete destroys the structure. The emittance scales quadratically and an electron energy reduction to 917 MeV reduces the emittance to $\varepsilon_x = 1.66\,\pi \cdot nm \cdot rad$. The energy spread decreases linearly to $\sigma_E = 3.67 \cdot 10^{-4}$. The spiral structure becomes observable, though, the influence of the emittance is still noticeable (Fig. 2).

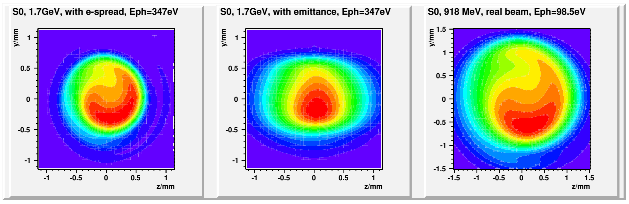

Figure 2: Evaluated interference patterns at 1.72 GeV (left and center) and at 917 MeV (right). Left: only energy spread included; center: only emittance included; right: emittance and energy spread included.

The interference experiment at BESSY II was very sensitive to the proper overlap of the two undulator beams. The resonance energy of the undulators was tuned to 100 eV where the divergence (2σ) is approximately 117 µrad. The monochromator was slightly detuned to 99 eV where the donut shaped spatial distribution starts to appear (Fig. 3). The spatial distribution of the 2nd harmonic of the helical undulator (1st harmonic at 49.5eV) is ring shaped with zero intensity on axis. The interference pattern is expected to be most pronounced if both beams have approximately the same intensity. Thus, the helical undulator was tuned such that the ring structure overlaps with the outermost part of the linearly polarized beam (Fig. 3).

The sensitivity of the interference pattern upon the gap setting of the helical undulator is demonstrated in Fig. 4. The OAM photons where observed at a magnetic gap of 23.94 mm. At larger gaps the overlap of the beams vanishes whereas for smaller gaps the ring diameter of the OAM beam shrinks which causes a decreasing intensity ratio of both beams and, eventually, the disappearance of the spiral structure. Even at zero emittance (Fig. 3) the spiral structure is observed only within a narrow band of +/-100 µm of gap detuning. Including emittance the observation of OAMs is even more sensitive to the correct gap setting.

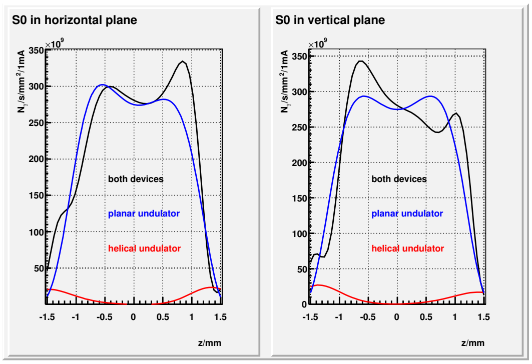

Figure 3: Horizontal and vertical cuts of the simulated intensity distributions at 13 m distance from the center of the double undulator (without emittance and energy spread). Cuts for the single beams and the combination of both beams are plotted. The asymmetry of the blue curve (left) is due to the horizontally deflecting phase shifter which is located between the two undulators.

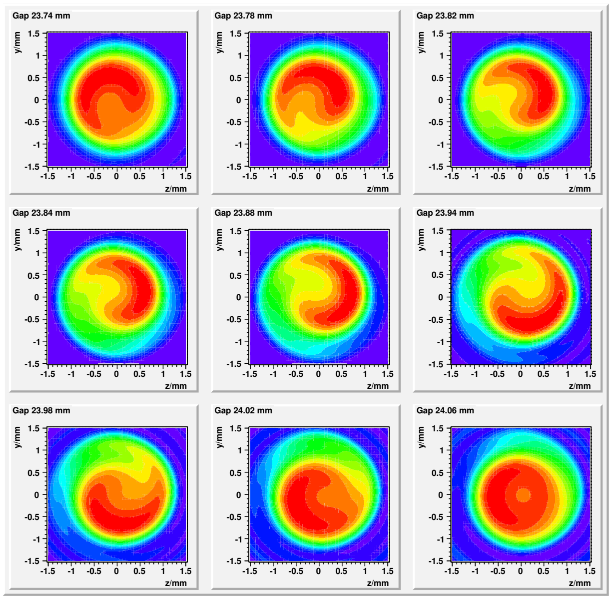

Figure 4: Intensity distribution of two interfering undulator beams generated by a helical undulator (2nd harmonic) and a linear undulator (1st harmonic). The magnetic gap of the helical undulator is varied linearly from 23.74mm (top left) to 24.06mm (bottom right). The gap of the linear undulator is fixed. The simulations are performed for zero emittance and energy spread.

ANOTHER DETECTION SCHEME FOR UNDULATOR OAM-PHOTONS

The conducted interference experiment was very sensitive to the appropriate undulator gap settings because the spatial distributions of the beams were completely different and the relative intensities were off by one order of magnitude. An interference experiment with two beams of similar shape and intensity is expected to be much simpler.

We studied numerically the interference of two helical beams with opposite helicity. The 1st harmonic of both devices is 49.5 eV and the interference was simulated for 99 eV. Thus, the difference of the topological charges of both beams is $n=2$. As expected the total intensity does not show any OAM specific feature. However, the intensity distribution of the Stokes parameter S_1 (i.e. the intensity difference between horizontal and vertical polarized contributions) shows a clear spiral structure with two-fold symmetry (Fig. 5). The features are already visible in the horizontally or vertically polarized intensities. At 917 MeV the structures are still visible if emittance and energy spread are included.

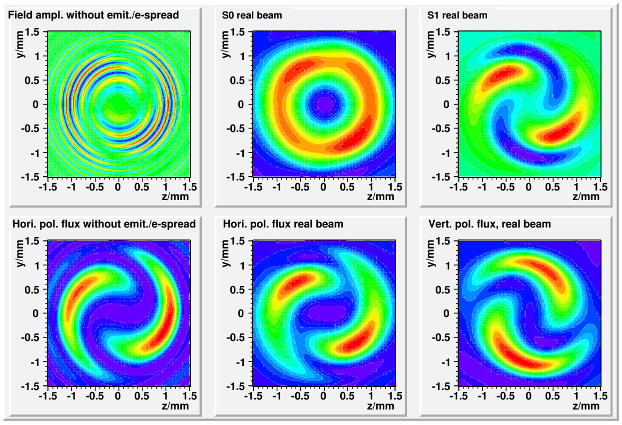

Figure 5: Interference patterns of the 2nd harmonics of two helical undulators with opposite helicity. Left column: zero emittance and energy spread; left top: real part of the horizontal field amplitude; left bottom: intensity; center and right column: emittance and energy spread included; center and right top: S_0 and S_1; center bottom: $(S_0+S_1)/2$; right bottom: $(S_0-S_1)/2$.

The adjustment of the undulators is much simpler in this setup because the devices are operated at the same polarization mode. On the other hand the detection needs more effort since a linear polarizer is required. The polarizer must be based on reflection optics and the coating and reflection angle must be chosen such that photons with parallel linear polarization are transmitted whereas photons of the orthogonal linear polarization are efficiently suppressed. The polarizer can be implemented behind the monochromator. Two independent measurements with two orthogonal polarizer orientations would deliver all data for the determination of S_1.

Interestingly, the (indirect) observation parameter S_1 is far more sensitive to OAM photons than the linearly polarized flux, since S_1 is defined via a difference of intensities where the common background vanishes. Simulations with a round beam (1.72 GeV, coupling = 1, emittance identical is both planes (only hypothetical)) show a pronounced spiral structure in S_1 whereas the pattern can hardly be seen in the linearly polarized flux.

CONCLUSION

Helical undulators for the production of OAM carrying photon beams are available in all modern 3rd generation storage ring based light sources. Though, the OAM photons are generated in these rings, the observation / usage of singular beams in the soft X-ray region is nearly impossible due to the large emittance. As demonstrated, the quadratic emittance scaling permits the preparation of existing rings for small emittance which enables the observation of OAM-photons already today.

Ultimate storage rings, USRs, (i.e. diffraction limited sources) based on multibend achromates will not be dominated by emittance anymore. The USR MAX IV [8] is under construction and first light is expected in two years from now. Certainly, OAM carrying beams will be utilized in the future for spectroscopy and many more applications as soon as they are available at normal user conditions.

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