# STRENGTH OF HORIZONTAL INTRINSIC SPIN RESONANCES IN THE AGS* 

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## Abstract

Crossing of horizontal intrinsic resonances is today the main source of polarization losses in the AGS, in its dual partial snakes configuration for polarized proton acceleration.

Polarization losses were greatly reduced by the AGS tune jump system. However $100 \%$ polarization transmission through the AGS cycle is not yet achieved, partially due to the horizontal intrinsic resonances. This paper will explore the effect of optical distortions and different horizontal tunes on the strength of horizontal intrinsic resonances. Various options will be presented and practicability will be addressed. Theoretical model and multiparticle trackings using the Zgoubi code will show the expected polarization gains of different scenarios.

## INTRODUCTION

The AGS is used to accelerate polarized protons from 2.3 GeV to 23.8 GeV . It is critical to preserve the beam polarization but multiple depolarization resonances are crossed during the acceleration cycle. Depolarizing spin resonances occur if vertical perturbing magnetic field kicks add-up along the accelerator. These resonances are arranged in two categories depending on the resonant condition satisfied [1]:

- Imperfection depolarization spin resonances arise from vertical closed orbit distortion. These resonances occur when $Q_{s}=I$ with $I$ an integer and $Q_{s}$ the spin tune.
- Intrinsic depolarization spin resonances are caused by the vertical betatron motion. The resonant condition associated is $Q_{s} \pm Q_{z}=I$ where $Q_{z}$ is the vertical betatron tune.

The dual partial snakes configuration of the AGS minimizes polarization losses during the acceleration cycle by opening a forbidden band in the spin tune around integer values. The AGS configuration for polarized protons acceleration forbids spin tune values closer than approximately $4.510^{-2}$ to integer values. This effectively suppresses imperfection depolarization resonances and by maintaining the vertical tune in the spin tune gap, intrinsic depolarizing resonances are also avoided.

However the partial snakes configuration also tilts the stable spin direction away from the vertical direction. Vertical magnetic field kicks adding up coherently due to horizontal betatron motion can induce depolarization. When the condition $Q_{s} \pm Q_{x}=I$, with $Q_{x}$ the horizontal betatron tune, is

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satisfied depolarization occurs. This is a horizontal intrinsic resonance and is considered today as the main source of polarization losses during the acceleration of polarized protons in the AGS.

## ANALYTIC MODEL

An analytical model of the depolarization through horizontal intrinsic resonances was developed by Thomas Roser [2]. Assuming that each horizontal intrinsic resonance is narrow and far enough from any other spin resonance the polarization of a particle before $\left(P_{i}\right)$ and after $\left(P_{f}\right)$ a resonance follows the Froissart-Stora formula :

$$
\begin{equation*}
\frac{P_{f}}{P_{i}}=2 e^{-\frac{\pi|\epsilon|^{2}}{2 \alpha}}-1 \tag{1}
\end{equation*}
$$

Where the strength of the resonance is $\epsilon$. In the AGS the crossing rate $\alpha$ is typically around $\alpha=510^{-5}$ for most of the AGS acceleration cycle and can be expressed as:

$$
\begin{equation*}
\alpha=\frac{\mathrm{d}}{\mathrm{~d} \theta}\left[Q_{s} \pm Q_{x}\right] \tag{2}
\end{equation*}
$$

From equation 1 for a Gaussian beam one can get the average depolarization $<P_{f} / P_{i}>$ of the beam [3] :

$$
\begin{equation*}
<\frac{P_{f}}{P_{i}}>=\frac{1-\frac{\pi\left|\epsilon\left(I_{0}\right)\right|^{2}}{\alpha}}{1+\frac{\pi\left|\epsilon\left(I_{0}\right)\right|^{2}}{\alpha}} \tag{3}
\end{equation*}
$$

with $\epsilon\left(I_{0}\right)$ the rms resonance strength and $I_{0}$ the normalized horizontal rms emittance. Finally the depolarization after $n$ crossings of horizontal intrinsic resonances is the product of the the depolarization across each resonance :

$$
\begin{equation*}
<\frac{P_{f}}{P_{i}}>=\prod_{n} \frac{1-\frac{\pi\left|\epsilon\left(I_{0, n}\right)\right|^{2}}{\alpha_{n}}}{1+\frac{\pi\left|\epsilon\left(I_{0, n}\right)\right|^{2}}{\alpha_{n}}} \tag{4}
\end{equation*}
$$

In the AGS the two snakes have different strengths and the resonance strength can be estimated by the effect of the spin perturbing field at the stronger of the two snakes : the cold snake. The strength of a horizontal intrinsic spin resonance is given by [2] :

$$
\begin{equation*}
\epsilon\left(I_{0}\right)=\frac{G \gamma}{2 \pi} P_{h}(G \gamma) \sqrt{\gamma_{x}} \sqrt{\frac{I_{0 x}}{\gamma}} \sin \left(\pi Q_{x}\right) \tag{5}
\end{equation*}
$$

Where $\gamma_{x}$ and $P_{h}(G \gamma)$ are respectively the gamma Twiss parameter in the horizontal plane and the horizontal component of the stable spin direction in front of the cold snake, $I_{0 x}$ the normalized horizontal rms emittance.

05 Beam Dynamics and Electromagnetic Fields

Until now the depolarization due to horizontal intrinsic resonances was mitigated by increasing the resonance crossing rate $\alpha$ in equation 4 .

The AGS tune jump system considerably reduces the polarization losses due to horizontal intrinsic resonances by changing the horizontal betatron tune $Q_{x}$ [4]. However equation 5 shows that the effect of horizontal intrinsic resonances could be further reduced through the manipulation of the resonance strength itself.

The possibility of changing the Twiss parameter $\gamma_{x}$ at the strong snake will be simulated using the Zgoubi code [5] and compared to the analytic model previously introduced.

## $\gamma_{x}$ PARAMETER EFFECT



Figure 1: Proton polarization transmission in the AGS between $G \gamma=4.5$ and $G \gamma=45.5$ predicted by the analytical model as a function of the $\gamma_{x}$ Twiss parameter at the strong snake, using realistic crossing rate without tune jumps. The top axis shows the extrema values of $\gamma_{x}$ in the AGS (SS16 and SS17) and at the strong snake (SS20) in the case of the unperturbed machine.

Figure 1 shows that the minimum $\gamma_{x}$ parameter is encountered in the straight section 17 for the unperturbed lattice while the snakes are located in straight section 20 . However it is not possible to relocate any of the snakes in other locations since the straight sections 20 are the only ones long enough for these magnets. Nevertheless changing the position of the snakes is possible in simulations, both with the analytical model and using the tracking code Zgoubi.

## Comparison of Analytical Model and Tracking Results

The Zgoubi code was used to track 1000 particles picked in a 6 D Gaussian distribution cut a $2 \sigma$ leading to transverse normalized $95 \%$ emittances of $8 \pi . \mathrm{mm} . \mathrm{mrad}$ and longitudinal emittance of $1 \mathrm{eV} . \mathrm{s}^{1}$. The tracking was done in the later part of the cycle from $G \gamma=17.5$ to $G \gamma=45.5$ and with realistic acceleration rate. Figure 2 shows the evolution of the average polarization with different positions of the snakes. Note that the distance between the two snakes remains the same to preserve the spin gap. The polarization is computed by averaging the projection of the spin vectors on the stable

[^1]

Figure 2: Evolution of the average polarization with different positions of the AGS snakes as a function of the energy.
spin direction $\overrightarrow{n_{0}}$. This averaging induces artificial polarization spikes when $G \gamma$ crosses an integer, due to the flip of $\overrightarrow{n_{0}}$ and the momentum spread of the beam. The polarization computed using the analytical model uses the same optical functions, acceleration rate and horizontal beam emittance as the Zgoubi tracking.

The agreement between analytical model and tracking results is very good when the snakes are located in their usual locations (SS20) or placed at the lowest $\gamma_{x}$ parameter (SS17). However when the snakes are placed at a higher $\gamma_{x}$ parameter (SS16) some small discrepancies appear. This could be due to small differences in the beam dynamics between the simulations, leading to different bunch shape and emittance.

Possible effect of the beam dynamics can be avoided by using synthetic model of the snakes instead of the computed field maps. The estimated emittance evolves differently with the position of the snakes. However the optical effect of the snakes is very small at energies higher than $G \gamma=17.5$, one can remove the field maps and use synthetic modelisation of the snakes. Figure 3 shows the comparison between Zgoubi tracking using synthetic model of the snakes and analytical model predictions. In this case the snakes are represented by a localized rotation of the spin and have no effect on the beam dynamics. Predicted final polarization is very similar to Figure 2.

## Optics Distortion

As seen above, reducing the $\gamma_{x}$ parameter at the snakes significantly reduces the losses through horizontal intrinsic


Figure 3: Evolution of the average polarization with different positions of the AGS snakes as a function of the energy, using synthetic snakes.
resonances. However it not possible to move the snakes in the real machines since the straight sections 20 are the only large enough for these magnets.
Since moving the snakes is not possible it was proposed to use the $18^{\text {th }} \theta$ quadrapoles: a special set of high field quadrupoles corrector coils [6] wired to control the $18^{\text {th }} \theta$ harmonic of the beta function along the ring. However the maximum current allowed in these coil being only 10 A , the resulting distortion of the optics cannot significantly modify the $\gamma_{x}$ parameter at the strong snake.

Another solution involving strong quadrupoles [7] used to compensate for the optical effects of the snakes at low energy was investigated. The Zgoubi code and the Zgoubi AGS online model [8] were used together with minimizing algorithms to find an AGS optics with nominal tunes and chromaticities but lower $\gamma_{x}$ parameter at the strong snake. The minimization of the $\gamma_{x}$ parameter was only done after


Figure 4: Evolution of the $\gamma_{x}$ parameter at the strong snake as a function of the energy.
transition energy since the optics is disturbed due to the transition quadrupoles. As seen in Figure 4 the final solution does only modify moderately reduces the $\gamma_{x}$ parameter at the cold snake. Both the analytical model and the Zgoubi tracking with this distorted optics showed a negligible effect on the final average polarization.

Since Zgoubi trackings as well as analytic model did not show any improvement of the final polarization using this method and with the current hardware, no experiment were carried out in the AGS.

## TUNE EFFECT

While we have seen that the strength of horizontal intrinsic resonances can be reduced by the machine optics, equation 5 also shows a dependence on the horizontal lattice tune $Q_{x}$. The horizontal betatron tune also influences the location of the horizontal intrinsic resonances, hence the horizontal component of the stable spin direction $P_{h}$ at the resonance. Using the analytical model presented earlier, Figure 5 shows the expected polarization transmission as a function of the horizontal tune. In the case of the AGS the horizontal tune is around 8.7 for most of the cycle.

Although small changes in the horizontal tune would not influence much the polarization transmission, Figure 5 shows that pushing the horizontal tune around 8.9 would significantly reduce the polarization transmission.


Figure 5: Polarization transmission of polarized protons in the AGS between $G \gamma=4.5$ and $G \gamma=45.5$ as a function of the horizontal tune, predicted by the analytical model.

More studies are yet to be done, in particular the prediction of the analytical model were not compared with Zgoubi tracking in this case.

## CONCLUSION

We have seen that the analytical model of polarization losses through horizontal intrinsic resonances agrees with the Zgoubi tracking. This allows to quickly estimate the effect on the polarization transmission of changes in the machine optics and tunes.

The changes proposed here do not seem to be effective in the case of the acceleration of polarized protons in the AGS. Particularly because the tune jumps, which considerably reduce the depolarization through horizontal intrinsic resonances, were not included in this studies.

However these propositions might become interesting in the case of stronger horizontal intrinsic resonances, for instance with polarized ${ }^{3} \mathrm{He}$. Although this remains to be investigated in details.

## REFERENCES

[1] S.Y. Lee, Spin Dynamics and Snakes in Synchrotrons, (World Scientific Publishing, 1997).
[2] F. Lin et al., Phys. Rev. ST Accel. Beams 10, 044001 (2007).
[3] H. Huang, T. Roser, A. Luccio, "Spin Tracking Study of the AGS", AGS/RHIC Spin Note No. 043
[4] V. Schoefer et al, "Increasing the AGS Beam Polarization with 80 Tune Jumps", IPAC'12, New Orleans, LA USA, http: //www.JACoW.org
[5] F. Méot, Zgoubi code, 2014, http://sourceforge.net/ projects/zgoubi/
[6] E. Bleser AGS Tune Quads Production Measurements AGS/AD/Tech. Note No. 455, January 14, 1997.
[7] N. Tsoupas et al, "Near Integer Tune for Polarization Preservation in the AGS", IPAC' 12 , New Orleans, LA USA, http: //www.JACoW.org
[8] F. Méot et al, "Modelling of the AGS using Zgoubi-Status", IPAC'12, New Orleans, May 2014, http://www. JACoW.org


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