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LONGITUDINAL ACCEPTANCE EVALUATION FROM HAMILTONIAN

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Abstract

An RF cavity is designed around a reference particle; if the energy or the phase of a real particle are too far from the reference, the particle is lost. The widest area of energy-phase that allows a particle to be transported by the cavity is called acceptance of the cavity. In simulations the acceptance is evaluated tracking several particles with different energies and phases and marking when a particle is transmitted or lost. This process can be time consuming because of the large amount of tracked particles requested to characterise the cavity acceptance. In this paper we propose an alternative method to evaluate the acceptance studying directly the Hamiltonian associated to the cavity.

INTRODUCTION

The longitudinal acceptance does not have a rigorous definition in literature and is in general considered as the largest area in the phase space that is transported from the beginning to the end of a transport line. Such a definition implies that a particle outside the acceptance's area does not arrive to the end of the transport line. We start wondering what is the mechanism that stops the run of the particle. If a particle is too far from the reference particle, in terms of distance in the longitudinal phase space, its magnetic rigidity is different from the one used to design the focusing lattice of the accelerator. The particle is then over focused or defocused and hits the vacuum pipe. It is clear that many variables are involved in this process like the configuration of the lattice or the aperture of the accelerator. This makes difficult an estimation of the longitudinal acceptance based only on the parameters of the cavities, nevertheless a technique can be used and it is here presented.

LONGITUDINAL ACCEPTANCE

Hamiltonian and Separatrix

The first step is to define the acceptance for one cavity. With the definition given above the acceptance of one cavity tends to infinity: all the particles are transported by one cavity even if they are very far from the reference particle. This is due to the short length of the cavity, in the order of few meters, that produces a weak transversal force. We propose, as definition of acceptance, to use the concept of oscillation around the synchronous particle. We ideally put the cavity in a ring and we assume a small acceleration with respect to the energy of the particle. It is known from the literature [1] that the phase space can be divided in a stable region, around the reference particle and an unstable region far from the reference particle. The curve that separates the two regions is called separatrix. We define the accepted area of one cavity the area within the separatrix. This definition is not perfect because a particle outside the separatrix can be transported along the line if it is within the stable area of the cavities downstream, but we use this definition because it is dependent only by the parameters of the cavity.

To write the Hamiltonian and calculate the equation of the separatrix we should use a convenient pair of conjugate variables. The natural choice for the longitudinal phase space should be $[\Delta E, \Delta t]$ where the Δ is referred to the difference from the synchronous particle. These variables are conjugate but the time is not the simplest variable we can use to describe the dynamics in a cavity. It is preferable to use the energy-phase such as: $[\Delta E, \Delta\phi]$. If we consider only the longitudinal plane these variables are like the conjugate, but in a full 3D description a transformation of the transversal coordinates is required to preserve the volume of the phase space.

If we use the index s for the synchronous particle, it can be convenient to use the following coordinates:

$$\begin{aligned} t_\Delta &= t - t_s \quad (1)\\ \phi_\Delta &= \phi - \phi_s \quad (2)\\ E_\Delta &= E - E_s \quad (3) \end{aligned}$$

that are respectively: the difference between times, phases and energies of a particle with respect to the synchronous particle.

The Hamiltonian that describes the dynamics is [2]:

$$H = -\frac{1}{2\Omega} {E_\Delta}^2 - eV(\sin(\phi_\Delta + \phi_s) - \phi_\Delta \cos(\phi_s)). \quad (4)$$

The independent variable is here z so the associated canonical equations are:

$$\begin{aligned} \frac{dE_\Delta}{dz} &= -\frac{\partial H}{\partial \phi_\Delta} \quad (5)\\ \frac{d\phi_\Delta}{dz} &= \frac{\partial H}{\partial E_\Delta}. \quad (6) \end{aligned}$$

Ω is the amplitude of the maximum energy oscillation and is in general not easy to calculate for a linac. It depends mainly on the mass of the particle, on the frequency of the cavity and on the kinetic energy (or a power of $\beta\gamma$). For the purpose of this paper the factor Ω contains also a correction due to the relative position of the cavities: if a particles exit from a cavity with a lower energy with respect to the synchronous particle, it will arrive to the next cavity with a delay in phase due to the time of flight between the two cavities. The value of Ω was calculated for one cavity and scaled with a semi-empirical algorithm. Probably a complete

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theoretical treatment is feasible and will be investigate in a future work.

Expanding the (5) and (6) we have:

$$\frac{dE_\Delta}{dz} = eV\left[\cos(\phi_\Delta+\phi_s)-\cos(\phi_s)\right] \quad (7)$$

$$\frac{d\phi_\Delta}{dz} = -\frac{1}{\Omega}E_\Delta, \quad (8)$$

evaluating the derivative of (8) and substituting the (7) we obtain:

$$\frac{d^2\phi_\Delta}{dz^2}+\frac{eV}{\Omega}\left[\cos(\phi_\Delta+\phi_s)-\cos(\phi_s)\right]=0. \quad (9)$$

For small acceleration, when it is possible to consider Ω constant with respect to z, the integral of the equation is:

$$\frac{1}{2}\left(\frac{d\phi_\Delta}{dz}\right)^2+\frac{eV}{\Omega}\left[\sin(\phi_\Delta+\phi_s)-\phi_\Delta\cos(\phi_s)\right]=C_1. \quad (10)$$

Substituting the (8) we obtain what we can consider the equation of the separatrix for a linac:

$$\frac{{E_\Delta}^2}{2}+\Omega eV\left[\sin(\phi_\Delta+\phi_s)-\phi_\Delta\cos(\phi_s)\right]=C_2. \quad (11)$$

To evaluate the constant C_2 it is enough to consider that the singular point of the separatrix is when $\phi_\Delta=-2\phi_s$. Finally the separatrix became:

$$\frac{{E_\Delta}^2}{2}+\Omega eV\left[\sin(\phi_\Delta+\phi_s)-\phi_\Delta\cos(\phi_s)\right]= -\Omega eV\left[\sin(\phi_s)-2\phi_s\cos(\phi_s)\right]. \quad (12)$$

From One to Many Cavities

The equation (12) identify an area where the particle is stable if it passes many times on the same cavity. To extend this equation to many cavities is not straightforward because the cavities are with different parameters and the equations that provide the transformation of the phase space between many cavities are very complicated. For the purpose of this study we will do an approximation that is reasonably fair. If a particle is within the separatrix of one cavity we consider it stable. If it is outside of the separatrix, but it is within the separatrix of the following cavity, we still consider that the particle will be accepted. So extending this way to think, a particle is accepted if it moves within the largest superposition of all the separatrices. The idea is probably correct if the particle goes from cavities with small acceptance to cavities with large acceptance. In the opposite case, from larger to smaller acceptance, probably there could be an overestimation of the overall acceptance. From this point of view it is important to consider that in a real case, like in the various sections of the ESS linac shown later, the area of the separatrices have a maximum variations of 20% between the smallest and the largest. This should insure that the superposition is a good estimation.

COMPARISON WITH TRACEWIN

In order to verify the accuracy of the separatrix method, we applied it to a real case and compared with the acceptance simulated with the TraceWin code. The accelerator used for the comparison is the ESS linac as described in [3] and shown in Fig. 1.

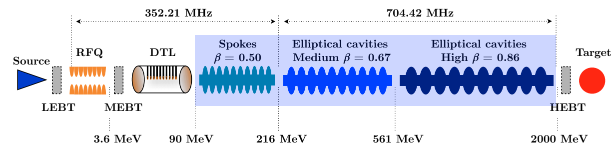

Figure 1: ESS linac layout: the sections evaluated in this paper are enclosed in the blue area.

The comparison was performed in the superconducting linac analysing separately the sections Spokes, Medium-β and High-β. For each section it was used the matched optics with the nominal beam; the acceptance was evaluate with the code TraceWin according to the procedure described in [4] on pages 19 and 20. The results of the simulations are evaluated versus the results obtained with the method of the separatrices and are presented in the following sections.

Spokes Section

The layout of the spokes section is summarised on Table 1.

Table 1: Spokes Section Parameters

Parameter	Value
Number of Cavities	26
Cells per Cavity	3
Input Energy	89.8114 MeV
Output Energy	216.593 MeV
Input ϕ_s	−28°
Output ϕ_s	−26°
Frequency	352.21 MHz
Geometrical β	0.50
Length	55.38 m

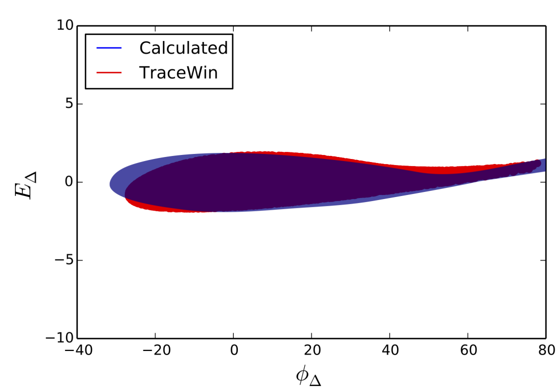

Figure 2: Acceptance for Spokes cavities section.

For the spokes case the overlapping between the acceptance calculated with the Hamiltonian and the one evaluated with the TraceWin simulation is around 90% as it is possible to see in Fig. 2. The difference of the shape in the left part

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of the figure is probably due to the approximation done in the calculation of the separatrix that neglect the acceleration for the integration in the Equation (9). The tail on the right part of the plot calculated with the separatrices is obtained overlapping all the positive tails of the separatrices. This area does not have any evident reason why it should be stable because it is outside of the stable area of the separatrix. Nevertheless it was added to see if the tail of the acceptance "club golf" shape can be related to the particles that lives around the tails of the separatrices. The overlapping of the tails seems good for the spokes cavities but not perfect for the Medium-β section and a further investigation is required.

Medium β Section

The layout of the Medium-β section is summarised on Table 2.

Table 2: Medium β Parameters

Parameter	Value
Number of Cavities	36
Cells per Cavity	6
Input Energy	216.593 MeV
Output Energy	570.492 MeV
Input ϕ_s	$-24°$
Output ϕ_s	$-15°$
Frequency	704.42 MHz
Geometrical β	0.67
Length	76.68 m

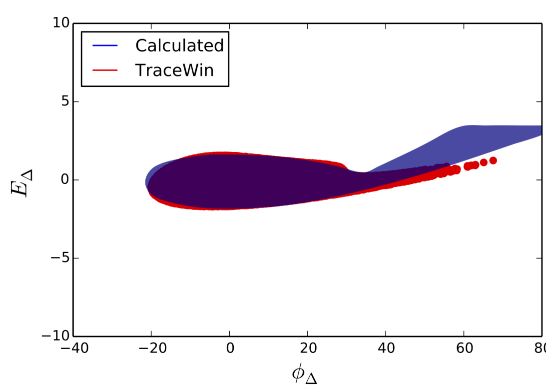

Figure 3: Acceptance for Medium β cavities section.

Here the overlapping of the core part of the acceptance is greater than 90% as shown in Fig. 3. The tail does not follow the same angle and overlap only partially.

High β Section

The layout of the High-β section is summarised on Table 3.

In the High-β section the difference between the Calculated and the simulated is greater than for the other two sections. The core part, before the singular points of the separatrices, overlaps well, but there is a large part of particles that are considered lost in the Hamiltonian evaluation that are in reality transported by the accelerator (see Fig. 4). It is possible that the shape of the separatrix, with the singular

Table 3: High β Parameters

Parameter	Value
Number of Cavities	84
Cells per Cavity	5
Input Energy	570.492 MeV
Output Energy	2000.035 MeV
Input ϕ_s	$-15°$
Output ϕ_s	$-14°$
Frequency	704.42 MHz
Geometrical β	0.86
Length	178.92 m

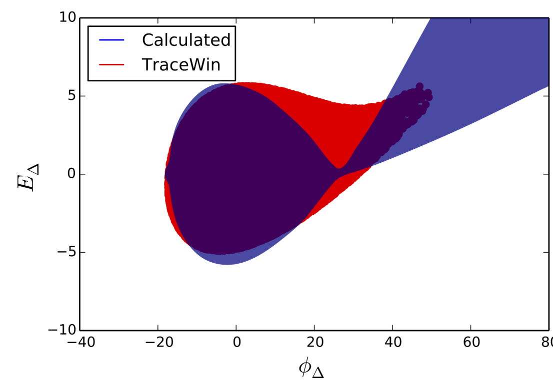

Figure 4: Acceptance for High β cavities section.

point, is too sharp in case of large acceptance, such as for the High-β section. The particles around the phase $-2\phi_s$ are probably less stable, and this is proved by the local minimum in the TraceWin acceptance, but the effect is less dramatic than predicted with the separatrices. A future work should investigate this issue in detail.

CONCLUSIONS

It was presented here a possible way to evaluate the longitudinal beam acceptance starting from the Hamiltonian of the RF cavities through an analytic calculation. The first batch of results shows a good agreement with the same acceptance calculated in the usual multi-particle simulation. In some cases, especially in the High-β section, the prediction with the Hamiltonian method is limited and requires additional studies to be better understood.

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