

# METHODS FOR THE OPTIMIZATION OF A TAPERED FREE-ELECTRON LASER

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## Abstract

In a free-electron laser (FEL), the technique of wiggler tapering enables the sustained growth of radiation power beyond the initial saturation. With the goal to develop an X-ray FEL in the terawatt power regime, it is important to utilize this technique and optimize the taper profile, giving the wiggler parameter as a function of the distance along the wiggler line. This article examines two methods of optimization, which are based on the theoretical analysis by Kroll, Morton and Rosenbluth (KMR). Using the numerical simulation code GENESIS, the methods are applied to a case for the possible future FEL at the MAX IV Laboratory, as well as a case for the LCLS-II.

## INTRODUCTION

Wiggler tapering is a technique used for enhancing the power extraction efficiency of a free-electron laser (FEL). With appropriate variations of the wiggler parameter along the wiggler line, the energy transfer from the electron beam to the radiation can be sustained beyond the initial saturation. As has been proven experimentally by Orzechowski et al. [1], this can lead to an increase of radiation power by orders of magnitude.

To exploit the full potential of this technique, it is necessary to optimize the taper profile, thereby extracting the largest possible amount of energy from the electron beam. The knowledge on taper optimization is important for the MAX IV Laboratory [2] in the development of an X-ray FEL, which has been laid out in the long-term strategic plan of the laboratory. The plan involves the extension of the MAX IV linear accelerator to 4–6 GeV, so as to produce radiation in the ångström wavelength regime.

In this work, two methods of optimization are studied on a case for the future FEL at the MAX IV Laboratory, using the numerical simulation code GENESIS [3] in the steady-state mode. The methods are based on the theoretical analysis of variable-wiggler FELs by Kroll, Morton and Rosenbluth [4], hereafter referred to as the KMR formulation.

In a 2012 work by Jiao et al. [5], the one-dimensional KMR formulation is extended to include the physics of diffraction and optical guiding. The work also includes numerical simulations, in which various optimization schemes are applied to a case for the LCLS-II [6]. One of such optimization schemes is the GINGER self-design taper algorithm [7], which is also based on the KMR formulation. The taper profile obtained by the GINGER self-design taper algorithm will be used for comparison in this article.

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## METHODS

### Ordinary KMR Method

The KMR formulation [4] defines a reference particle (often known as the synchronous particle), and the tapered FEL can be optimized by imposing certain conditions on the reference particle. This results in a taper profile, giving the RMS wiggler parameter  $a_w \equiv e\lambda_w B_w / (2\sqrt{2}\pi m_e c)$  as a function of the distance  $z$  along the wiggler line.

In the ordinary KMR method, the reference particle is made to maintain the resonant energy  $\gamma_R(z) = \sqrt{\frac{\lambda_w}{2\lambda} [1 + a_w^2(z)]}$  and a constant ponderomotive phase  $\psi_R(z) = \psi_R(0)$  throughout the wiggler line. The particle's trajectory in the  $(\psi, \gamma)$  phase space is governed by the following equation of motion [4, 8]:

$$\frac{d\gamma_R}{dz} = -\frac{e}{\sqrt{2} m_e c^2} \frac{a_w(z) f_B(z) E_0(z)}{\gamma_R(z)} \sin[\psi_R(z)]$$

where  $E_0$  is the amplitude of the optical field and  $f_B = J_0\left(\frac{a_w^2}{2+2a_w^2}\right) - J_1\left(\frac{a_w^2}{2+2a_w^2}\right)$  is the Bessel function factor for planar wigglers. Meanwhile, particles with small energy deviations from  $\gamma_R$  perform synchrotron oscillations around the reference particle.

Using these equations, numerical simulations are carried out with GENESIS to determine the taper profile  $a_w(z)$  that satisfies the desired conditions for the reference particle.

### Modified KMR Method

The conditions imposed on the reference particle by the ordinary KMR method are arbitrary [9]. In particular, there is no reason why  $\psi_R$  has to be constant a priori. Therefore, a modified KMR method is proposed in this article. While the reference particle still maintains the resonant energy throughout the wiggler line, its ponderomotive phase increases linearly, so that  $\psi_R(z) = gz$  for some constant  $g > 0$ . The modification is motivated by the consideration of the FEL field equations.

## APPLICATION TO A MAX IV CASE

The two methods of taper optimization are applied to a case for the future MAX IV FEL. The electron beam has an energy of 4 GeV, an energy spread of 40 keV, a normalized emittance of 0.2 mm mrad and a peak current of 4 kA. The radiation wavelength is 4 Å, while the input radiation (seed) power is 0.1 MW. The wiggler period is 20 mm and the initial  $a_w$  value is 1.2.

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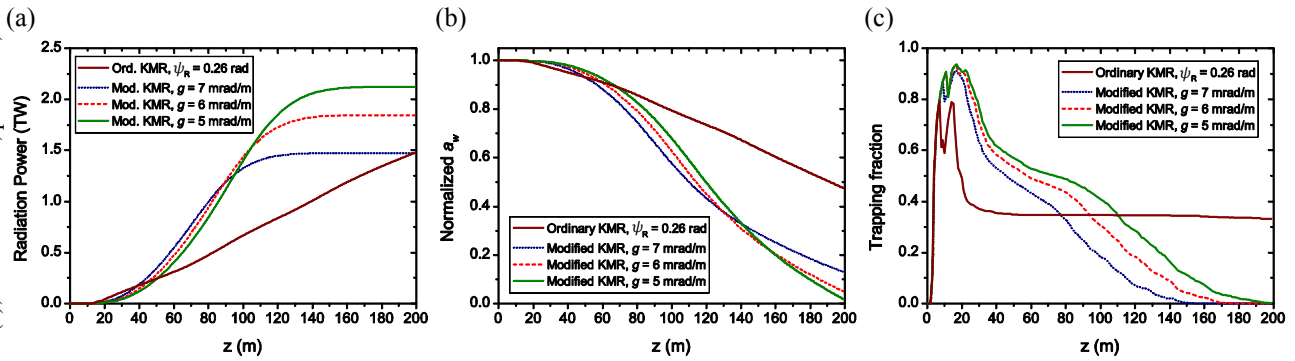


Figure 1: Application of the ordinary KMR method and the modified KMR method to the MAX IV case. (a) The radiation power as a function of distance  $z$  along the wiggler line. (b) The taper profile, with  $a_w$  normalized to the initial value. (c) The fraction of particles trapped in the ponderomotive bucket, plotted as a function of  $z$ .

To study the behaviour of the electron beam and the radiation over a long distance, a total wiggler length of 200 m is simulated. For the ordinary KMR method, simulations reveal that the radiation power is maximized by choosing  $\psi_R = 0.26$  rad. The resulting radiation power, taper profile and trapping fraction are shown in Fig. 1. For the modified KMR method, the corresponding results for three selected  $g$  values, namely 5 mrad/m, 6 mrad/m and 7 mrad/m, are shown in the same figure.

The ordinary KMR method features a slow, steady growth of radiation power throughout the wiggler line. At  $z = 200$  m, the radiation power reaches 1.5 TW.

For the modified KMR method with the three selected  $g$  values, the radiation power grows much more rapidly, and final saturation is reached long before  $z = 200$  m. With  $g = 7$  mrad/m, the radiation power reaches 2.1 TW at  $z = 200$  m, which is 1.4 times the power given by the ordinary KMR method.

In reality, it is often infeasible to construct a wiggler line as long as 200 m. If the wiggler line is shortened to 100 m, the modified KMR method will still give a power of 1.3–1.4 TW, while the ordinary KMR method will give 0.67 TW. In this case, the modified KMR method is also more preferable than the ordinary KMR method.

The modified KMR method yields a more aggressive taper profile than the ordinary KMR method does (see Fig. 1b). With the ordinary KMR method, the  $a_w$  parameter at  $z = 200$  m is about half of the initial value. With the modified KMR method, the  $a_w$  parameter decreases so rapidly that the value at  $z = 200$  m is less than 20% of the initial value.

The two methods also result in very different trapping fractions (see Fig. 1c). With the ordinary KMR method, the trapping fraction soon reaches a plateau, meaning that the number of particles in the ponderomotive bucket is almost constant. With the modified KMR method, the trapping fraction is initially higher, but decreases gradually to zero along the wiggler line.

These phenomena are also reflected in Fig. 2, which shows the  $(\psi, \gamma)$  phase space at four different positions along the wiggler line. For the ordinary KMR method, the width of the ponderomotive bucket is almost the same at

all the four  $z$ -positions. For the modified KMR method, the bucket width decreases continually and the bucket centre shifts towards larger  $\psi$ . As the bucket width decreases, some particles fall out of the bucket, leading to a reduction of the trapping fraction.

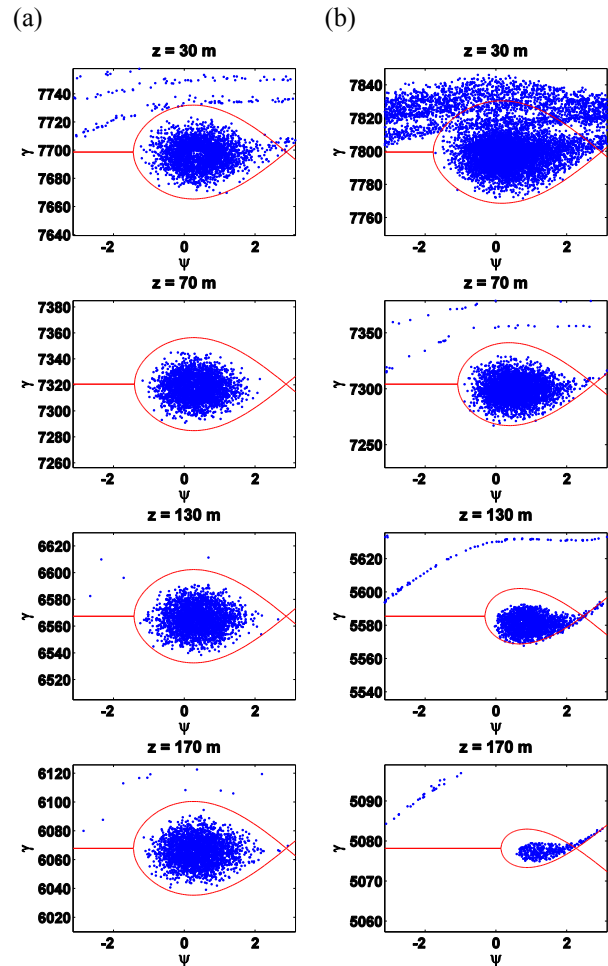


Figure 2: The  $(\psi, \gamma)$  phase space at four different positions along the wiggler line for the (a) ordinary KMR method and (b) modified KMR method. The separatrices of the ponderomotive bucket are shown in red.

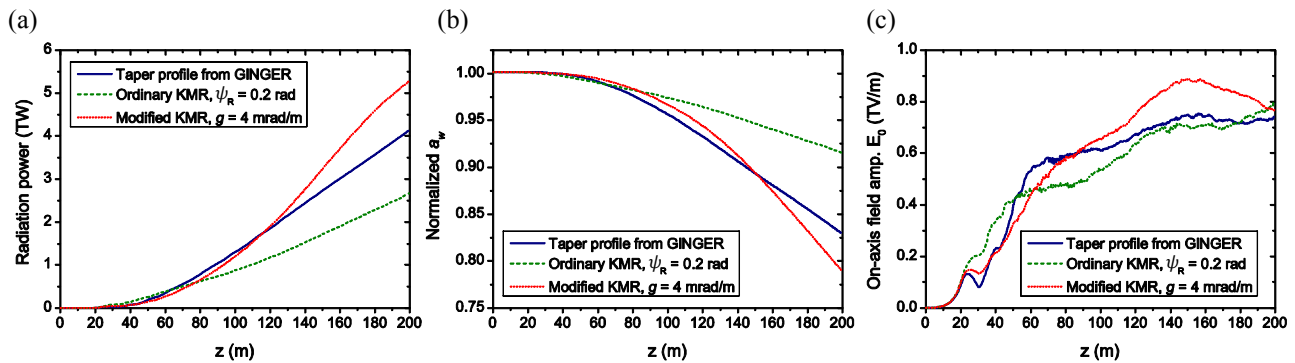


Figure 3: Application of the ordinary KMR method, the modified KMR method and the Jiao *et al.* taper profile (obtained from the GINGER self-design taper algorithm) to the LCLS-II case. (a) The radiation power as a function of distance  $z$  along the wiggler line. (b) The taper profile, with  $a_w$  normalized to the initial value. (c) The on-axis field amplitude  $E_0$  as a function of  $z$ .

## APPLICATION TO AN LCLS-II CASE

The two methods of taper optimization are also applied to a case for the LCLS-II, as defined in Ref. [5]. As before, a total wiggler length of 200 m is simulated.

For the ordinary KMR method, simulations reveal that the radiation power is maximized by choosing  $\psi_R = 0.2$  rad. For the modified KMR method, the  $g$  value 4 mrad/m is selected. The resulting radiation power, taper profile and on-axis field amplitude are shown in Fig. 3.

Reference [5] shows a taper profile obtained from the GINGER self-design taper algorithm [7], which is also based on the KMR formulation. For comparison with our two methods, this taper profile is extracted and input to GENESIS for a steady-state simulation. The results are also shown in Fig. 3.

Note that the precise configuration of the strong-focusing lattice is not provided in Ref. [5]. Therefore, an arbitrary strong-focusing lattice is used for all the three methods here, so as to achieve the same average beam radius of  $17.5 \mu\text{m}$ .

As seen in Fig. 3a, the radiation power at  $z = 200$  m is 2.7 TW for the ordinary KMR method, 4.1 TW for the GINGER self-design taper profile and 5.3 TW for the modified KMR method. Out of the three methods, the modified KMR method (with  $g = 4$  mrad/m) gives the highest radiation power.

The modified KMR method has the most aggressive taper profile among the three methods (see Fig. 3b). At  $z = 200$  m, the normalized  $a_w$  value is 0.79 for the ordinary KMR method, 0.83 for the GINGER self-design taper profile and 0.91 for the modified KMR method.

The modified KMR method also gives the largest on-axis field amplitude among the three methods from  $z = 85$  m to 195 m (see Fig. 3c).

## CONCLUSIONS

This article examines two methods for the optimization of tapered FELs, namely the ordinary KMR method and the modified KMR method. Using the numerical simulation code GENESIS, the two methods are applied

to a case for the future MAX IV FEL, as well as a case for the LCLS-II. In the latter case, a taper profile obtained by the GINGER self-design taper algorithm is extracted from Ref. [5] for comparison. In both cases, it is found that the modified KMR method produces the highest radiation power.

While the ordinary KMR method results in radiation power in the terawatt regime, the modified KMR method provides a possibility to enhance the power even further. Beyond this work, further studies are to be performed, so as to provide a better understanding of the tolerance and sensitivity of the taper optimization methods.

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