CONVERGENT CHERENKOV RADIATION FROM DIELECTRIC TARGETS*

Sergey N. Galyamin[†], Ekaterina S. Belonogaya, Andrey V. Tyukhtin, Viktor V. Vorobev,
St. Petersburg State University, St. Petersburg, 198504, Russia

Abstract

Dielectric targets that concentrate Cherenkov radiation (CR) are presented. We suppose that CR is produced by a small bunch moving along the axis of cylindrical channel inside targets. The first case is a part of cone that concentrates CR near the line being the symmetry axis of the target. The second case is the target with hyperbolic profile that concentrates CR in a small vicinity of given point (focus). Ray optics laws and aperture integration method are used for the calculation of the field. We show that the hyperbolic concentrator working at terahertz frequencies can increase the field at least up to two orders in comparison with that on the surface of the target.

INTRODUCTION

Cherenkov radiation (CR) is a convenient tool for detection of charged particle [1] and bunch diagnostics [2]. Other promising applications of this effect are wakefield acceleration [3], terahertz radiation sources [4,5], Cherenkov luminescence tomography [6] etc. However, due to the complexity of real radiator geometry, different approximate techniques are elaborated for investigation of the excited radiation [5,7]. Here we develop recently reported approximate method for calculating Cherenkov radiation of a charge flying near a dielectric target having two main boundaries (the first interacts with a charge field and the second mainly refracts a generated radiation) [8]. This method combines the exact solution of certain "key" problem and ray optics laws. Since it is frequently important to concentrate the CR energy in certain small area of space, we focus here on cases where the radiation outside the target is convergent. In other words, we suggest target geometries that combine radiator and concentrator into a single device. Because ray optics fails near the focal points, we also involve the aperture integration technique for calculating the field.

CONICAL TARGET

CR from a conical target with cylindrical channel was investigated recently [8]. Here we discuss in more detail the situation where a conical target can concentrate CR. We deal with the problem where a point charge q moves with constant velocity $\vec{v} = \beta c \vec{e}_z$ along the axis of cylindrical channel in the target [Fig. 1(a)]. As was shown
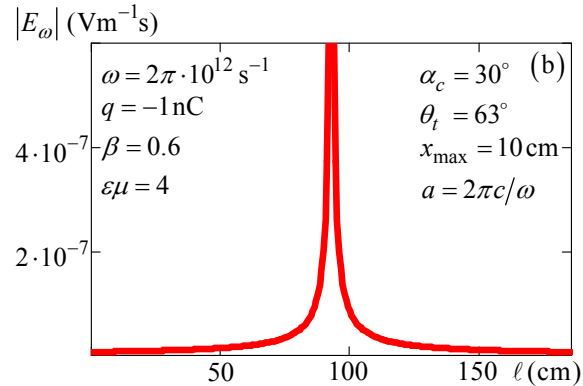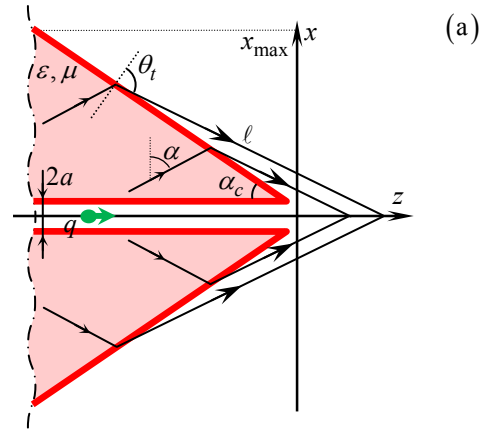
Figure 1: (a) Geometry of a cross-section of a conical target and convergent rays, $\sin\alpha = (\sqrt{\varepsilon\mu}\beta)^{-1}$. (b) Behavior of the Fourier transform of the electric field along a ray.

in [8], if the angle of cone α_c in not too large, namely

$$\tan\alpha_c < (1-\beta)/\sqrt{\varepsilon\mu\beta^2 - 1}, \qquad (1)$$

then we obtain the convergent geometry of refracted rays. As the ray optics calculation shows, the field increases along a ray [Fig. 1(b)] and possesses an infinite peak in the point where a ray intersects the symmetry axis. This means that real field (which can be calculated using, for example, the aperture integration approach [9]) increases significantly. However, as formula (1) shows, a conical target has several serious limitations for practice. First, the expected concentration is not strong because energy is spread over the axis of symmetry. Second, for ultra-relativistic particles ($\beta \to 1$) one should have extremely small angle α_c to satisfy the condition for concentration (1). These are the reasons to consider other target geometries.

HYPERBOLIC TARGET

Here we solve the problem of finding the target shape that concentrates the charge's radiation in the focus point z_f situated on the axis of symmetry of the problem [Fig. 2(a)]. First, to find the CR field inside the target, we utilize the approximate method suggested in [8]. In short, this field is supposed to be the same as in corresponding "key" problem: a point charge moves along the vacuum channel in unbounded medium with ε and μ. Exact solution for the Fourier transform of the field can be found in [10]. The interaction of this field with the target surface can be described in the frame of ray optics. Using far-field approximation for the field inside the target, the wave front and corresponding rays inside the target can be found. Refracted ray is obtained using the Snell's law and the field is multiplied by the corresponding Fresnel transmission coefficient.
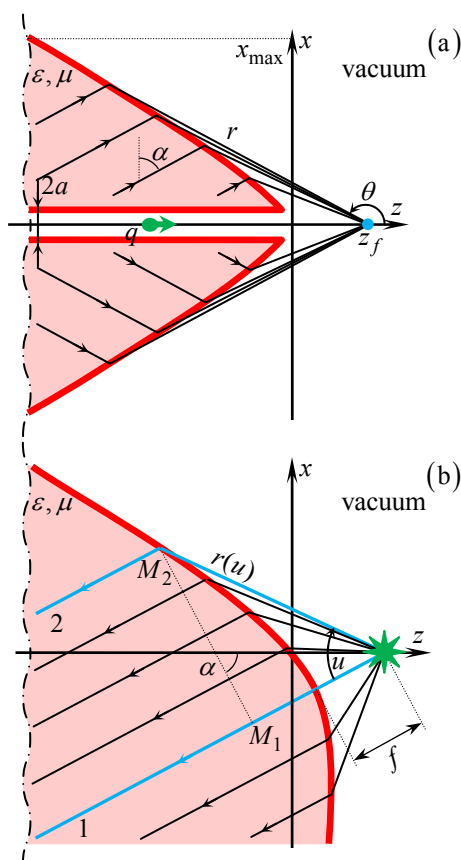

Figure 2: (a) Geometry of the problem for the target providing CR concentration in a point. (b) Problem on the lens antenna to which the initial problem is reduced.

To find the shape of the surface that provides ray convergence to the focus after transmitting the boundary, we consider this problem from another side. We expand the ray picture from the "upper" part of the target to the whole plane and turn over the rays by supposing that a point source is located in the focus point [Fig. 2(b)].

After that, we obtain the known problem of finding the surface of the lens antenna [9], and the solution is hyperbolic surface,

$$r(u) = f(1 - \sqrt{\varepsilon\mu})/[1 - \sqrt{\varepsilon\mu}\cos u], \quad (2)$$

where f is the shortest distance from the source to the surface, other abbreviations are shown in Fig. 2. From the physical point of view, the fact that rays go parallel after refraction means that the optical path difference for two arbitrary rays 1 and 2 equals zero in points M_1 and M_2. Now we cut from the whole surface the corresponding piece, rotate it over z axis and obtain the required target.

In the frame of ray optics, nonzero components of the field in the vacuum area are calculated as follows:

$$H_{\varphi\omega} = -E_{\theta\omega} = H^*_{\varphi\omega}T_\parallel\sqrt{D(0)/D(\ell)}\exp(i\omega\ell/c), \quad (3)$$

where ℓ is the ray length in vacuum, $H^*_{\varphi\omega}$ is the field in the point of the ray start u^*, $T_\parallel$ is the Fresnel transmission coefficient,

$$T_\parallel = 2\sqrt{\varepsilon\mu}\cos\theta_i[\sqrt{\varepsilon\mu}\cos\theta_i + \cos\theta_t]^{-1}, \quad (4)$$

θ_i and θ_t are angles of incidence and transmission,

$$\cos\theta_t = \frac{n\cos u - 1}{\sqrt{1 - 2n\cos u + n^2}}, \sin\theta_i = \sin\theta_t/n, \quad (5)$$

$$\sqrt{D(0)/D(\ell)} = |1 - \ell/r(u^*)|^{-1}. \quad (6)$$

The value (6) describes the convergence of the ray tube. As one can see from (3) and (6), ray optics approach gives the infinite field in the focus since $\ell = r(u^*)$ in this point. In other words, this approach is not applicable here.

To overcome this difficulty, we involve the aperture integration technique [9]. For example, the field in a point ρ, φ, $z > 0$ can be presented as an integral over aperture S_a (an area "illuminated" by refracted rays in the plane $z = 0$)

$$4\pi\vec{E}_\omega = \frac{i}{k}\iint_{S_a}\left(\left[\vec{e}_z, \vec{H}^a_\omega\right], \nabla'\right)\nabla'g\,d\Sigma' +$$
$$+ik\iint_{S_a}g\left[\vec{e}_z, \vec{H}^a_\omega\right]d\Sigma' - \iint_{S_a}\left[\left[\vec{E}^a_\omega, \vec{e}_z\right], \nabla'g\right]d\Sigma', \quad (7)$$

where $d\Sigma' = \rho'd\rho'd\varphi'$, the prime sign means that differentiation and integration are performed over primed coordinates of the point at the aperture, $g = \exp(ik\tilde{R})/\tilde{R}$,

$$\tilde{R} = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\varphi - \varphi') + (z - z')^2}, \quad (8)$$

$\vec{H}^a_\omega$ and $\vec{E}^a_\omega$ are the filed components on S_a calculated via (3). Integral (7) should be calculated numerically.

Figures 3(a) and 3(b) show the field behavior along a ray calculated using two discussed techniques. As was mentioned, ray optics (RO) shows the infinite field in the focus, while the aperture integration (AI) shows the finite one. The curves are in a good agreement (excluding narrow vicinity of the peak's center). The larger wave number $k = \omega/c$, the better agreement between curves. Expressed oscillations in the AI curve are explained by interference of fields of the equivalent elementary radiators of the aperture [9]. As one can see, height of the
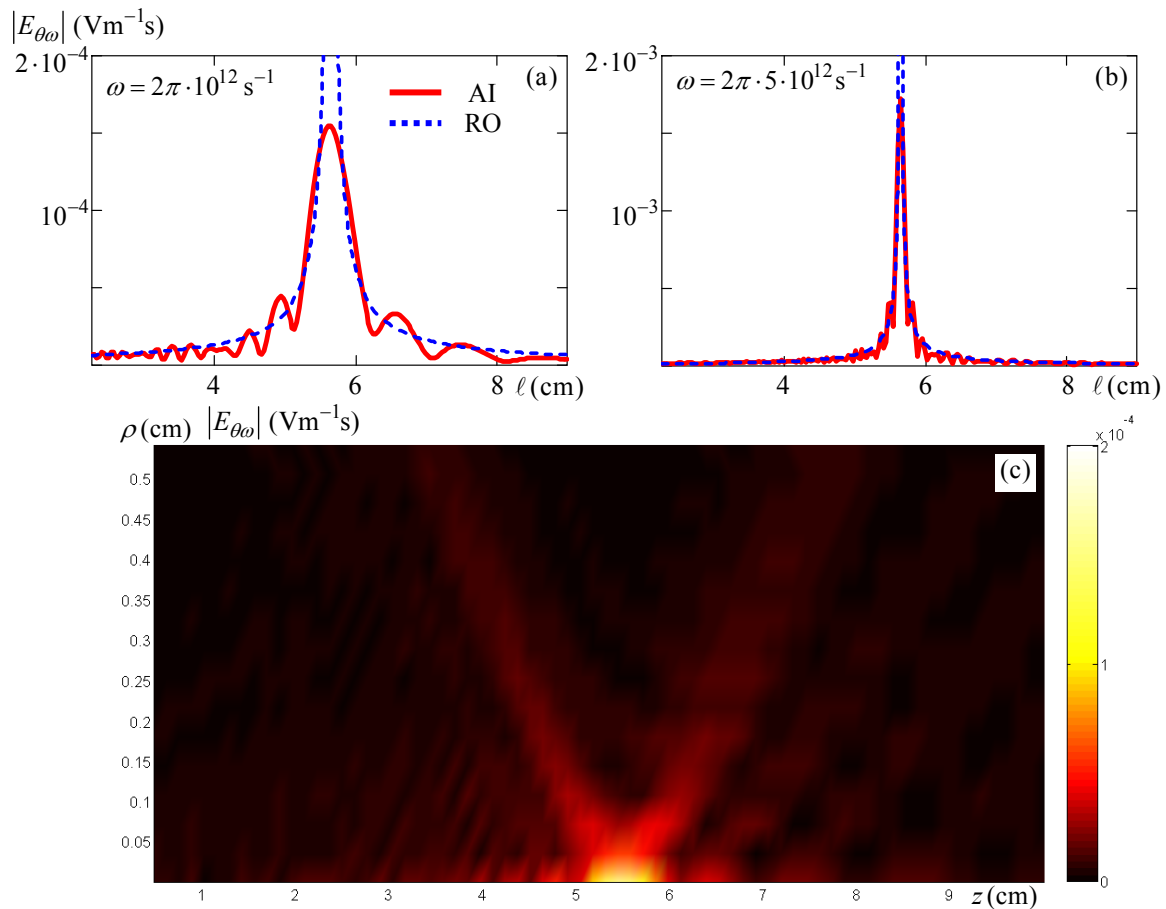
Figure 3: (a), (b) Field behavior along the ray determined by the angle $\theta = 179°$ calculated via the ray optical formulas (3) (RO) and the aperture integration approach (7) (AI). Problem parameters: $q = -1\text{nC}$, $\sqrt{\varepsilon\mu} = 1.12$, $\beta = 0.9$, $\alpha = 2\pi c/\omega$, $x_{\max} = 26\text{cm}$, $z_f = 5.5\text{cm}$, $f = 5\text{cm}$. Value $\ell = 5.6\text{cm}$ corresponds to the focus point. (c) Distribution of the absolute value of the field $|E_\theta|$ over a cross-section calculated with AI approach (7). Frequency $\omega = 2\pi \cdot 10^{12}\text{s}^{-1}$.

peak increases and the width if the peak decreases with an increase in k. Moreover, the field in the focus is around two orders larger compared with the field on the surface of the target.

Figure 3(c) shows the two-dimensional field distribution over a cross-section of the problem. As we see, for a decimeter-size target, the focal spot (where the field is larger than $10^{-4}\,\text{Vm}^{-1}\text{s}$) is around $1\,\text{cm}$ in longitudinal direction and $0.1\,\text{cm}$ in orthogonal direction. Moreover, we should expect that an increase in frequency will lead to further decrease of the focal spot dimensions.

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